Admission requirements of mathematics for Bachelor Econometrics, Actuarial Science and Operations Research.

ALGEBRA

Integers, rational numbers and the real number system.

Integer exponents:
$a^n = a \times a \times \cdots \times a$ (n factors)
$a^{-n} = \dfrac{1}{a^n}$ and $a^0 = 1$
Fractional exponents: $a^{\frac{p}{q}} = \sqrt[q]{a^p}$ ($a > 0$)

Rules for exponents:
$a^n \cdot a^m = a^{n+m}$ $\qquad \dfrac{a^n}{a^m} = a^{n-m}$

$(ab)^n = a^n b^n$ $\qquad \left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$

$(a^n)^m = a^{n \cdot m}$

Illustration:
Simplify: $\dfrac{x^4 y^{-3}}{(x^2 y^{-3})^2}$, $\dfrac{y^2 \cdot \sqrt[3]{x^4}}{x^2 y^{-1} \cdot \sqrt[3]{x}}$, $\left(\dfrac{1}{3^{-2}}\right)^{-2}$

Rules of algebra:

$(a+b)(c+d) = ac + ad + bc + bd$

$a(b+c) = (b+c)a = ab + ac$

$(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

$(a-b)(a+b) = a^2 - b^2$

$(x+a)(x+b) = x^2 + (a+b)x + ab$

Illustration: expand: $(2x - 7y)^2$, $(x + \dfrac{2}{x})^2$

Remove parentheses and collect terms.
Illustration:
$(2pq - 3p^2)(p + 2q) - (q^2 - 2pq)(2p - q) = -3p^3 + q^3$
$(2t - 3)(5t^2 + 3t - 1) = 10t^3 - 9t^2 - 11t + 3$
Factoring.
Illustration:

$$8x^2y^2-16xy=8xy(xy-2)$$
$$p^2q-q^2p=pq(p-q)$$
$$x^2-6x+9=(x-3)^2$$
$$x^2+7x+10=(x+2)(x+5)$$
$$x^2-1=(x+1)(x-1)$$
$$P(1+r)+P(1+r)r=P(1+r)(1+r)=P(1+r)^2$$

Fractions:

$$\frac{a}{b}=\frac{a\times c}{b\times c}$$

$$-\frac{a}{b}=\frac{-a}{b}=\frac{a}{-b}$$

$$\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}$$

$$\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}$$

$$\frac{a}{b}\cdot\frac{c}{d}=\frac{ac}{bd}$$

$$\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}$$

Illustration:

Sum: $\frac{1}{2}+\frac{1}{3}$ without a calculator.

Simplify: $\frac{x^2+x-5}{x-7}-\frac{x^2-2}{x-7}+\frac{-4x+8}{x^2-9x+14}$ and $\frac{\frac{y^2-4}{y^2+2y}}{\frac{y-2}{y^2}}$

Simplify: $\frac{(x+1)\cdot\frac{1}{2\sqrt{x}}-\sqrt{x}}{(x+1)^2}=\frac{1-x}{2\sqrt{x}\cdot(x+1)^2}$

Simplify: $\frac{\frac{1}{x+h}-\frac{1}{x}}{h}$

Roots and radicals:

$\sqrt{12}=2\sqrt{3}$, $\frac{1}{\sqrt{2}}=\frac{1}{2}\sqrt{2}$, $\frac{2\pm\sqrt{12}}{2}=1\pm\sqrt{3}$ enz.

EQUATIONS:

Equivalent operations:
Add or subtract the same (polynomial) expression to (from) both sides.
Multiplying or dividing both sides of an equation by the same nonzero constant.

Linear equations: $ax + b = 0$

Quadratic equations: $ax^2 + bx + c = 0 \quad (a \neq 0)$

The quadratic formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Illustration:

Solve:

$$\frac{p}{3} + \frac{3}{4}p = \frac{9}{2}(p-1)$$

$$\frac{1}{3x+1} = \frac{2}{x+2}$$

$$x^2 + x - 12 = 0$$

$$(x - 1.17)^2 = 0$$

$$x^2 - 3x - 7 = 0$$

$$\frac{2}{x-2} - \frac{x+1}{x+4} = 0$$

Equations with roots:

$$q + 2 = 2\sqrt{4q - 7}$$

$$\sqrt{x+2} = x - 40$$

INEQUALITIES
Linear and quadratic inequalities. Sign diagram.
Linear fractional inequalities.
Illustration:

$$5(x-4) - 6(x+2) > 4$$

$$x^2 < 2x$$

$$\frac{2x-3}{x-1} > 3 - x$$

ABSOLUTE VALUE :

$$|3 - 2x| = 7$$

$$|4t - 1| < 1$$

INTERVALS
Open interval: $a < x < b$ and closed interval: $a \leq x \leq b$
Open/closed intervals.

EQUATIONS WITH PARAMETERS (LITERAL EQUATIONS)
Solve for P: $S = \alpha + \beta P$
Solve for z: $\dfrac{x - 2y + xz}{x - z} = 4y$

FUNCTIONS AND GRAPHS
(x,y)-axis system. Domain and range. Notation: $y = f(x) = \cdots$
The concepts of dependent and independent variables.
Combinations of functions:
Sum, difference, product and quotient of two functions.
Composition: $(f \circ g)(x) = f(g(x))$.
Linear functions. The slope. Point of intersection of two linear functions.
Quadratic functions, parabolas. The graphs of :
$y = x^2,\ y = x^3,\ y = \sqrt{x},\ \ y = \dfrac{1}{x},\ \ y = |x|$
Polynomial functions.
Exponential functions: $f(x) = a^x$ with $a > 0$ and $a \neq 1$
The number $e \approx 2,72$ and the graph: $y = e^x$

Inverse functions: $y = f(x) \ \Leftrightarrow \ x = f^{inv}(y)$

Illustration: $y = f(x) = 3x - 2 \quad \Leftrightarrow \quad x = f^{inv}(y) = \dfrac{y+2}{3}$

and: $q = \dfrac{400}{p^2} \quad \Rightarrow \quad p = \dfrac{20}{\sqrt{q}} = 20 \cdot q^{-\frac{1}{2}}$

Logarithms and logarithmic functions:

$\log_a(x) = \log_a x$: the power we have to raise a, to get x. (Some countries use the notation: ${}^a\log x$)
$\log_a x = y \Leftrightarrow x = a^y$, met $x > 0,\ a > 0$ en $a \neq 1$

Notations: $\log_{10} x = \log x$ and $\log_e x = \ln x$

Illustration: find without a calculator:
$\log_2 \frac{1}{16}$, $\log 0.001$
Formulas:
$\log(ab) = \log a + \log b$

$\log(\dfrac{a}{b}) = \log a - \log b$

$\log(a^b) = b \log a$

The natural logarithm: $\ln x = \log_e(x)$.
So for every $a > 0$ holds: $a = e^{\ln a}$, so : $a^x = e^{x \ln a}$
$\log_a x = \dfrac{\ln x}{\ln a}$

Logarithmic and exponential equations:

Solve:

$\log x + \log(x - 15) = 2$

$200 \cdot (1,06)^t = 500 \cdot (0,96)^t$

$8^x = \frac{1}{4}$

Trigonometric functions:
Angles and degree measure. Radian measure.
Sin, cos and tan in a rectangular triangle.
Sin, cos and tan in the unit-circle.
The graphs of sin x, cos x and tan x as functions of a real variable (x in rad.)

$\tan x = \dfrac{\sin x}{\cos x}$

$\sin^2 x + \cos^2 x = 1$

$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$

$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$

Amplitude, frequency and shift:
Graphs of $y = A \cdot \sin x$, $y = A \cdot \sin 2x$ and $y = A \cdot \sin 2(x - \varphi)$

DIFFERENTIAL CALCULUS

A simple treatment of limits: $\lim_{x \to a} f(x) = b$

" $f(x)$ comes arbitrary close to b if x comes sufficiently close to a but $x \neq a$."

Horizontal asymptote: $\lim_{x \to \pm\infty} f(x) = b$

Vertical asymptote: $\lim_{x \to a} f(x) = \pm\infty$

Nonvertical (oblique) asymptote: $f(x) \to ax + b$ as $x \to \pm\infty$

The derivative function: $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$f'(a)$ is the slope of the tangent at the graph of f in the point $(a, f(a))$

And: $f'(a) = \tan(\alpha)$

List of derivatives:

$f(x)$	$f'(x)$
x^a	$a \cdot x^{a-1}$
e^x	e^x
a^x	$a^x \ln a$
$\ln x$	$\dfrac{1}{x}$
$\log_g x$	$\dfrac{1}{x \ln g}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$

Rules of differentiation:

Sum, difference, product and quotient rule.
Chain rule: If $y = f(g(x))$ then $y' = f'(g(x)) \cdot g'(x)$

Or, if $g(x) = u$ then $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

Special cases:
The general power rule: $f(x) = [u(x)]^a \;\Rightarrow\; f'(x) = a \cdot [u(x)]^{a-1} \cdot u'(x)$

and for exponential and logarithmic functions:

$$f(x) = e^{u(x)} \;\Rightarrow\; f'(x) = e^{u(x)} \cdot u'(x)$$

$$f(x) = \ln u(x) \;\Rightarrow\; f'(x) = \frac{1}{u(x)} \cdot u'(x)$$

Applications: curve sketching, extreme values, tangents.
Illustration:

Given: $f(x) = \dfrac{2}{x - \sqrt{x}}$

Find the equation of the tangent at the graph of the function in the point where $x = 4$.

Given the function: $f(x) = \dfrac{x-1}{x^2 - 2x + 2}$

Find the domain of the function.
Find the extreme values of the function and say: maximum or minimum.
Also boundary extreme values.

For logarithmic and exponential functions:
With domain $x \neq 0$ is given the function: $f(x) = \dfrac{e^x}{x^2}$

Find the extreme values and classify : maximum or minimum.

The second derivative. Inflection points. Convex and concave parts of a graph (concave up, concave down).

INTEGRAL CALCULUS
Antiderivatives. The indefinite integral. List of antiderivatives:

$f(x)$	one $F(x)$
k	kx
x^a	$\frac{1}{a+1}x^{a+1} \quad (a \neq -1)$
x^{-1}	$\ln \mid x \mid \quad (x \neq 0)$
e^x	e^x
a^x	$\dfrac{a^x}{\ln a}$
a^{bx}	$\dfrac{a^{bx}}{b \ln a}$
$\sin(ax)$	$-\frac{1}{a}\cos(ax)$
$\cos(ax)$	$\frac{1}{a}\sin(ax)$

$$\int (f(x) \pm g(x))dx = \int f(x)dx \pm \int g(x)dx$$

$$\int A \cdot f(x)\,dx = A \cdot \int f(x)\,dx$$

The method of substitution:

$\int f(g(x)) \cdot g'(x)\,dx = F(g(x)) + c$ with $g(x) = u$ and $g'(x)\,dx = du$

Illustration:

$$\int x\sqrt{x^2+5}\,dx = \tfrac{1}{3}(x^2+5)^{\frac{3}{2}} + C$$

$$\int x \cdot e^{-x^2}\,dx = -\tfrac{1}{2}e^{-x^2} + C$$

$$\int \frac{x^2}{1+x^3}\,dx = \tfrac{1}{3}\ln(1+x^3) + C$$

Integration by parts: $\int u\,dv = uv - \int v\,du$

Illustration:

$\int x\,e^x\,dx = x\,e^x - x + C \qquad \int \ln x\,dx = x\ln x - x + C$

The definite integral as the limit of a sum: $\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum f(x_i) \cdot \Delta x$

Area, as the limit of a sum if $f(x) \geq 0$ on $[a,b]$

The fundamental theorem of the integral calculus: $\int_a^b f(x)\,dx = F(b) - F(a)$

Illustration:

Find the area below the graph of the function $f(x) = 1 + e^{3x}$, the x-axis and the lines $x = 0$ and $x = 2$.

Area between curves:

Find the area of the region bounded by the curves: $y^2 = x + 1$ and $x + y = -1$

Literature:
All subjects except trigonometric functions can be found in:

INTRODUCTORY MATHEMATICAL ANALYSIS
For business, economics, and the life and social sciences, 14^{th} edition
By: *Haeussler, Paul and Wood* (Pearson Education 2019)
ISBN 978 – 0 – 13 – 414110 - 7

For trigonometric functions, see the internet: e.g. Wikipedia or the Khan Academy.