Admission requirements of mathematics for Bachelor Econometrics, Actuarial Science and Operations Research.

ALGEBRA

Integers, rational numbers and the real number system.

Integer exponents: 
\[ a^n = a \times a \times \cdots \times a \ (n \text{ factors}) \]
\[ a^{-n} = \frac{1}{a^n} \quad \text{and} \quad a^0 = 1 \]

Fractional exponents: 
\[ a^{\frac{1}{n}} = \sqrt[n]{a} \quad (a > 0) \]

Rules for exponents:
\[ a^n \cdot a^m = a^{n+m} \quad \frac{a^n}{a^m} = a^{n-m} \]
\[ (ab)^n = a^n b^n \quad \left( \frac{a}{b} \right)^n = \frac{a^n}{b^n} \]
\[ (a^n)^m = a^{nm} \]

Illustration:
Simplify: 
\[ 4^{\frac{3}{2}} - 2^{\frac{3}{2}} \]

Rules of algebra:
\[ (a + b)(c + d) = ac + ad + bc + bd \]
\[ a(b + c) = (b + c)a = ab + ac \]
\[ (a + b)^2 = a^2 + 2ab + b^2 \]
\[ (a - b)^2 = a^2 - 2ab + b^2 \]
\[ (a - b)(a + b) = a^2 - b^2 \]
\[ (x + a)(x + b) = x^2 + (a + b)x + ab \]

Illustration: expand: 
\[ (2x - 7y)^2 \quad \left( x + \frac{2}{x} \right)^2 \]

Remove parentheses and collect terms.
Illustration:
\[ (2pq - 3p^2)(p + 2q) - (q^2 - 2pq)(2p - q) = -3p^3 + q^3 \]
\[ (2t - 3)(5t^2 + 3t - 1) = 10t^3 - 9t^2 - 11t + 3 \]

Factoring.
Illustration:
\[8x^2y^2 - 16xy = 8xy(xy - 2)\]
\[p^2q - q^2p = pq(p - q)\]
\[x^2 - 6x + 9 = (x - 3)^2\]
\[x^2 + 7x + 10 = (x + 2)(x + 5)\]
\[x^2 - 1 = (x + 1)(x - 1)\]
\[P(1 + r) + P(1 + r)r = P(1 + r)(1 + r) = P(1 + r)^2\]

Fractions:
\[
\frac{a}{b} = \frac{a \times c}{b \times c}
\]
\[
-\frac{a}{b} = -\frac{-a}{b} = \frac{a}{-b}
\]
\[
\frac{a + b}{c} = \frac{a + b}{c}
\]
\[
\frac{a + c}{b + d} = \frac{ad + bc}{bd}
\]
\[
\frac{a}{b} \div \frac{c}{d} = \frac{ac}{bd}
\]
\[
\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}
\]

Illustration:
Sum: \(\frac{1}{2} + \frac{1}{3}\) without a calculator.

Simplify:
\[
\frac{x^2 + x - 5}{x - 7} - \frac{x^2 - 2}{x - 7} + \frac{-4x + 8}{x^2 - 9x + 14}
\]
and
\[
\frac{y^2 - 4}{y - 2} + \frac{2y}{y^2}
\]

Simplify:
\[
\frac{(x + 1) \cdot \frac{1}{2\sqrt{x}} - \sqrt{x}}{(x + 1)^2} = \frac{1 - x}{2\sqrt{x} \cdot (x + 1)^2}
\]

Simplify:
\[
\frac{1}{x + h} \div \frac{1}{h}
\]

Roots and radicals:
\[
\sqrt{12} = 2\sqrt{3}, \quad \frac{1}{\sqrt{2}} = \frac{1}{2} \sqrt{2}, \quad \frac{2 \pm \sqrt{12}}{2} = 1 \pm \sqrt{3} \text{ enz.}
\]
EQUATIONS:

Equivalent operations:
Add or subtract the same (polynomial) expression to (from) both sides.
Multiplying or dividing both sides of an equation by the same nonzero constant.

Linear equations: \( ax + b = 0 \)
Quadratic equations: \( ax^2 + bx + c = 0 \quad (a \neq 0) \)

The quadratic formula: \( x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \)

Illustration:
Solve:
\[
\frac{p}{3} + \frac{3}{4} p = \frac{9}{2} (p - 1) \\
1 = \frac{2}{x + 2} \\
x + 2 = 0 \\
(x - 1.17)^2 = 0 \\
x^2 - 3x - 7 = 0 \\
\frac{2}{x - 2} - \frac{x + 1}{x + 4} = 0
\]

Equations with roots:
\[
q + 2 = 2 \sqrt{4q - 7} \\
\sqrt{x + 2} = x - 40
\]

INEQUALITIES
Linear and quadratic inequalities. Sign diagram.
Linear fractional inequalities.
Illustration:
\[
5(x - 4) - 6(x + 2) > 4 \\
x^2 < 2x \\
\frac{2x - 3}{x - 1} > 3 - x
\]

ABSOLUTE VALUE:
\[
|3 - 2x| = 7 \\
|4t - 1| < 1
\]

INTERVALS
Open interval: \( a < x < b \) and closed interval: \( a \leq x \leq b \)
Open/closed intervals.
EQUATIONS WITH PARAMETERS (LITERAL EQUATIONS)

Solve for $P$: $S = \alpha + \beta P$

Solve for $z$: $\frac{x - 2y + xz}{x - z} = 4y$

FUNCTIONS AND GRAPHS

$(x, y)$-axis system. Domain and range. Notation: $y = f(x) = \cdots$

The concepts of dependent and independent variables.

Combinations of functions:

Sum, difference, product and quotient of two functions.

Composition: $(f \circ g)(x) = f(g(x))$.

Linear functions. The slope. Point of intersection of two linear functions.

Quadratic functions, parabolas. The graphs of:

$y = x^2$, $y = x^3$, $y = \sqrt{x}$, $y = \frac{1}{x}$, $y = |x|$

Polynomial functions.

Exponential functions: $f(x) = a^x$ with $a > 0$ and $a \neq 1$

The number $e \approx 2.72$ and the graph: $y = e^x$

Inverse functions: $y = f(x) \iff x = f^{-1}(y)$

Illustration: $y = f(x) = 3x - 2 \iff x = f^{-1}(y) = \frac{y + 2}{3}$

and: $q = \frac{400}{p^2} \iff p = \frac{20}{\sqrt{q}} = 20 \cdot q^{-\frac{1}{2}}$

Logarithms and logarithmic functions:

$log_a(x) = \log_a x :$ the power we have to raise $a$, to get $x$. (Some countries use the notation: $^a \log x$)

$log_a x = y \iff x = a^y$, met $x > 0$, $a > 0$ en $a \neq 1$

Notations: $\log_{10} x = \log x$ and $\log_e x = \ln x$

Illustration: find without a calculator:

$log_2 \frac{1}{16}$, $\log 0.001$

Formulas:

$log(ab) = \log a + \log b$

$log\left(\frac{a}{b}\right) = \log a - \log b$

$log(a^b) = b \log a$

The natural logarithm: $\ln x = \log_e(x)$.

So for every $a > 0$ holds: $a = e^{\ln a}$, so: $a^x = e^{x \ln a}$

$log_a x = \frac{\ln x}{\ln a}$

Logarithmic and exponential equations:
Solve:
\[ \log x + \log(x - 15) = 2 \]
\[ 200 \cdot (1.06)^t = 500 \cdot (0.96)^t \]
\[ 8^x = \frac{1}{2} \]

Trigonometric functions:
Angles and degree measure. Radian measure.
Sin, cos and tan in a rectangular triangle.
Sin, cos and tan in the unit-circle.
The graphs of \( \sin x \), \( \cos x \) and \( \tan x \) as functions of a real variable \( x \) in rad.

\[ \tan x = \frac{\sin x}{\cos x} \]
\[ \sin^2 x + \cos^2 x = 1 \]
\[ \sin(x \pm y) = \sin x \cos y \pm \cos x \sin y \]
\[ \cos(x \pm y) = \cos x \cos y \mp \sin x \sin y \]

Amplitude, frequency and shift:
Graphs of \( y = A \cdot \sin x \), \( y = A \cdot \sin 2x \) and \( y = A \cdot \sin (x - \varphi) \)

**DIFFERENTIAL CALCULUS**
A simple treatment of limits: \( \lim_{x \to a} f(x) = b \)
"\( f(x) \) comes arbitrary close to \( b \) if \( x \) comes sufficiently close to \( a \) but \( x \neq a \)."
Horizontal asymptote: \( \lim_{x \to \pm \infty} f(x) = b \)
Vertical asymptote: \( \lim_{x \to a} f(x) = \pm \infty \)
Nonvertical (oblique) asymptote: \( f(x) \to ax + b \) as \( x \to \pm \infty \)

The derivative function:
\[ f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \]
\( f'(a) \) is the slope of the tangent at the graph of \( f \) in the point \( (a, f(a)) \)
And: \( f'(a) = \tan(a) \)

List of derivatives:

\[
\begin{align*}
  f(x) & \quad f'(x) \\
  x^a & \quad a \cdot x^{a-1} \\
  e^x & \quad e^x \\
  a^x & \quad a^x \ln a \\
  \ln x & \quad \frac{1}{x} \\
  \log_\varphi x & \quad \frac{1}{x \ln \varphi} \\
  \sin x & \quad \cos x \\
  \cos x & \quad -\sin x
\end{align*}
\]

Rules of differentiation:
Sum, difference, product and quotient rule.

Chain rule: If \( y = f(g(x)) \) then 
\[
y' = f'(g(x)) \cdot g'(x)
\]
Or, if \( g(x) = u \) then \[
\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}
\]

Special cases:
The general power rule: \( f(x) = [u(x)]^n \Rightarrow f'(x) = a \cdot [u(x)]^{n-1} \cdot u'(x) \)

And for exponential and logarithmic functions:
\[
f(x) = e^{u(x)} \Rightarrow f'(x) = e^{u(x)} \cdot u'(x)
\]
\[
f(x) = \ln u(x) \Rightarrow f'(x) = \frac{1}{u(x)} \cdot u'(x)
\]

Applications: curve sketching, extreme values, tangents.
Illustration:

Given: \( f(x) = \frac{2}{x - \sqrt{x}} \)
Find the equation of the tangent at the graph of the function in the point where \( x = 4 \).

Given the function: \( f(x) = \frac{x - 1}{x^2 - 2x + 2} \)
Find the domain of the function.
Find the extreme values of the function and say: maximum or minimum.
Also boundary extreme values.

For logarithmic and exponential functions:
With domain \( x \neq 0 \) is given the function: \( f(x) = \frac{e^x}{x^2} \)
Find the extreme values and classify: maximum or minimum.

The second derivative. Inflection points. Convex and concave parts of a graph (concave up, concave down).

INTEGRAL CALCULUS
Antiderivatives. The indefinite integral. List of antiderivatives:

\[
\begin{align*}
\int f(x) \, dx &= F(x) \\
k &\rightarrow kx \\
x^a &\rightarrow \frac{1}{a+1}x^{a+1} \quad (a \neq -1) \\
x^{-1} &\rightarrow \ln |x| \quad (x \neq 0) \\
e^x &\rightarrow e^x \\
a^x &\rightarrow \frac{a^x}{\ln a} \\
a^{bx} &\rightarrow \frac{a^{bx}}{b \ln a} \\
\sin(ax) &\rightarrow -\frac{1}{a} \cos(ax) \\
\cos(ax) &\rightarrow \frac{1}{a} \sin(ax)
\end{align*}
\]

\[
\int (f(x) \pm g(x)) \, dx = \int f(x) \, dx \pm \int g(x) \, dx
\]
\[ \int A \cdot f(x) \, dx = A \cdot \int f(x) \, dx \]

The method of substitution:
\[ \int f(g(x)) \cdot g'(x) \, dx = F(g(x)) + c \quad \text{with} \quad g(x) = u \quad \text{and} \quad g'(x) \, dx = du \]

Illustration:
\[ \int x\sqrt{x^2 + 5} \, dx = \frac{1}{3}(x^2 + 5)^{\frac{3}{2}} + C \]
\[ \int x \cdot e^{-x^2} \, dx = -\frac{1}{2}e^{-x^2} + C \]
\[ \int \frac{x^2}{1 + x^3} \, dx = \frac{1}{3} \ln(1 + x^3) + C \]

Integration by parts:
\[ \int u \, dv = uv - \int v \, du \]

Illustration:
\[ \int x e^x \, dx = x e^x - x + C \quad \int \ln x \, dx = x \ln x - x + C \]

The definite integral as the limit of a sum:
\[ \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i) \cdot \Delta x \]

Area, as the limit of a sum if \( f(x) \geq 0 \) on \([a, b]\)

The fundamental theorem of the integral calculus:
\[ \int_{a}^{b} f(x) \, dx = F(b) - F(a) \]

Illustration:
Find the area below the graph of the function \( f(x) = 1 + e^{3x} \), the x-axis and the lines \( x = 0 \) and \( x = 2 \).

Area between curves:
Find the area of the region bounded by the curves: \( y^2 = x + 1 \) and \( x + y = -1 \)

Literature:
All subjects except trigonometric functions can be found in:

INTRODUCTORY MATHEMATICAL ANALYSIS
For business, economics, and the life and social sciences, 14th edition
By: Haeussler, Paul and Wood (Pearson Education 2019)

For trigonometric functions, see the internet: e.g. Wikipedia or the Khan Academy.