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Voter Turnout as a Participation Game: An Experimental Investigation[1]

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1 Introduction

The paradox of voter turnout has intrigued economists for decades. The problem was first formulated by Downs (1957), when he confronted the theoretical irrationality of an individual vote with the empirical observation of massive turnout in general elections. This apparent lack of rationality in human behavior is something (mainly public choice) economists could not accept (see Schram and van Winden, 1991, for a discussion of the literature). Many theoretical and empirical papers have been published on the subject, but only in the last decade have game theoretic models been developed which show turning out to vote might be rational in an instrumental sense under specific circumstances. In a game theoretic setting, the decision to vote or abstain is seen as a case of strategic interaction between individuals. The general idea is that if everyone knows that it is rational for everyone to abstain, one might expect zero turnout, in which case it may be rational to vote.

One of the first important steps in this direction was made by Palfrey and Rosenthal (1983). In their model, the turnout decision is presented as a participation game. In this game, players in two or more teams have to make a decision: whether they should participate in an action that is beneficial to everyone in the team or not. Participation is costly but increases payoffs in one's own team and decreases payoffs in the other team, *ceteris paribus*. Payoffs (net of costs) are equal for everyone in the same team, and Palfrey and Rosenthal assume identical costs for all players. Moreover, payoffs to the teams are assumed to be symmetric and of a 'winner-takes-all' nature. The relationship between this game and the turnout decision is obvious: every supporter of a party may gain from every vote for that party. They show that in many cases equilibria with substantial turnout exist. Their result that in some cases turnout may rise as the costs of voting rise is well-known.

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This paper presents an extension of the Palfrey and Rosenthal (1983) approach to voter turnout by applying the participation game to systems with proportional representation. The major part of the paper presents an experimental analysis of participation games. Attention will be focussed on the game theoretic predictions, with emphasis on the comparative statics. A situation representing winner-takes-all elections will be compared to a proportional representation case, and the effect of group size in winner-takes-all will be investigated. We find that participation is larger in a winner-takes-all situation than in proportional representation and that the size of the groups in the game does not appear to affect participation rates. Finally our experiments show that 'strangers' (Andreoni, 1988) participate less than 'partners'.

In this paper, we focus on game theoretic models of behavior, and their implications. In the concluding discussion, a little will be said about other analyses undertaken. The organization is as follows. The following section presents the experimental design. In doing so, the participation game is described. In addition some general results will be presented. Section 3 presents a brief game theoretic analysis of the game at hand. It also confronts the theory presented with our experimental results. Finally, section 4 presents a concluding discussion.

2 Experimental Design and General Results

2.1 Turnout as a Participation Game

When a voter turns out to vote, (s)he endures costs, c. On the benefit side, the gains from casting a vote depend on the behavior of others. Here, we assume there are two groups (parties). There are n supporters of party I and m of party II. The benefits for a supporter of I can be represented in an $(n+1)x(m+1)$ matrix A. If $i(\leq n)$ votes in I and $j\ (\leq m)$ in II cast a vote, the benefits for members of I are $A\,[i,j]$. The corresponding matrix for group II is denoted by A^*. For example, if an electoral victory by I (II) yields benefits equal to 1 (0), then in a winner-takes all system where ties are broken by a random draw, $A[i,j]=1$ (0.5,0), if $i>j (i=j, i<j)$.

This set-up describes the participation game used to analyze voter turnout. In the game, a strategy for any player is either to participate or not. If (s)he participates, the number of participants in her or his group is increased by 1. Given the interpretation of $A\,[i,j]$ as the benefits from voting, A is assumed to be non-decreasing in i and non-increasing in j (cf. Palfrey and Rosenthal, 1983).

For the experimental design, we must choose n, m, c, and the matrices A and A^*. The choices are presented in the following subsection. In §3 and the appendix, these are used to derive (game-)theoretic predictions for the outcome of the participation games at hand.

2.2 Experimental Design

The (computerized) experiments were conducted in the laboratory of the Center for Research in Experimental Economics and Political Decisionmaking (CREED) at the University of Amsterdam. Subjects were recruited from the (undergraduate) student population of the University of Amsterdam. The experimental procedures are described in detail in an appendix which is available from the authors and via the ESL electronic archive at http://www.econlab.arizona.edu, here we only discuss the main features of the design. This paper reports on the results of 16 sessions with 20 periods each.

For the experiment participants were split into two teams, named yellow and blue. The size of the teams (n, m in §2.1) is one of the control variables in the design. In sessions 1–12, each group consisted of six subjects, in sessions 13 and 14 the yellow group consisted of eight and the blue group of six subjects. Finally, in sessions 15 and 16, each group had fourteen subjects. In sessions 1–4, the group composition changed randomly from one period to the next, so no two teams were identical in any two periods. In Andreoni's (1988) terms, these sessions consisted of 'strangers'. These sessions are labeled *mixed* throughout this paper. In all other sessions the groups remained constant over all 20 periods (the subjects were 'partners').

Two payoff configurations (matrices A and A^* in §2.1) were used, one representing an election in a system of proportional representation (*propres*) and one representing a winner-takes-all (*winall*) election. Table 1 summarizes the various conditions (mixed versus non-mixed; propes versus winall; group size) over the sessions.

In each of twenty periods, each participant had to decide whether to buy an imaginary disc or not. The price of a disc (c, in §2.1) was common knowledge and equal for everyone. In propres, the price of a disc was 0.7 guilders, in winall it was 1 guilder.[2] Thus, in this experiment, contributing in a participation game and turning out to vote in elections are represented by the value free decision to buy a disc.

The number of discs bought in each group determines the payoffs in A. In *propres* the payoff to any group-member is equal to the number of discs bought in one's own group divided by the total number of discs bought times *ƒ* 2.22. In *winall*, each member of the group that buys the most discs receives a payoff of

Table 1. Conditions

condition	propres 6×6-mixed	winall 6×6-mixed	propres 6×6	winall 6×6	winall 8×6	winall 14×14
sessions	1, 2	5, 6, 9, 10	3, 4	7, 8, 11, 12	13, 14	15, 16

[2] One guilder (denoted by *ƒ*) is worth approximately $0,55–$0,65.

Table 2. Parameter configuration

2a. PROPRES

Costs of a disc: 70 cents:

Payoff matrix in cents:

		number of discs in the other group						
		0	1	2	3	4	5	6
number of	0	111	0	0	0	0	0	0
discs in your	1	222	111	74	55	44	37	32
group	2	222	148	111	89	74	63	55
	3	222	167	133	111	95	83	74
	4	222	178	148	127	111	99	89
	5	222	185	159	139	123	111	101
	6	222	190	167	148	133	121	111

2b. WINALL

Costs of a disc: 100 cents:

Payoff matrix in cents:

		number of discs in the other group						
		0	1	2	3	4	56	
number	0	0 or 250	0	0	0	0	0	0
discs in your	1	250	0 or 250	250	0	0	0	0
group	2	250	250	0 or 250	0	0	0	0
	3	250	250	250	0 or 250	0	0	0
	4	250	250	250	250	0 or 250	0	0
	5	250	250	250	250	250	0 or 250	0
	6	250	250	250	250	250	250	0 or 250

f 2.50 and the payoff for the other group equals zero. In case of an equal number of discs in both groups the observer randomly draws a yellow or blue ball from an urn (with a probability of $\frac{1}{2}$ either way) to determine which group receives a payoff of *f* 2.50. Table 2 shows the format in which subjects received the information about A in the 6×6 sessions. Note that for these sessions, $A = A^*$. The matrices for the other sessions were straightforward extensions.

It should be mentioned that there was a slight difference in design between sessions 9–12 and the rest. In the former, it was made public who was in which group, though decisions remained anonymous, of course. This was done as part of a study concerning the effect of group identification on voter turnout. It turns out, that this particular manipulation has no effect. Therefore, the sessions are treated as if they were identical in the analysis in the present paper.[3]

[3] The reader may find the detailed results in Schram and Sonnemans (1996).

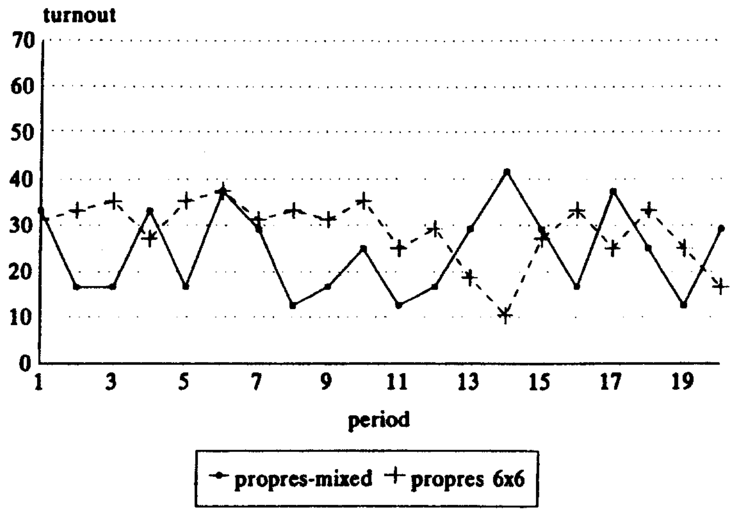

Fig. 1. Propres, mixed versus non-mixed

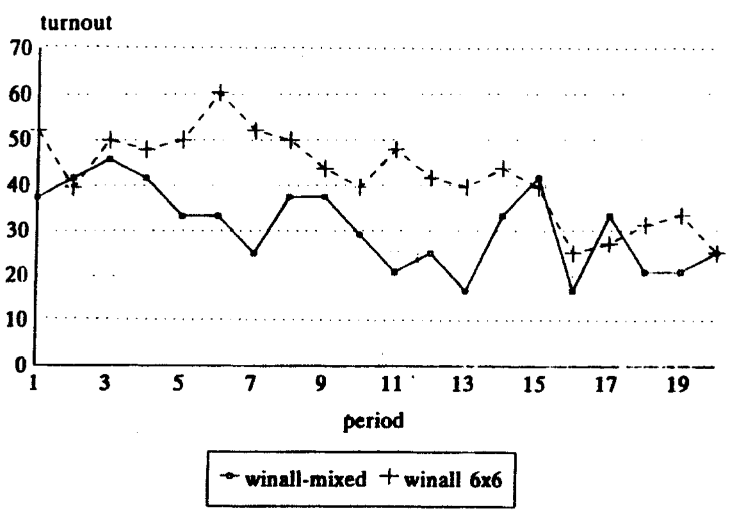

Fig. 2. Winall, mixed versus non-mixed

2.3 General Results

All of the results of all sessions may be obtained from the authors, and are available via the ESL web-site mentioned above. The aggregate results are presented in figures 1–4, which show the average 'participation-' or 'turnout rate' (number of discs bought divided by the number of participants) over the various

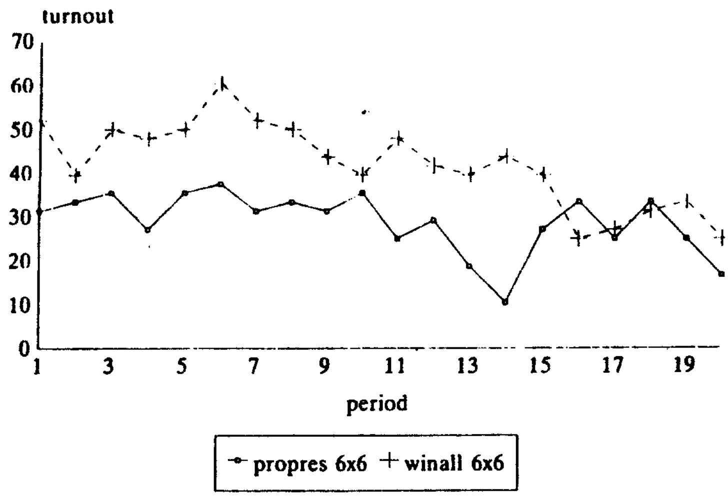

Fig. 3. Propres versus winall

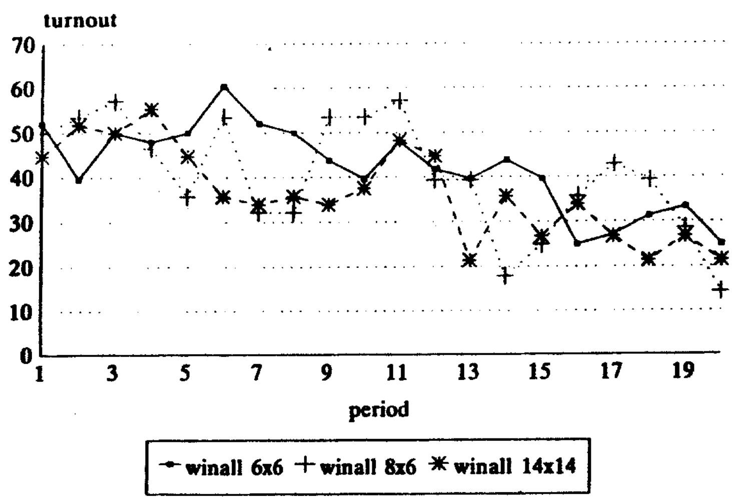

Fig. 4. Group size effects

sessions for each condition. Figure 1 compares the results for the 2 propres-mixed sessions with the 4 propres-non-mixed sessions. Figure 2 compares mixed (2 sessions) and non-mixed (2) for winall. Figure 3 compares the non mixed propres and winall sessions (4 each). Finally, figure 4 compares the winall conditions 6×6 (4 sessions) with the 8×6 (2) and 14×14 (2) sessions.

A first glance at these figures indicates that (1) turnout is lower in the *mixed* sessions than in the others (more so for winall than for propres); (2) turnout seems higher in winall than in propres, though some convergence towards the final

Table 3. Average number of discs and average earnings per individual

configuration	propres *mixed*	winall *mixed*	propres 6×6	winall 6×6	winall 8×6	winall 14×14
number of discs per individual	4.88	6.17	5.75	8.40	8.08	7.36[a]
earnings per individual[b]	18.79	18.83	18.18	16.65	16.93	17.18

[a] The average number in session 16 was corrected for the fact that only 19 periods were played due to computer problems.
[b] Net of start-off fee (cf. Appendix A).

periods is observed; (3) turnout declines towards the end of a session, especially in winall; and (4) the 8×6 and 14×14 conditions do not seen to differ much from the 6×6 condition.

A different way of summarizing the results is at an individual level. Table 3 (first row) presents the average number of discs bought per individual, in the various conditions.

Much of our statistical analysis will take place at an individual level (as opposed to a session-level). This is a consequence of the fact that most of the research questions and hypotheses we have are at the individual level. Because the observations in any one session are not completely independent (the choices of 'other' participants may vary systematically over sessions), one needs to test and correct for a 'session-effect'. No such effect could be distinguished in the data.

The net earnings in *propres*, excluding the *f* 15 subject received at the start, was between *f* 10.28 and *f* 25.57 with an average of *f* 18.38. For *winall* the subjects started off with *f* 20 and the net earnings were between *f* 0,00 and *f* 32,50 with an average of *f* 17,21. The average earnings for the various group conditions are presented in the second row of table 3. Relating the average earnings in both parameter configurations to the number of discs bought shows a negative relationship (stronger for *propres* than for *winall*). The average earnings do not decline monotonically, with the number of discs bought, however.

These general results only serve to provide a first impression of the results of the experiments. We now turn to a more systematic analysis of theories of individual behavior for the participation game and the theory will be confronted with the data observed.

3 Hypotheses on Individual Behavior

In this section, a game theoretic analysis for the experimental participation games is presented and used to develop hypotheses about individual behavior. For the parameter configurations described in table 2, we will derive various types of (Nash-)equilibrium. We will distinguish static (one shot) equilibria from dynamic

Table 4. Static game theoretic equilibria[a]

	pure strategies	mixed strategies	efficient
propres	1 1	(0.098, 0.098	0 0
winall 6×6	6 6	(0.051, 0.051) (0.949, 0.949)	0 0
winall 8×6	—	(0.037, 0.050)	0 0
winall 14×14	14 14	(0.019, 0.019) (0.981, 0.981)	0 0

[a] The numbers in the cell represent the total number of voters in each group for the Pareto optima and strategies. For mixed strategies, the equilibrium probability of voting (in both groups) is given.

equilibria for the complete 20 periods of an experiment. In addition, we will investigate what the consequences are of assuming incomplete information concerning the payoff functions of fellow players. Note that for both *propres* and *winall* the efficient outcome (defined as maximization of aggregate payoffs) is for no one to vote.[4]

3.1 Static Game Theoretic Equilibria

Assuming complete information, we have determined both pure and mixed strategy equilibria. For mixed strategies, attention is initially focused on totally quasi-symmetric equilibria (cf. Palfrey and Rosenthal, 1983), where all voters in one group are assumed to vote with the same probability (though this probability may differ between groups). A derivation of the equilibria is presented in the appendix. In the $(n, m) = (6, 6)$ and $(14, 14)$ cases, for the parameters used, all the mixed strategies are symmetric.[5] Table 4 presents the pure and mixed strategy equilibria for the parameters of our experiments.

Table 4 shows that the pure strategy individual-level equilibria are completely different for propres and winall. In propres, equilibrium turnout is very low, in winall it is 100% for groups of equal size. No pure strategy equilibrium exists for the 8×6 configuration. Note that the pure strategy equilibria for propres would seem to be very difficult to arrive at. It requires coordination as to which individual in each group should vote (there are 36 of these equilibria). The mixed strategy individual-level equilibria make low turnout possible in equilibrium in both cases. The efficient zero turnout is never an equilibrium.

[4] The various situations will be described using terminology referring to voting instead of buying discs.

[5] Thus, $(p, 1 - p)$ is not an equilibrium in groups of equal size. For example, in the winall 6×6 case $(0.051, 0.949)$ is not an equilibrium.

It is also possible to have equilibria where some subjects play a (symmetric) mixed strategy, and others vote or abstain with certainty.[6] Using the appendix, the following cases can be distinguished for this type of pure/mixed strategy equilibria. In group I (II) k_2 out of n (l_2 out of m) subjects play mixed strategy p (q), others (k_1 in one group, l_1 in the other) play a pure strategy of abstention. Finally, k_3 individuals in one group and l_3 in the other vote for sure. In the appendix it is argued that for winall, $k_3 \cdot l_1 = 0$ and $k_1 \cdot l_3 = 0$. If this condition is not met in the data, the observed behavior cannot be this type of equilibrium. When discussing the results, below, we shall check whether the observed behavior in the last ten periods of each session constitutes an equilibrium of this type.

3.2 Dynamic Aspects of the Game

If behavior is analyzed in the setting of a dynamic (repeated) game, there are two solution concepts that could be used for the present case. Which concept is relevant depends on the assumptions made concerning the individual information. In the static analysis above, complete information on other players' payoffs was assumed. In a repeated game, complete (but imperfect) information implies that a subgame perfect Nash-equilibrium is the appropriate concept to use. It can be shown that a repetition of the pure strategy-equilibria derived above for winall and propres are subgame perfect Nash equilibria for the participation games.[7]

To investigate the effects of incomplete information, it is assumed that non-monetary awards are added to the monetary payoffs of participants (people might just feel good about doing something nice for someone else). The formal incorporation of these rewards in the appendix is similar to the altruism model by Palfrey and Rosenthal (1988).[8] They rightfully stress that there are two effects following the introduction of these non-monetary rewards. First of all, a trivial consequence of a higher reward attributed to ones own contribution is that one is more likely to participate. This is not the interesting consequence, however. The second consequence worth studying, is that uncertainty concerning altruism of other participants yields different behavior (and different equilibria) then in a case of complete information. The analysis in the appendix shows that in equilibrium, voter turnout of 50% or more in winall will, in general, be followed by an increase of turnout in the next period.

[6] Actually, these equilibria are rather ad hoc, if one does not have any objective reason a priori to predict which individuals will play a pure strategy and which will play a mixed strategy. This matter is a topic of current research of the authors.

[7] It should be noted that, in theory, other subgame-perfect Nash equilibria could exist, where the outcome of a period need not be a Nash-equilibrium. We restrict the analysis to repetitions of the (same) static equilibria.

[8] Palfrey and Rosenthal (1988) apply their analysis to voluntary provision of public goods games. They derive a Bayesian Nash Equilibrium, i.e., no intertemporal behavior (including Bayesian updating) is investigated.

The game theoretic analysis will now be used to develop hypotheses about individual behavior in participation games. These hypotheses are confronted with the experimental data.

3.3 Hypotheses and Results

Experimentalists often find that game theoretic point predictions are not very accurate where actual behavior is concerned. On the other hand, comparative statics derived from game theory are often supported by experimental data. We shall have a look at both types of predictions.

The following notation is used. The variable x_{it} is a dummy equal to 1 if individual i buys a disc in period t, and 0 otherwise. Furthermore, let X denote the total number of discs bought by i: $X_i = \sum_t x_{it}$, and let n be the total number of discs bought in any one period: $n_t = \sum_i x_{it}$.

Point Predictions

We start with the equilibria presented in table 4. Because the group condition mixed (sessions 1–4) does not constitute a repeated game for fixed groups, the decisions made there will be related to these equilibria predictions straightforwardly. Though the coordination problems for the pure strategy equilibria in propres-mixed are immense, the hypothesis simply predicts that one of the equilibria will be found. If mixed strategies are assumed individual behavior over 20 periods must be in accordance with the probabilities constituting the strategy. Given that a repetition of stage-game equilibria is a subgame-perfect Nash equilibrium in case of complete information[9], the hypotheses can be extended to the other sessions. The most simple prediction from the static analysis and the dynamic analysis with complete information is therefore:

Hypothesis 1: In all cases n_t corresponds to a pure strategy equilibrium in table 4 or $X_i/20$ corresponds to a mixed strategy in table 4.

Results: To test this hypothesis, we simply counted the number of pure strategy equilibria observed. The equilibrium of every subject participating was *never* observed in any of the winall cases. In propres the equilibrium of one participant in either group was observed in 10 out of 120 possible rounds (7 of which occurred in the 40 'mixed' rounds). Using the Kolmogorov-Smirnov one-sample test we checked whether the participation probabilities corresponding to the mixed strategy equilibria in table 4 were observed. In all cases, these equilibria were rejected at a 1%-level.

[9] The analysis of incomplete information only yields a comparative static result. This is analyzed below.

Next, we checked whether the results observed in the various sessions constitute a mixed/pure strategy equilibrium. Thus, we test:

Hypothesis 2: The choices made in a session constitute a mixed/pure strategy equilibrium as defined in the appendix i.e. the conditions in eqs (4)–(9) are fulfilled.

Hypothesis 2 is more general than hypothesis 1. If only pure strategies or only mixed strategies are played ($k_2 = l_2 = 0$, $k_1 = l_1 = k_3 = i_3 = 0$), hypothesis 2 simply tests hypothesis 1.

Results: The following procedure was followed. First, to give the theory an optimal chance, only the last ten periods were considered, then the occurrence of various strategies were counted: the numbers k_1, k_3, l_1, and l_3 were determined. Next, winall sessions where $k_3 \cdot l_1$ or $k_3 \cdot l_1$ was not equal to zero were dropped (because they are not an equilibrium of this type). For all other sessions the corresponding equilibrium values of p and q (for k_2 and l_2) were determined and it was checked whether the behavior of the pure strategy players was consistent with these values (i.e., whether always voting or always abstaining was a best response). Finally, it was investigated whether the other participants behaved according to the mixed strategies p and q. This last tested was done as follows. We counted the total number of discs bought by all subjects (both teams) in the last ten periods. We then used the equilibrium probabilities to determine confidence intervals for this total number and tested the observed numbers using these intervals. It should be noted that this procedure might lead us to accept observations that should be rejected, because deviations from equilibrium in both groups might compensate each other however, in our tests, behavior according to equilibrium strategies was always rejected.

The results for the sessions 5–16 are summarized in Table 5. Sessions 1–4 are not considered because this type of equilibrium does not seem appropriate for the condition mixed. The note to the table explains the content of the columns. The conclusion from this table is that none of the observed behavior in the 12 sessions is an equilibrium of the mixed/pure type. In spite of the fact that the data were given an optimal chance (only the last ten periods were observed, and no a priori restrictions were introduced with respect to which subjects could play which type of strategy) this type of equilibrium is not supported by the data. Note that the mixed strategies observed are always higher than the probabilities in equilibrium.

Comparative Statics

We start with a hypothesis that deals with the three treatments of the experiments: winall versus propres, the effect of group size, and the effect of constant groups as opposed to mixed groups. The game theoretic predictions of table 4, especially those concerning pure strategy equilibria, will be used to formulate hypotheses.

Table 5. Results w.r.t. pure/mixed strategy equilibria[a]

Session	observed values k_1	k_2	k_3	l_1	l_2	l_3	$k_1 l_3 = l_1 k_3 = 0$? (necessary for for equi-librium in winall)	equilibrium value mixed strategies p	q	check equilibrium conditions (see appendix A) (4)	(6)	(7)	(9)	observed values mixed strategies p	q	Do players in k_2 and l_2 play the mixed equilibrium strategies?	Does observed behavior constitute an equilibrium?
5	2	4	0	1	5	0	yes	0,16	0,12	yes	x	yes	x	0.33	0.26	no	NO
6	1	5	0	1	5	0	yes	0.12	0.12	yes	x	yes	x	0.48	0.30	no	NO
7	1	5	0	1	5	0	yes	0.06	0.06	yes	x	yes	x	0.44	0.28	no	NO
8	1	4	1	1	5	0	no	—	—	—	—	—	—	0.60	0.44	—	NO
9	2	4	0	1	5	0	yes	0.16	0.12	yes	x	yes	x	0.33	0.28	no	NO
10	1	5	0	1	5	0	yes	0.12	0.12	yes	x	yes	x	0.26	0.28	no	NO
11	0	6	0	1	5	0	yes	0.05	0.06	x	x	yes	x	0.22	0.34	no	NO
12	0	6	0	2	3	1	yes	0.17	0.00	x	x	yes	no	0.38	0.30	—	NO
13	2	6	0	0	6	0	yes	0.05	0.05	yes	x	x	x	0.33	0.42	no	NO
14	1	7	0	1	5	0	yes	0.04	0.06	yes	x	yes	x	0.43	0.40	no	NO
15	3	11	0	1	12	1	no	—	—	—	—	—	—	0.54	0.35	—	NO
16	6	8	0	2	12	0	yes	0.00	0.02	no	—	—	—	0.56	0.22	—	NO

[a] The columns, respectively, present the session number; the observed values of k_i and l_j in the last ten periods; a check for the conditions $k_1 \cdot l_3 = l_1 \cdot k_3 = 0$; the equilibrium values of p and q (derived using eqs 5 and 8 in appendix A; in case of multiple equilibria, the one closest to observed behavior is presented); a test whether the equilibrium conditions in eqs. (4), (6), (7), and (9) are fulfilled; the observed values of p and q; a test whether the equilibrium strategies p and q are played (see the main text); and the conclusion whether the observed values are an equilibrium. A'—' denotes that a condition was not checked, because equilibrium was already rejected. An 'x' denotes that a condition is not relevant (because no subjects are observed for which it should hold).

The comparison of winner-takes-all and proportional representation is expected to show that turnout is higher in the former, because 100% turnout is the equilibrium there, whereas it is not an equilibrium in propres. Naturally, this is only tested for the 6 × 6 conditions, because propres was only run for this case. This yields:

Hypothesis 3a: Individual turnout X_i is larger for winall than for propres in 6 × 6.

Results: Average turnout in propres is 5.46 (out of 20 rounds), in winall it is 7.65. Using a two-tailed *t*-test, the difference is statistically significant ($p = 0.001$).Therefore, the hypothesis is supported by the data.[10]

As for group size, we can make a comparison between an increase from 6 × 6 to 14 × 14 for winall as well as a comparison between equal size groups (6 × 6) and unequal size groups (8 × 6). In case the size of both groups are increased from 6 to 14 there is only a size-effect. Isaac, Walker, and Thomas (1984), and Isaac and Walker (1988) have found that (in voluntary contribution games) free riding is not affected by group size, if the marginal return to participants is kept constant. In the present case their analysis is difficult to apply, because of the interaction with the subjects in the other team. From the equilibria in table 4, we conclude that increasing the group size equally in both groups will not affect the turnout *rate*, because always voting is an equilibrium in all cases. On the basis of the same table, the comparison between the 8 × 6 and 6 × 6 cases is expected to yield somewhat lower contributions for 8 × 6, especially where the subjects in the group of 8 are concerned. In absolute numbers, the difference to be expected using table 4 is small (mixed strategies of 0.051 and 0.037). We are not looking at point predictions however. In comparative static terms, we expect an increase in contributions of over 33% when p is raised from 0.037 to 0.051. For winall 6 × 6, we only considered the non-mixed cases.

Hypothesis 3b: Individual turnout X_i is equal in the (non mixed) winall 14 × 14 and 6 × 6 cases. It is higher in the winall 6 × 6 case than for the group of 8 in 8 × 6 case.

Result: Average turnout in the winall 6 × 6, winall 14 × 14, winall 8 × 6 (groups of 8), and winall 8 × 6 (groups of 6) is 8.40, 7.36, 7.63, and 8.67, respectively (cf. table 3). None of the differences is statistically significant. The fact that the difference between 14 × 14 (7.36) and 6 × 6 (8.40) is not statistically different from zero supports the hypothesis. On the other hand, the average contributions by members of the '8-groups' (7.63) were expected to be smaller than the 8.40 observed for 6 × 6-subjects. It is smaller, but now the lack of statistical signifi-

[10] Note from figure 3, that the difference between winall and propres declines towards the end. The average number of discs bought in the last five periods is 1.42 for winall and 1.33 for propres. This difference is not statistically significant. In this paper, we do not address the question why the decline occurs in winall. It is not simply the fact that subjects are learning about the situation. They are actually moving away from the pure strategy equilibrium.

cance is a lack of support for the hypothesis. Therefore, the hypothesis is only partly supported by these results.

Now consider a comparison between the mixed condition (cf. Andreoni's, 1988, 'strangers') and the other conditions ('partners'). Though Andreoni finds that partners free ride more than strangers, our game theoretic analysis suggests that there should be no difference between these two conditions, because the static equilibria in table 4 also constitute the stage-game outcomes of the subgame perfect equilibrium for the repeated game (cf. Appendix B).

Hypothesis 3c: Individual turnout X_i is equal in the mixed and non-mixed cases, if the difference between propres and winall is taken into account.

Results: For winall, average turnout in mixed is 6.17, for propres it is 4.88. These figures are 8.40, 5.75, respectively, for the non-mixed cases. Using a F-test the difference between strangers and partners is statistically significant, after correcting for the difference between winall and propres. Thus, the hypothesis is rejected. Note that we find the opposite result from Andreoni: strangers contribute less than partners do. An important difference between our setting and Andreoni's, is that in our case not only the group one is a member of remains constant but also a group that one is playing against. Andreoni's experiments did not have any interaction between groups. Whether this can explain our observations will be explored more deeply in future research.

A final comparative static result follows from the analysis concerning incomplete information. It is only applicable to the non-mixed cases, and only for winall. The result with respect to the development of the turnout level yields the following hypothesis:

Hypothesis 4: In winall 6×6 and 14×14, a turnout higher than or equal to 50% will, on average, be followed by a higher turnout in the next period.

The term 'on average' has been added, because an increase in turnout probability for every participant need not always yield a higher turnout.

Results: We observed 45 rounds where total turnout was higher than 50%. In 29 of these rounds (64%) turnout in the next round was lower, in 8 cases it was the same and in 8 cases the hypothesis was supported. All in all, the theory developed for incomplete information finds little support in the data.

4 Concluding Remarks

Schram and Sonnemans (1996) presents various other topics relative to the type of experiments discussed here. A central theme is the effect of group identification and communication on voter turnout. It turns out that both have a (separate) effect. For group identification, this is support for theoretical results by Morton (1991) and Schram and van Winden (1991). Finally, in that other paper, we also present an analysis of possible learning theories as explanations for the behavior of individuals in our experiments. These do not appear to explain very much, however.

We have learned various things from the analysis presented in this paper. Besides the fact that the experimental design seems to work, these mainly concern the game theoretic analysis. First of all, as expected, the comparative static results from game theory find more support in the data than the point predictions. This is a stronger result than it might seem at first sight. The processes involved in the decisionmaking are very complex, and the game theory has been kept very basic. Therefore, accurate point predictions may be too much to expect and support for comparative static results is encouraging.

A conclusion one may reach from comparing the point prediction results to the comparative static results is that the theory used may assume too much symmetry across individuals. Of course, the mixed/pure strategy equilibria are an exception to this observation, because there, not all subjects in a team are treated symmetrically. In current research, we are looking for ways in which a priori information that might help explain differences in strategies can be gathered on individual types.

One thing that we have learned is that participation in a winner-takes-all situation is larger than in a system of proportional representation. Of course, it is tempting to try to find real world data to compare this result with. These data are mixed, however. For example, turnout for presidential elections in the USA is generally between 53% and 65%. For all elections in the Netherlands, a system of proportional representation is used. In Dutch elections for the European Parliament turnout is usually less than 50%. In elections for national parliament turnout is about 80%. Finally, turnout for municipal elections is similar to turnout for presidential elections in the U.S.

We have also found that increasing groups size does not have a clear effect on participation. This may be due to the fact that both groups are increased. In equilibrium, nothing changes. A priori the effect of increasing group size on the marginal return of participation is even harder to grasp for participants than in public good experiments, because now the reactions in the other group has to be taken into account.

Finally, we have observed that participation by 'strangers' is smaller than participation by 'partners', contrary to the result that Andreoni (1988) found for VCM's. Again, the most important difference between Andreoni's setting and ours, is that in our case not only stability in one's own group is provided in partners, but also stability in the other team. The inter- and intragroup incentives are opposite. If both teams could coordinate, they might try to end up at the efficient point of no participation. If coordination is restricted to one's own team, participation might be encouraged in order to 'beat the other team'.

This brings us beyond the scope of the present paper, however. Here we have simply tried to establish the merits of the participation game as a framework for the description of individual behavior in turnout decisions. The comparative static results presented provide some support for this framework.

Appendix: Game Theoretic Analysis

Randomly labeling the groups I and II, denote the probability of an individual in I(II) voting by $p(q)$. Assume $n(m)$ members in I(II). Furthermore, denote the $(n+1, m+1)$ matrix defining the benefits from an election result for an individual in I by A (cf. Table 3, for examples). Let the $(m+1, n+1)$ matrix A^* denote the benefits for an individual in II (note that for $m = n$, symmetry demands $A = A^*$). Finally, the costs of voting are assumed equal for everyone and denoted by c.

We distinguish the following three types of strategy in equilibrium: some people in a group vote for sure, some do not vote, and the others play (the same) mixed strategy. Assume that there are k_1 individuals in I that do note vote ($p_{i1} = 0$), k_2 play a mixed strategy ($p_{i2} = p$), and k_3 vote for sure ($p_{i3} = 1$). For group II distinguish l_1 individuals with $q_{i1} = 0$, l_2 with $q_{i2} = q$, and l_3 with $q_{i3} = 1$. Of course, $k_1 + k_2 + k_3 = n$, and $l_1 + l_2 + l_3 = m$.

To start with the pure strategy equilibria ($k_2 = l_2 = 0$) for the games determined by Table 2, it can easily be seen that for *propres*, all (36) situations where one participant in each group votes consistitute a Nash equilibrium ($k_1 = l_1 = 5$, $k_3 = l_3 = 1$). In *winall* only the situation where everyone votes represent a Nash equilibrium in pure strategies in the 6×6 or 14×14 games ($k_1 = l_1 = 0$). In the 8×6 game, no Nash-equilibrium in pure strategies exists.[11]

Turning to equilibria in mixed strategies, we start with totally quasi-symmetric equilibria, where everyone in a group votes with the same probability ($k_1 = k_3 = l_1 = l_3 = 0$). For a member of I, the expected value of voting or not voting depends on the number of other individuals i in I, and the number of individuals j in II casting a vote. The probability of i and j occurring for group size n, m can be denoted by Pr_{ij}^{nm}. For notational convenience, the superscripts n and m will be dropped. Given i and j voters in I and II, the benefit from voting is.[12]

$$A[i+1, j] - c.$$

The benefit from not voting is:

$$A[i, j].$$

[11] We are assuming that risk attitudes are such, that an equal probability of earning f 1, 5 or losing f 1 is preferred over a sure profit of f 0.

[12] For notational convenience, the payoffs for zero turnout in a group are presented in the first row (column) of A, which is labeled row zero (column zero). Thus, A has $n + 1$ rows (from 0 to n) and $m + 1$ columns (from 0 to m).

Given the probabilities Pr_{ij}, this yields the expected benefit to be obtained from voting as opposed to abstaining:

$$E_{\text{vote}} - E_{\text{abstain}} = \sum_{i=0}^{n-1} \sum_{j=0}^{m} \mathrm{Pr}_{ij} \cdot (A[i+1, j] - c - A[i, j]), \tag{1}$$

where

$$\mathrm{Pr}_{ij} = \binom{n-1}{i} \cdot p^i \cdot (1-p)^{n-i-1} \cdot \binom{m}{j} \cdot q^j \cdot (1-q)^{m-j}.$$

For any matrix A, a mixed equilibrium strategy for an individual in I will occur, if:

$$E_{\text{vote}} - E_{\text{abstain}} = 0. \tag{2}$$

The set (p, q) constitutes an equilibrium if eq. (2) holds and q is an equilibrium strategy for any voter in II.[13] The latter requires:

$$\sum_{i=0}^{n} \sum_{j=0}^{m-1} \mathrm{Pr}^*_{ji} \cdot (A^*[j+1, i] - c - A^*[j, i]) = 0, \tag{3}$$

where

$$\mathrm{Pr}^*_{ji} = \binom{n}{i} \cdot p^i \cdot (1-p)^{n-i} \cdot \binom{m-1}{j} \cdot q^j \cdot (1-q)^{m-j-1}.$$

Solving eqs. (2) and (3) yields the mixed strategy equilibria for the games constituted by various values of m and n. In our cases, $(n, m) = (6, 6)$, $(8, 6)$, and $(14, 14)$. These equilibria are presented in Table 4.

Next we can look at mixed/pure equilibria where some individuals play a pure strategy and some play a mixed strategy ($k_i, l_j > 0$, $i, j = 1, 2, 3$). For each group, one can determine the condition that must hold for the assumed strategy to be an equilibrium.

For the individuals in I that don't vote in equilibrium, the expected value of abstention must exceed the expected value of voting:

$$F_1 \equiv \sum_{i=0}^{k_2} \sum_{j=0}^{l_2} \mathrm{Pr}^1_{ij} \cdot (A[k_3 + i + 1, l_3 + j] - c - A[k_3 + i, l_3 + j]) < 0, \tag{4}$$

where

$$\mathrm{Pr}^1_{ij} = \binom{k_2}{i} \cdot p^i \cdot (1-p)^{k_2-i} \cdot \binom{l_2}{j} \cdot q^j \cdot (1-q)^{l_2-j}.$$

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[13] Note that eqs. 2 and 3 are a generalization of eqs. 11 and 12 in Palfrey and Rosenthal (1983).

For the individuals in k_2 eq. (2) holds, so:

$$\sum_{i=0}^{k_2-1}\sum_{j=0}^{l_2} \mathrm{Pr}_{ij}^2 \cdot (A[k_3+i+1, l_3+j] - c - A[k_3+i, l_3+j]) = 0, \tag{5}$$

where

$$\mathrm{Pr}_{ij}^2 = \binom{k_2-1}{i} \cdot p^i \cdot (1-p)^{k_2-i-1} \cdot \binom{l_2}{j} \cdot q^j \cdot (1-q)^{l_2-j}.$$

Finally, for the group in k_3, the expected value of voting exceeds the expected value of abstaining:

$$F_2 \equiv \sum_{i=0}^{k_2}\sum_{j=0}^{l_2} \mathrm{Pr}_{ij}^3 \cdot (A[k_3+i, l_3+j] - c - A[k_3+i-1, l_3+j]) > 0, \tag{6}$$

where

$$\mathrm{Pr}_{ij}^3 = \mathrm{Pr}_{ij}^1.$$

Of course, similar conditions hold for group II:

$$F_1^* \equiv \sum_{i=0}^{k_2}\sum_{j=0}^{l_2} \mathrm{Pr}_{ji}^{*1} \cdot (A^*[l_3+j+1, k_3+i] - c - A^*[l_3+j, k_3+i]) < 0 \tag{7}$$

$$\sum_{j=0}^{l_2-1}\sum_{i=0}^{k_2} \mathrm{Pr}_{ji}^{*2} \cdot (A^*[l_3+j+1, k_3+i] - c - A^*[l_3+j, k_3+i]) = 0, \tag{8}$$

$$F_2^* \equiv \sum_{i=0}^{k_2}\sum_{j=0}^{l_2} \mathrm{Pr}_{ji}^{*3} \cdot (A^*[l_3+j, k_3+i] - c - A^*[l_3+j-1, k_3+i]) > 0 \tag{9}$$

where

$$\mathrm{Pr}_{ji}^{*1} = \mathrm{Pr}_{ji}^{*3} = \mathrm{Pr}_{ij}^1.$$

$$\mathrm{Pr}_{ji}^{*2} = \binom{k_2}{i} \cdot p^i \cdot (1-p)^{k_2-i} \cdot \binom{l_2-1}{j} \cdot q^j \cdot (1-q)^{l_2-j-1}.$$

Equations (4)–(9) can be used to test whether any observed values of $k_i, l_j, i, j = 1, 2, 3$, constitute an equilibrium. This is undertaken in §3.3. One thing can be derived from these equations straight away, however. Using the relationship that exists between A and A^*, it can be shown that for the *winall* case, $F_1 = F_2^*$, and $F_2 = F_1^*$. Therefore, for *winall* it must hold that $k_1 \cdot l_3 = l_1 \cdot k_3 = 0$. In other words, in this type of equilibrium it cannot occur that in one group some individuals play a pure strategy of abstention and at the same time some voters in the other group play a pure strategy of voting.[14]

Turning to the dynamic case with incomplete information, non-monetary rewards are incorporated in the analysis by adding a parameter d_i to eqs. (1) and (3). Given the results in Table 4, that for groups of equal size, mixed strategies were found to be equal across groups, we restrict attention to symmetry within and between groups: everyone has the same probability of participating. Thus, the analysis that follows does not hold for the 8×6 condition. An extension to asymmetric cases is straightforward, however.

For the symmetric cases, we only need to adapt (1), which now reads:

$$E_{\text{vote}} - E_{\text{abstain}} = \sum_{i=0}^{n-1} \sum_{j=0}^{m} \text{Pr}_{ij} \cdot (A[i+1,j] - A[i,j] + d_i - c). \qquad (1')$$

Note that if $d_i > c - \min\{A[i+1,j] - A[i,j]\}$, the individual will always vote, and if $d_i < c - \max\{A[i+1,j] - A[i,j]\}$, (s)he will always abstain. These are individuals whose sense of duty is so high or low that a dominant pure strategy exists. Here, we focus on intermediate cases.

A perfect Bayesian equilibrium for this game consists of strategies and beliefs (concerning the levels of altruism of other participants) that satisfy (roughly speaking):[15] (1) each player maximizes expected payoffs, given her or his beliefs and the strategies of others; (2) beliefs are updated using Bayes' rule. The latter requirement is very tricky in the game at hand. Because participants have no feedback concerning other participants' behavior at an individual level, Bayesian updating would necessarily concern some aggregate measure concerning the altruism of others. Assume that this aggregate measure determines the distribution function of $d_j, j \neq i$, as individual i believes it to be. The turnout observed then provides information concerning the value of this measure. There are now two possibilities.

First of all, i may believe that her or his altruism stems from the same distribution. In this case, i (assuming (s)he knows d_i) has more direct information concerning the altruism distribution and it is not clear how (s)he should use both types of information optimally. The second case is where i believes that her or his altruism contains no information with respect to the altruism of other participants. Because d_i is relevant for the beliefs of others, this yields a complex system of interrelated beliefs about the distribution of altruism of 'the others'.

In both cases, optimizing rational individuals will be forward looking in the sense, that they will take account of the fact that their behavior will not only affect the behavior, but also the beliefs of other players. This adds yet another complexity to the perfect Bayesian outcome. Given all these complexities, we shall not set out on a quest for the perfect Bayesian outcome. Instead, two

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14 It is noted that the only corresponding pure/mixed strategy equilibrium Palfrey and Rosenthal discuss has $k_1, k_3, l_2 > 0$, $k_2 = l_1 = l_3 = 0$.

15 For a more formal definition, see Gibbons (1991, 177–180).

simplifying assumptions will be made. First, players are not assumed to be forward looking, in the way described. A participant is assumed to maximize payoffs, given beliefs, without taking account of the possibility of affecting future beliefs. Second, it is assumed that players disregard their own altruism when updating their belief about the distribution of altruism on the basis of observed behavior. As mentioned above, a final characteristic of the analysis is that only symmetric equilibria are under investigation.

Focusing on symmetric equilibria, strategies consist of equal participation probabilities for both groups, (p^*, p^*), and beliefs concerning the distribution of altruism are shared equally by everyone. Following Palfrey and Rosenthal (1988), an equilibrium to the game can be characterized by a cutoff-point d^*, (equal for all individuals), such that individual i will vote if $d_i > d^*$, abstain if $d_i < d^*$ and be indifferent if $d_i = d^*$. This can be seen from eq. (1′) by noting that one individual with a mixed strategy and $d_i = d^*$ implies that people with lower d_i have $E_{\text{vote}} < E_{\text{abstain}}$ and people with higher d_i have $E_{\text{vote}} > E_{\text{abstain}}$. The symmetric mixed strategy equilibria (p^*, p^*) can now be derived from (1′). The equilibrium strategies will depend on d^*.

The value of d_i for other individuals represent incomplete information for the participants. It is assumed that each player has an idea as to the distribution of d_i accross individuals; this is represented by the distribution function $F_\mu(d)$. This function is known to the individual up to its mean μ. In any period of the game, individuals are assumed to have a belief concerning the value of μ. This belief represents the incomplete information aspect of the game. As the individual gathers more information about choices by other participants this belief is updated using Bayes' rule. Given our definition of (p^*, p^*) as equilibrium parameters and d^* as cutoff-points, we have:

$$p^* = 1 - F_\mu(d^*). \tag{10}$$

Note that the same distribution function F_μ is assumed for both groups. It would mean a straightforward extension to the analysis to let the function vary.

Next, we make use of the fact that at the cutoff-points d^* an individual is indifferent between voting and abstaining, i.e., the expected benefit of both is equal (eq. 2). From (1′) it can be derived:

$$d^* = c - \frac{1}{(n-1)\cdot m} \cdot \sum_{i=0}^{n-1} \sum_{j=0}^{m} \mathrm{Pr}_{ij} \cdot (A[i+1, j] - A[i, j]), \tag{11}$$

where Pr_{ij} is given as a function of p and $q(=p)$ below eq. (1).

Together, eqs. (10) and (11) constitute two equations for the unknown variables p^* and d^*, if F_μ is known. In any static game, this (Bayesian) Nash equilibrium can be solved. In a dynamic setting, one needs to address the updating of beliefs. We shall not do so formally, but restrict attention to comparitive statics that follow from this updating.

To start with, simply assume that μ is updated because of observed behavior. The effect of this change in μ on the equilibrium value of p can be derived by substituting (11) in (10) and totally differentiating:

$$\frac{dp^*}{d\mu} = \frac{-\frac{\partial F}{\partial \mu}(d^*)}{1 + f_\mu(d^*)\cdot G(p^*)}, \tag{12}$$

where

$$G(p) = \frac{1}{(n-1)m} \sum_{i=1}^{n-1} \sum_{j=1}^{m} \frac{\partial \mathrm{Pr}_{ij}}{\partial p}(p)\cdot(A[i+1,j] - A[i,j]).$$

Using the definition of $\mathrm{Pr}_{ij}(p)$, it can be derived that:

$$\frac{\partial \mathrm{Pr}_{ij}}{\partial p} = \frac{i+j-(m+n-1)p}{p(1-p)}\cdot \mathrm{Pr}_{ij}(p).$$

In (12), f_μ represents the density function of μ. Furthermore, for any d, the derivative of F to μ is negative, so the numerator is positive. Therefore the sign of the change in the equilibrium value of p following an updating of μ is determined by the sign of the denominator. Given that $f_\mu > 0$, a sufficient condition for p to move with μ is that $G(p) > 0$. It can be shown for $n = m = 6$ that $G(p)$ is a monotonically increasing function in both of our cases, and is larger than 0 for $p > 0.5$ for *winall* (the same holds for the 14×14 case of winall), but $G(p) < 0$ for *propres*.

The final step to take is to determine the sign of $d\mu$, i.e., in what direction μ is updated. We wish to do so for a general case, without specifying the distribution F. The value of μ will decline if individuals are confronted with unexpected low turnout, and will increase if turnout is unexpected high. The higher an observed turnout is in any given period, the more likely that it will be unexpectedly high, and therefore the more likely it is that μ will be adapted upwards. Moreover, from the previous paragraph, we know that a sufficiently high equilibrium value of p^* will mean that the upward movement in μ will be followed by an upward movement in p^*, at least for *winall*, where $p^* > 0.5$. This heuristic reasoning is tested in the main text (§3.3).

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