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Estimating Demand in Treasury Auctions:

A Normal Cumulative Distribution Function Approach

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Abstract

In this paper I introduce a new approach to modeling aggregate bidding functions (demand functions) submitted by bidders in share auctions, one based on (scaled) normal cumulative distribution functions (cdf). First, I provide a simple model that illustrates how normal cdf-shaped demand arises in case of normally distributed signals available to the bidders. Second, using new data from the Polish Treasury securities auctions, I show that normal cdf approach generates a better fit than approximation by logistic function utilized so far. This paper contributes also to the discussion on performance of auction mechanisms. I compute the average difference between yield on primary and secondary market (which is found to be substantially higher than in most other studies) and relate the resulting rents expropriated by primary dealers to economic variables known prior to the auction. As this is obtained via prediction of the shape of the bidding function, we obtain a simple and powerful tool for early detection of slumps in performance of a particular auction design.

Keywords: Treasury auctions, multi-unit auctions, discriminatory auctions, demand functions, normal cdf, logistic function

JEL Classification Codes: C51, C53, D44, G28

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1 Introduction

Despite the spectacular progress in our knowledge on divisible goods auctions in the last years, it appears that researchers have not been able to deliver a clear answer to some of the questions that appear crucial from a policy-maker's point of view. The most spectacular examples are the four-decades-long discussions regarding relative performance of different pricing rules and magnitude of underpricing in Treasury auctions when compared to the secondary market. Even though these inter-related issues were (for a good reason) in the centre of the discussion on Treasury auctions, the answers are anything but clear-cut; theoretical results seem to be quite sensitive to particular assumptions. It seems especially difficult to derive general form of equilibrium strategies in discriminatory share auctions (see however Back and Zender (1993), Hortaçsu (2002) and Viswanathan, Wang and Witelski (2000) for special cases).

Worse still, it can be questioned that equilibria which are almost impossible to compute, have any practical relevance. While computing optimal response to given strategies of other players might be much easier than finding (all) equilibria, these strategies are plainly *not* given. Even assuming that bidders have access to individual bids submitted in the past, which need not be the case, they typically cannot learn what the informational basis for particular behavior was. Further, intentional randomization may blur the picture. It is thus certainly not clear whether real bidders actually best-respond. Besides, general conclusion from the theoretical literature is also that in such a complex environment multiple equilibria are possible. It seems then that the most promising way to get insight into impacts of possible changes in the auction design, economic situation etc. is to analyze actual bidding functions. In this way the burden of investigation is transferred to the empiricists.

Unfortunately, the quest to measure and compare underpricing across different auction types also faced substantial difficulties. Cross-countries studies are of limited value, due to several institutional differences that may be confused with the pricing rule effect. Within-country comparisons are only possible in the rare cases when a Treasury decided to change the auction format. Even then, it cannot be taken for granted that relevant economic variables did not change over the period. Further, it can be argued that it takes time for the bidders to adjust to the new system, thus period directly following introduction of the new rule may be untypical. Worse still, change of pricing rule is likely to be endogenous – if, e.g. its performance is subject to random variation and the change of the pricing rule results after a longer period of dissatisfying results, the new system will probably outperform the old one due to regression to the mean rather than any substantial advantage (just as rain summoning works quite fine if you wait long enough and duration of droughts exhibit increasing hazard rate).

Given these obstacles, it should not be surprising that results are not univocal. To mention just a few examples, Umlauf (1993) finds underpricing of 0.018% of face value in case discriminatory auctions of Mexican Treasury securities and no significant underpricing in uniform price auctions. Nyborg and Sundaresan (1996) and Goldreich (2003) also report higher revenue for uniform pricing rule, introduction of which lowered underpricing by some 0.2 basis points in the USA, a significant improvement. Using different technique Heller and Lengwiler (1997) find qualitatively similar results for Swiss government auctions.

On the other hand Bjonnes (2001, 2002) finds opposite evidence in Norway. Also Hortaçsu (2003), using a smart counterfactual analysis based on bootstrap estimation of unobservable true valuations, concludes that discriminatory auctions yield higher revenues than uniform auctions in Turkey. Fevrier et al. (2002) come to a similar conclusion using French data. Hamao and Jagadeesh (1998) find no significant underpricing in discriminatory auctions in Japan.

In view of the above-mentioned difficulties and generally mixed results, new approach to investigation into Treasury auctions has emerged in the last ten years, which, rather than aggregate statistics, analyzes actual bid functions submitted by the buyers. In this way the researchers seek to identify determinants of actual bidding behavior and resulting proceeds from the auction. The main research route is to model the, typically S-shaped, bid functions as logistic functions. This functional form is attractive due to its flexibility and the fact that it might be obtained as an integral of a bell-shaped distribution of yield rates in individual bids. The variations of estimated parameters of the logistic function over time can be though of as random (Boukai and Landsberger 1999, Berg et al. 1999), or depending on other economic variables (Preget & Waelbroeck 2005, Özcan 2004). The latter approach seems more promising as it allows generating out-of-sample predictions of bid functions, and thus cut-off price conditional on changes in explanatory variables (Preget & Waelbroeck 2005). As shown by Özcan (2004) the logistic function approach can help us compare performance of different pricing rules. His strategy is to estimate the relationships between certain economic variables and (parameters of) bid functions under uniform and discriminatory pricing rules separately (which is possible due to a switch from one mechanism to the other that occurred in his sample of Turkish Treasury auctions) and simulate the hypothetical bids that would have been submitted under counter-factual pricing rule. He concludes that discriminatory pricing rule would have outperformed the uniform rule. Preget and Waelbroeck (2005), who only have data on discriminatory auctions, investigate potential results of hypothetical design changes within this pricing rule. They find i.a. that the Treasury should avoid running too many auctions on the same day and that reopening given line of bills generates additional costs, compared to launching a new issue. Vargas (2003) uses estimation of bidding functions in uniform price Treasury auctions in Argentina to compute the (revenue-relevant) level of risk-aversion prevalent among the bidders.

This paper continues this line of research, yet making some substantial changes in the methodology. First, I approximate the bids using normal cumulative distribution function (cdf) rather than logistic function. While logistic approach might be justified on the grounds that the two functions differ only slightly and logistic function is somewhat more handy from computational viewpoint, I argue that normal cdf is more appropriate as aggregation of individual demand functions. To illustrate the point, I sketch a model of dealer-specific bid functions that is consistent with the data and lends support to the normal cdf specification of aggregate demand functions. I also show that normal cdf model actually outperforms logistic form in that it approximates actual demand functions more closely. Finally, I am able to contribute to the discussion on performance of particular Treasury auctions mechanisms and, related, rents obtained by the primary dealers, by predicting profits basing on information available prior to the auction. Any substantial deviations from forecasted values would indicate a systemic change in the behavior of primary dealers. This can e.g. result from emergence of a collusive agreement. Likewise, behavioral results of institutional modifications (i.e. changes in the auction design) can be assessed in a handy way in terms of corresponding shifts in demand parameters. In the case of Poland, introduction of new regulations for supplementary fixed-price tenders in 2005 calls for such analysis and will be addressed elsewhere.

It is also worth noting that, to the best of my knowledge, this is one of the few papers on Treasury auctions in a former communist country (and the only one that models individual bids data). Given that features of the auction design, secondary market thickness, market power and links between primary dealers etc. differ a lot between various countries and may substantially affect auction results, in-depth analysis of data from economies with varying background is highly desirable.

The rest of the paper is organized as follows. Section 2 describes the data on Polish Treasury auctions, including evidence of substantial underpricing relative to the secondary market. Section 3 explains the methodology of fitting logistic and normal cdf curves to the aggregated demand functions and estimating their parameters. Also in that section I discuss a model of individual bidder behavior supporting the normal cdf curve approach. Section 4 covers results of the estimation procedures and investigates relationship between the bidding functions and profits obtained by primary dealers. Section 5 concludes.

2 Description of data

Current paper makes use of two data sources: the primary market data set reporting individual bids in two-year bond auctions and twenty-two week bill auctions and the secondary market data set containing yields of securities with same durations. In this section I shall briefly describe both.

Fifty-two week Treasury bills are the most important short-term government security in Poland. The Ministry of Finance (MF), represented by the Central Bank, auctions app. $250 million of those every Monday. Tenders of 2-year zero-coupon bonds are organized on monthly basis with face value of app. $750 million at every auction. Bids are required to be submitted before 11.00 am of the specified day (a Wednesday or Thursday) and results of the auctions are published within an hour. Since the beginning of the year 2003 only 12 primary dealers have right (and obligation) to submit sealed bids at the auction and resell the securities on the secondary market. Bids are formulated in terms of price per zl. 10000 (52 week bills) or zl. 1000 (2Y bonds) of face value. No deposit against the submitted bid is required. Payments follow within two days after the auction in case of Treasury bills and up to two weeks in case of 2Y bonds. The minimal bid is zl. 1 million (app. $280 thousand) and the number of bids is unlimited. The MF adopts the discriminating (multi-price) rule and noncompetitive bids are not allowed. In general the supply is known in advance. It is to MF's discretion to reduce the amount being sold in case of dissatisfying demand, but it occurs utmost rarely. In case of 2Y bonds the MF may however and frequently does, offer additional bonds on the next day, at fixed price equal to the weighted average of the accepted bids.

The primary market data set contains all individual bids (price-quantity pairs) submitted by primary dealers in auctions of 52-week Treasury bills from April 2003 to September 2004 and 2-year bonds from January 2003 to December 2004. As Treasury bills are offered weekly, this time span encloses 74 auctions. In case of 2Y bonds sold on monthly basis, there are 36 observations, 12 of which were supplementary, fixed price tenders. The total value of 2Y bonds sold within the analyzed period amounts to app. zl. 62 billion (or over $17 billion) and this of bills, to zl. 75 billion (app. $20.6 billion).

Selected statistics of the market and bidding functions of the bidders are covered in table 1.

Table 1. Selected summary statistics of the primary market data

	52W bills	**2Y bonds**
Number of competitive auctions.	74	24
Mean bid/cover ratio.	2.43	2.63
Total number of bids.	8988	3356
Bids per auction, min.	54	69
Bids per auction, min.	118.7	145.6
Bids per auction, max.	198	217
Accepted bids per auction, min.	2	2
Accepted bids per auction, mean.	60.0	62.8
Accepted bids per auction, max.	109	108
Active dealers per auction, mean.	12.3	12.2
Bids per dealer in auction, mean.	9.6	12.5

Observations that are noteworthy from a researcher's viewpoint are following. First, competition as measured by bid/cover ratio (the ratio of sum of all bids to the amount sold) is relatively low. In the analyzed period it amounted to 2.43 in case of Treasury bills and 2.63 in case of 2Y bonds. It was however significantly higher in fixed price tenders.

Regarding the individual bids, it is clear that buyers submit non-trivial demand functions; average number of bids submitted by individual dealer active in given auction was equal to 12.6 in case of 2Y bonds and 9.6 for 52W bills. This is a clear message that it is desirable to model individual demand functions as downward-sloping continuous or multi-step function rather than single-step function. This also implies making use of divisible goods auction theory rather unit-demand extensions of the standard one-item auction theory.

Potential reasons for large number of bids per bidder considered in the literature include risk aversion, adjustment to winner's curse (Gordy 1999) or collusive practice (see e.g. Back & Zender 1993). The latter, however is restricted to the case of uniform pricing rule. Further, major banks – primary dealers can be convincingly argued to display risk neutrality.[1]

Analysis of the institutional features of the Polish market, supplemented by information gathered from the talks with the traders and Finance Ministry representatives suggest yet another reason for spreading the bids. While primary dealers make purchases on their own account and when-issued market is non-existent,[2] every auction is preceded by meetings of the primary dealers with representatives of other large financial institutions. These convey information regarding the demand in the market and are thus very valuable from the dealers' viewpoint. On the other hand, the clients signalling early their willingness to buy the security are able to negotiate a price below the official post-auction ask price posted by the dealers.

The secondary market data was obtained from the Warsaw Stock Exchange. This is not entirely satisfactory given that WSE only represents a small fraction of the secondary Treasury securities market in Poland. Volume of transactions is by far greatest on the unregulated market of negotiated inter-bank transactions. Unfortunately, no data on those can be obtained. The other segment of the secondary market, Electronic Treasury Securities Market (ERSPW), while having higher average volumes of transactions than WSE, is however quite often too thin on the short end of the curve.

The secondary market data set used in this paper contains bid and ask yields of benchmark 1-year and 2-year securities posted on the day of the auction and one (working) day before the auction. Further, to capture the level of market volatility, I compute sample variance of logged

[1] This "preference" might however not be faithfully implemented by the manager. Her incentive scheme may induce risk-aversion.

[2] As in many other countries where relatively small market is likely to be short-squeezed.

daily price changes within 22 trading days (or approximately one month) preceding each auction.

2.1 Underpricing

Following the standard approach, I compute the mean spread between the weighted average yield of an accepted bid in the auction and the midpoint of the bid-ask spread of benchmark security at the end of the auction day. The numbers reported in this subsection have to be treated with caution due to data problems signaled in the previous subsection. In particular, the bid-ask spread is relatively wide (around 6 basis points on average). If average price in negotiated transactions on the inter-bank market differs systematically from the mean bid-ask spread on the WSE, the aggregate profits may be over- or under-estimated.

Underpricing measured in this way amounts to 5.41 basis points in case of 2Y bonds and 5.7 basis points in case of 52W bills. This translates into profit of 9.5 cents or 5.4 cents per 100$ respectively, numbers statistically different from zero.

These figures are substantially higher than those reported in previous studies (see e.g. Keloharju et al. 2004 for an overview). This might not be surprising given that Polish security market is still in its development phase. In particular, primary dealers system was only launched in 2002. To the extent that brokers responsible for submitting demand functions are more concerned about possible overbidding (which is immediately seen as a loss from the bank management viewpoint) than underbidding (resulting in less obvious a loss due to missed opportunity), relative underpricing might have resulted from their willingness to deal cautiously with the new system. Investigation into possible collusion as a potential reason for substantial gap between yields in primary and secondary markets seems utmost difficult.

We are also able to compute individual, dealer-specific profits, aggregating over all successful bids made by given dealer. Of particular interest is the relationship between profits and aggregate purchases, as it indicates to what extent smaller bidders are in a position to compete with the large ones. It is quite clear that large players have incentive and means to pursue more detailed research regarding possible shifts in the value of the security. Further, controlling a substantial part of the market, dealer faces relatively less uncertainty regarding the aggregate bid function. In other words, two separate entities could, in general, raise their joint profit by joining forces and thus, taking into account what previously had been an external effect. Thus, one would expect profits to grow more than proportionally with the scope of purchases.

The data does not provide substantial support for this hypothesis. Relationship between overall amount of purchases and profits in 52W bill auctions is presented in Figure 1, along with a linear approximation fitted by ordinary least squares method. As the line fits the data rather well and can be extended to (nearly) cross the origin, it clearly suggests that dealers' rents are proportional to the amount bought. This can be confirmed by running a log-linear regression showing that elasticity of profit with respect to purchase is equal to 1.06 and not significantly different from 1 ($p = 0.339$). In case of bond auctions, log-linear regression can only be run if we exclude a single outlier observation with negative profit,[3] just to find that elasticity of profit with respect to amount purchased is 1.27, significantly more than 1 ($p = 0.032$). Without dropping any observation, we can only compute the ratio of profits to purchases and regress it on the amount purchased, concluding that null hypothesis that explanatory variable is a plain constant cannot be rejected.

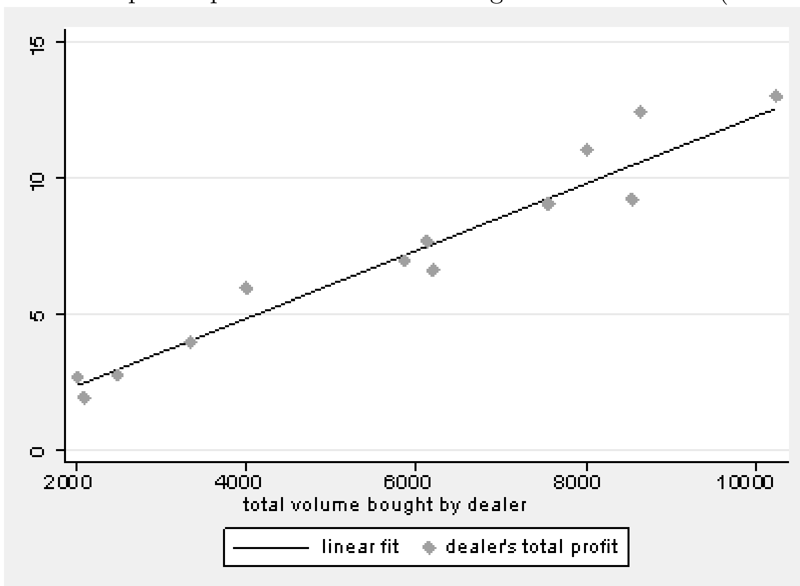
To sum up, obtained results indirectly indicate that dealers posses information of comparable precision and that they compete on roughly equal terms – extraordinary relative profits made due to sheer scope of operation are not observed in bill auctions and modest, at best, in the case of bond auctions.

3 Methodology

Estimation of economic determinants of aggregated demand functions and resulting profits proceeds in following steps. First, I fit the logistic and normal cdf functions to the aggregate demand.

[3] This is justified by the fact that this dealer's poor performance resulted predominantly from a catastrophic loss in a single auction early after the introduction of the system.

Figure 1: Dealer-specific profits and volumes bought in bill auctions (in zl. million).



Next, I regress the obtained parameters on the background economic variables that are known at the beginning of the auction. Finally, I'm able to reconstruct expected demand functions conditional on these variables and compute resulting underpricing and dealers' profits. Before we turn to the detailed description of the estimations procedures, let us first consider a simple model of individual behavior that lends support to the normal cdf formulation.

3.1 Rationale for the normal cdf model

The major theoretical advantage of the normal cdf specification is that it arises naturally under assumption of normality imposed on the distribution of signals available to the dealers. To illustrate the point with a simple example, we can assume that each bidder $i \in I$ observes a normally distributed signal s_i in the yield space[4] with identical mean μ_0 and variance σ_0^2. Further, each bidder submits a single bid of value A_i (this being independent of the signal, but possibly differently distributed for different i's) "shading"[5] the observed yield rate s_i by a fixed amount K (as in

[4] Throughout the paper I speak of bids, demand functions etc. being made in terms of yield to maturity rather than price. Given that the relation between the two is locally linear, this choice seems rather innocuous.

[5] Shading of the price corresponds to inflating the yield rates.

Goldreich 2005; linear shading would lead to identical results). Then, expected value of demand at yield rate y is equal to:

$$E(D(y)) = \Sigma_i d_i(y) = \Sigma_i A_i \Pr(s_i - K < p) = \Sigma_i A_i \Pr(s_i < p + K) = \tag{1}$$

$$= E(\Sigma_i A_i)\Phi(\frac{p + K - \mu_0}{\sigma_0}), \tag{2}$$

where $d_i(y)$ denotes individual demand submitted by bidder i and Φ is the standard normal distribution function. If number of bidders is sufficiently large, demand function can then be well described by the scaled normal cdf specification (see also next subsection)

$$D(y) = \beta\Phi(\frac{y - \mu}{\sigma}) \tag{3}$$

with

$$\begin{aligned} \beta &= E(\Sigma_i A_i), \\ \mu &= \mu_0 - K, \\ \sigma &= \sigma_0. \end{aligned} \tag{4}$$

In this simple environment estimated parameters of the aggregate demand function can be readily translated into parameters of the distribution of signals observed by the players and amounts they seek to purchase.

Similar reasoning can be followed for cases of multiple bids per participant and more complex signal-contingent behavior. In our data set, as shown previously, the assumption that bidders' information is equally precise can be sustained. The single-bid feature of the simple model sketched above, on the other hand, cannot. If we are however willing to accept the supposition that primary dealers submit multiple bids mostly basing on the demand signals received from potential contractors, a following extension of the model can account for normal cdf shape of aggregate demand function.

Each bidder $i \in I$ observes $n_i \in N$ quantity-yield pair signals $(a_{ij}, y_{ij}), j = 1, 2, ...n_i$ from potential investors; n_i being identically and independently distributed, y_{ij} following normal distribution with mean μ_0 and variance σ_0^2. Regarding a_{ij} we only assume it is positive and independent of the vector y with typical element y_{ij}. Bidder's strategy is to submit m_i bids (suppose for simplicity that $m_i = m$ is fixed), $(Y_{ik}, A_{ik}), k = 1, 2, ...m_i$ with yield rates being linear combinations of price signals observed by this bidder, possibly shifted by a constant:

$$Y_{ik} = w_{k0} + \Sigma_{j=1}^{n_i} w_{kj} p_{ij}, \tag{5}$$

where $w_{kj}, j = 0, ...n_i$ are pre-specified weights. Further we assume that amounts sought are functions of the observed quantity signals:

$$A_{ik} = f_i(a_{i1}...a_{in_i}). \tag{6}$$

The weights w_{kj} are assumed to be identical across bidders, yet may be conditional on any information publicly available prior to the auction and, obviously, depend on n_i.Functions f_i are allowed to vary also across bidders. It is very easy to see then that submitted yield rates have normal distribution with mean μ_0 and identical variance and each A_{ik} is independent of Y_{ik}. Derivation analogous to (2) leads also in this case to the conclusion that aggregate demand function should converge to the normal cdf curve.

While this formulation is quite general, it is tempting to consider a simple and intuitively appealing example. The dealer may for instance, upon observing the signals from potential investors compute the average signaled yield rate $\overline{y}_i = \frac{1}{n_i} \Sigma_{j=1}^{j-n_i} y_{ij}$ and submit bids for quantities identical to those originally signaled and yield rates being weighted averages of original signals and $\overline{y}_i$, possibly shaded by a fixed amount:

$$A_{ik} = a_{ik}, \tag{7}$$

$$Y_{ik} = w_{ik0} + w_{\overline{y}}\overline{y}_i + (1 - w_{\overline{y}})y_{ik}, \tag{8}$$

for $k = 1, 2, ...n_i$. Amount-weighted average of the terms w_{ik0} can then be interpreted as shading. The dealer may also submit an additional bid (or bids), not aimed at any particular investor, e.g. $A_{i0} = z\Sigma_{j=1}^{n_i} a_{ik}$, $Y_{i0} = w_{k0} + \overline{y}_i$ where z denotes a positive constant. It is very easy to check that this strategy is a special case of the model given by Equations (5) and (6).

Note that this model implies that also dealer-specific individual demand curves may be approximated by normal cdf specification. Indeed, running the estimation procedure described in the following subsection for each bidder in each auction separately,[6] I find on average much better fit of logistic and normal functions than of a linear-quadratic function (which also has three parameters). This analysis is problematic of course, given relatively small number of observations (bids per dealer). Even when this number is greater than 10 however, I observe a remarkably good fit. This lends support to the simple model described above.

3.2 Fitting aggregated demand functions

To compare the two approaches in fitting the demand function, namely the logistic function approach and the normal cdf approach, I first estimate the three-parameter logistic function given by the formula:

$$D(y) = \frac{\alpha}{1 + \exp(-\frac{y-\tau}{\lambda})}, \tag{9}$$

where $D(y)$ stands for demand (aggregate bid) at given yield[7] of the security y.

Interpretation of the parameters is following:

- α is the maximal demand, that is, asymptotic demand for increasing y,

[6] Due to scarcity of place and the fact that this analysis does not have direct practical importance, I do not reproduce the detailed results here. These are available from the author upon request.

[7] Modeling relationship between demand and price is also customary. See footnote 4 .

- λ is a scale parameter that captures dispersion of the bids,
- τ determines the point of inflection of the logistic curve, thus corresponds to the general yield level (and resulting price level).

To make sure that the iterated non-linear least squares estimation procedure converges to the global minimum, it is essential to start with appropriate initial values of parameters. To this end I first estimate the linearized version of the equation given above:

$$\log(\frac{D(y)}{\alpha - D(y)}) = \frac{1}{\lambda}y - \frac{\tau}{\lambda}. \tag{10}$$

To transform this relationship into a estimable linear model, one has to set some value for α. In view of the interpretation of this parameter as theoretical maximal demand, it is natural to set α equal to a number somewhat higher than the actual maximal demand (or demand for lowest price) in a given auction. While this is admittedly an arbitrary decision, I follow Preget & Waelbroeck (2005) in this respect and set α equal to exactly 1.01 of the maximal demand. It is then possible to estimate the linearized equation and obtain initial values for τ and λ:

$$\tau = -\frac{a}{b}, \lambda = \frac{1}{b}, \tag{11}$$

where a denotes the estimate of intercept in Eq. (10), while b is the estimated slope.

With these initial values I perform the non-linear least squares estimation of Eq. (9) to obtain the parameters and goodness-of-fit statistics of the logistic approximation.

Next, a scaled normal cdf,

$$D(y) = \beta\Phi(\frac{p - \mu}{\sigma}), \tag{12}$$

is fitted. Given that interpretation of β is identical to α, I set identical initial value for this parameter. Also μ, which indicates point of inflection, is set equal to τ obtained from regression model (10). As for σ, one has to take into account the relationship between variance of logistic and standard normal distributions. If random variable X follows logistic distribution with cdf $F_X(x) = \frac{1}{1+e^x}$, then

$$VAR(X) = \frac{1}{3}\pi^2. \tag{13}$$

Thus I set initial value of σ as:

$$\sigma = (\frac{1}{3})^{0.5}\pi\lambda \tag{14}$$

and analogously estimate parameters of the normal cdf specification by means of iterated non-linear least squares method. Choice of this procedure stems on one hand from the fact that normal and logistic curves are so similar; on the other hand, there is no useful linearization of the normal model. I shall hasten to say, however, that this somewhat mechanical application of estimation procedure most suitable for the logistic model to the normal model may lead, if any, to worsened fitting of the latter.

3.3 Explaining parameters of demand functions

To be able to relate the shape of aggregated demand functions to the underlying economic conditions and subsequently make predictions, I estimate a model by means of a Seemingly Unrelated Regression (SUR). This choice of estimation method is justified by the fact that within-period error terms affecting value of particular parameters of the aggregated demand functions are likely to be correlated (as these parameters are jointly determined by strategic decisions of players).

Formally I assume the following model:

$$\mathbf{y}_i = \mathbf{X}_i\beta_i + \mathbf{u}_i, E(\mathbf{u}_i\mathbf{u}_i^T) = \sigma_{ii}I_{n,} \tag{15}$$

where:

$\mathbf{y}_i$ stands for n-vector of observations on the i^{th} dependant variable (parameter of logistic or normal cumulative distribution function) and

$\mathbf{X}_i$ is a n x k_i matrix of explanatory variables.

The assumption of no autocorrelation can be defended on the grounds that the time lag between subsequent auctions is relatively long. As mentioned before however, we allow for correlation of error terms across equations within set time period:

$$E(u_{ti}u_{tj}) = \sigma_{ij} \text{ for all } t, E(u_{ti}u_{tj}) = 0 \text{ for all } t \neq s. \tag{16}$$

The Σ matrix with typical element σ_{ij} is referred to as contemporaneous covariance matrix.

We further assume weak exogeneity

$$E(U_t|X_t) = 0. \tag{17}$$

Under these assumptions performing separate regressions for each of the explained variables yields consistent but inefficient estimates. Two important exceptions from the latter result are when the contemporaneous covariance Σ matrix is diagonal (but the diagonal elements σ_{ii} need not be identical) and when each of the X_i matrices of explanatory variables for variable $\mathbf{y}_i$ are identical. In both cases OLS can be shown to be numerically identical to SUR (see e.g. Davidson & MacKinnon (2004), pp. 508-509). It is noteworthy that the latter is precisely the case e.g. in Preget & Waelbroeck (2005) and Özcan (2004), thus rendering use of the SUR technique unnecessary. In the current paper, employing different sets of explanatory variables to each of the equations seems desirable in view of small number of observations and, as discussed in the next section, justified on grounds of economic reasoning.

As Σ matrix is in general unknown, thus SUR model must be estimated by means of Feasible Generalized Least Squares (FGLS). Alternative approach would be to make use of Maximum Likelihood estimation, based on the assumption of normality of error terms. In the FGLS one first obtains consistent estimators of $\mathbf{u}_i$ performing OLS and then runs GLS with the weighting matrix

$$\widehat{\Sigma} \equiv \frac{1}{n}\widehat{U}^T\widehat{U}, \tag{18}$$

where $\widehat{U}$ denotes an n x g matrix with i^{th} column $\widehat{u_i}$. The FGLS estimator is then:

$$\widehat{\beta}_{\bullet}^{F} = (X_{\bullet}^T(\widehat{\Sigma}^{-1} \otimes I_n)X_{\bullet})^{-1}X_{\bullet}^T(\widehat{\Sigma}^{-1} \otimes I_n)y_{\bullet}, \tag{19}$$

where $\otimes$ denotes Kronecker product and the dot (as in $X_\bullet$) indicates stacking matrices:

$y_\bullet$ is a gn-vector obtained from stacking y_1 through y_g,

$X_\bullet$ is a gn x k, $k = \Sigma_{i=1}^{g} k_i$ block-diagonal matrix

$$X_\bullet = \left\{ \begin{matrix} X_1 & 0 & \cdots & 0 \\ 0 & X_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & X_g \end{matrix} \right\}$$

and $\beta_\bullet$ is a k-vector of stacked vectors β_1 through β_g.

To assess statistical significance of particular variables and precision of parameters estimation in the relatively small sample at hand, I make use of non-parametric bootstrap technique to compute standard errors and confidence intervals (the latter based on quantiles of the bootstrap statistic distribution).

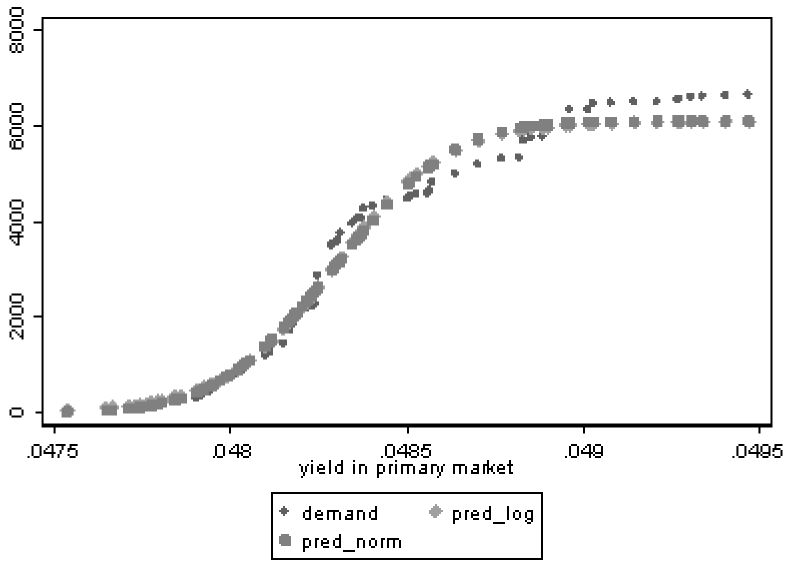
4 Results of estimation of parameters.

4.1 Fitting aggregated demand functions

Upon running the procedure described in subsection 3.2, we note following findings[8] (which, unless specified otherwise, apply to both types of securities).

[8] A rather large table of auction-specific estimates and measures of goodness-of-fit, as well as diagnostic plots are available from the author upon request.

Figure 2: Actual demand and predicted values from logistic and normal CDF models, a 2Y bonds auction (demand in millions of zl.).



First, both logistic and normal functions do a very good job in fitting the empirical demand function. The (uncentered) R^2 of the regressions is hardly ever below 0.98 in particular auctions; overall, it exceeds 0.995 in bill auctions and 0.997 in bond auctions.

Second, both models generate very similar predictions. Sum of squared differences between the predicted values from the logistic model and the normal model in case of 2Y (52W bills) is close to 0.005% (0.007%) of the total sum of squares of the predicted values from the logistic model. Put differently, this sum of squares amounts to just 2% of the Residual Sum of Squares from the normal model. Related to this, we find a strikingly high correlation of over 99% between corresponding parameters of both models: α and β, τ and μ and λ and σ.

Both of these regularities are clearly seen on a scatter plot (Figure 2) presenting actual demand function and predictions from both models for one of the auctions. Visual inspection of plots from remaining auctions confirms the excellent fit.

Auction-specific differences between goodness-of-fit of the two methods are rather moderate:.ratios of Residual Sum of Squares generated in normal model to RSS from the logistic model

Figure 3: Q-Q plot for actual and fitted cut-off prices in 2Y bond auctions (price in \$ per 1\$ of face value).

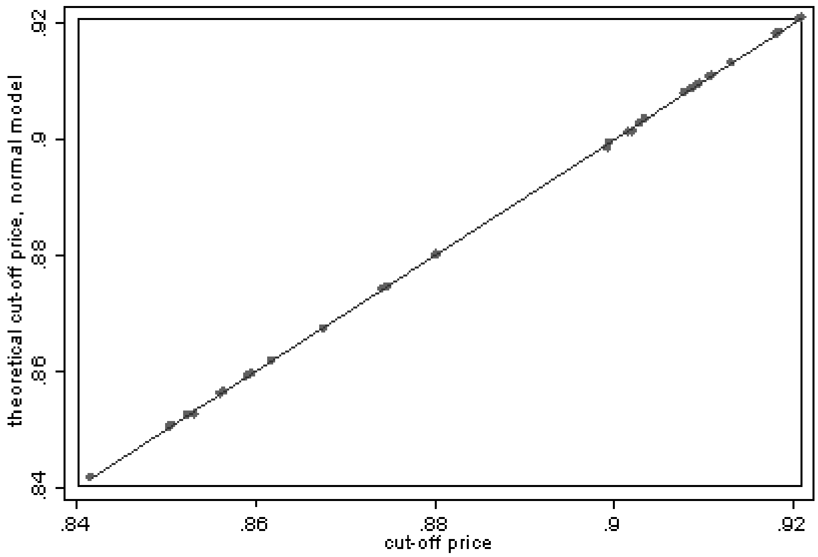


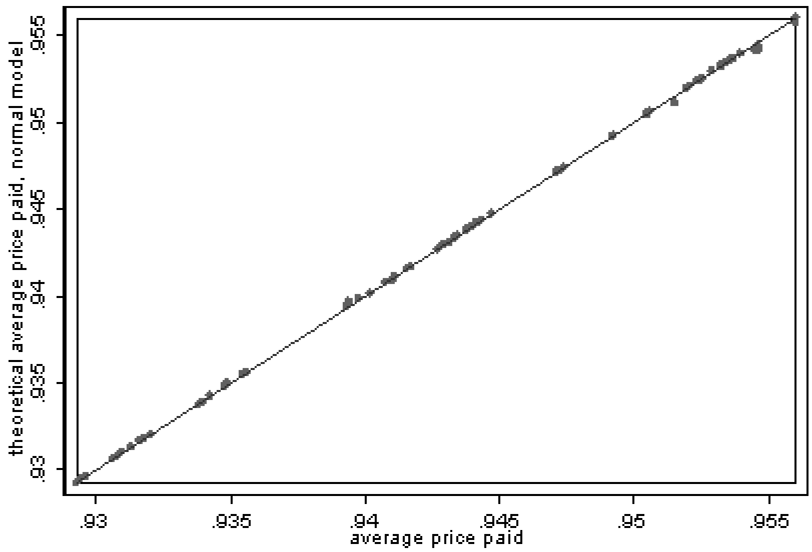
Figure 4:

vary from 0.72 to 1.10 in the case of 2-year bonds and from 0.84 to 1.17 for 52 week bills.

The extraordinary good fit of the normal cdf model may be further confirmed by reconstructing the cutoff-price that would emerge have predicted demand functions been submitted under the actual volume of sales. Figures 4 and 5 show that normal cdf model makes essentially perfect predictions of the cut-off price (here: for 2Y bonds) and average price paid (here: for 52 weeks) respectively. Pictures for logistic model are similar.

On average, normal model appears to perform better in bond auctions. It generates lower residuals in 16 cases (2/3 of the sample), whereas the opposite is true in 8 auctions. Total sum of squared residuals aggregated over all auctions (which, given common unit, seems to be a fair measure of overall performance of the model) is some 4.7% lower in the normal model than in the logistic model. Further, as we have noted previously, normal model appears to be "less risky" in that it is generates, at worst, RSS 10% higher than the other model, whereas using logistic model may result in inflating RSS by 39% relative to the normal model.

Figure 5: Q-Q plot for actual and fitted cut-off prices in 52W bills auctions (price in \$ per 1\$ of face value).



In the case of Treasury bills auctions, both models do almost equally good a job, with logistic model being fitted more closely in exactly half (37) of the auctions. Still, the sum of squared residuals is 1.0% lower in the latter.

We conclude that the normal cdf model, being theoretically more appropriate, also performs at least as well, indeed better in one of the samples, as the logistic model. Thus use of the normal cdf model in fitting aggregate demand functions is advisable. In the following I focus on this approach, reporting estimation results for parameters of the normal cdf specification. Needless to say, however, all the techniques mentioned could have been equally well applied to the logistic form.

5 Estimation of economic determinants of demand function parameters

The set of initially considered auction-specific variables (scaled to a common order of magnitude) is given in Table 2.

Table 2. Explanatory variables in SUR estimation: description, mean and standard deviation

variable	description	m(2Y)	sd(2Y)	m(52W)	sd(52W)
a_supply	volume on offer (in zl. bill.)	2.488	0.445	0.985	0.188
a_wig_1	WSE* index at $t-1$	2.042	0.433	2.084	0.331
a_y_1	yield in sec. market at $t-1$	0.062	0.010	0.058	0.009
a_v22**	volatility. in sec. market	1.469	3.806	1.252	4.143
a_nb_01	"1" for first issue	0.25	0.442	1	1

*WSE=Warsaw Stock Exchange, WIG22 Index; multiplied by 10^{-3}.

** I used volatility of yields within last month, or exactly 22 last working days; multiplied by 10^{3}.

Results of the estimation of the model in the sample of bond auctions and bill auctions are presented in Tables 3 and 4 respectively.[9] As is readily seen, total demand in bond auctions, as measured by β, depends in the first place on the total volume offered in the auction (a_supply). This is in line with findings reported in Preget & Waelbroeck (2005) and especially Boukai & Landsberger (1998) and Berg et al. (1999). In the latter two models, the investors bid for a fraction of total supply rather than particular amount. The inflection point μ is best predicted by the recent secondary market yield a_y_1.

[9] I present only results for normal model parameters. Those for logistic model are rather similar. Note that, for the sake of the estimation process, explanatory variables have been scaled in such a way to obtain coefficients of same order of magnitude.

Table 3. Determinants of demand function parameters in 2Y bond auctions (SUR estimates).

beta/1000	**coeff.**	**st. er.***	**conf. interval****	
$a\_supply * 10^{-2}$	0.408	0.166	0.115	0.640
$a\_wig\_1 * 10^{-4}$	0.207	1.169	−0.009	0.385
$cons * 10$	−0.446	0.397	−1.040	0.227
mu	**coeff.**	**st. er.***	**conf. interval****	
$a\_supply * 10^{-6}$	−0.831	0.900	−2.053	0.750
$a\_y * 10^{-1}$	0.109	0.003.	0.105	0.114
$a\_vol22 * 10$	−0.148	0.603	−0.356	−0.067
$a\_wig\_1 * 10^{-6}$	−0.272	0.098	−0.418	−0.111
$a\_nb\_01 * 10^{3}$	−0.166	0.658	−1.345	0.793
$cons * 10^{-2}$	0.302	0.322	−0.278	0.729
sigma	**coeff.**	**st. er.***	**conf. interval****	
$a\_y\_1 * 10^{-1}$	0.184	0.080	0.046	0.303
a_vol22	0.325	0.764	−0.013	1.195
$a\_wig\_1 * 10^{-7}$	−0.511	0.227	−0.881	−0.121
$cons * 10^{-3}$	0.436	0.407	−0.128	1.140

* Non-parametric bootstrap, $M = 1000$ replications.

* 90% percentile non-parametric bootstrap, $M = 1000$ replications.

Table 4. Determinants of demand function parameters in 52W bill auctions (SUR estimates).

beta/1000	**coeff.**	**st. er.***	**conf. interval****	
$a\_supply * 10^{-2}$	0.107	0.046	0.036	0.189
$a\_vol22 * 10^{3}$	-0.417	6.458	-0.763	-0.094
$a\_wig\_1 * 10^{-3}$	0.038	0.337	-0.527	0.581
$cons * 10$	0.122	0.078	-0.005	0.254
mu	**coeff.**	**st. er.***	**conf. interval****	
$a\_supply * 10^{-5}$	0.220	0.087	0.094	0.374
$a\_y\_1 * 10^{-1}$	0.100	0.001	0.098	0.102
a_vol22	0.203	60.75	-0.069	0.424
$a\_wig\_1 * 10^{-7}$	0.892	0.476	0.182	1.676
$cons * 10^{-2}$	-0.227	0.086	-0.375	-0.093
sigma	**coeff.**	**st. er.***	**conf. interval****	
$a\_supply * 10^{-6}$	0.480	0.173	0.239	0.797
$a\_vol22 * 10^{-1}$	-0.959	79.630	-2.140	0.234
$a\_wig\_1 * 10^{-7}$	-0.298	0.127	-0.528	-0.119
$cons * 10^{-3}$	0.625	0.272	0.203	1.104

* Non-parametric bootstrap, $M = 1000$ replications.

* 90% percentile non-parametric bootstrap, $M = 1000$ replications.

General climate on the financial markets captured by the WIG22 index of the Warsaw Stock Exchange has no significant effect on the maximal demand β. The bull market leads however to lower inflection point in the yield space μ which indicates generally higher prices or more optimistic bidding. Just like in Preget & Waelbroeck (2005) it also significantly lowers the dispersion of bids measured by σ.

It is noteworthy that μ correlates negatively with volatility – thus higher uncertainty in the market leads to lower yield rates and higher prices. While this effect is moderate – increase in volatility equal to one standard deviation results in μ decreasing by approximately 3.5 basis points – it is still significant. This is a surprising result: if there is little uncertainty, auction price should coincide with the secondary market price (adjusted for administrative costs), whereas in case of great uncertainty, winner curse is expected to shift demand downwards. I also investigate suggestion made i.a. by Fleming (2002), that reopening is associated with higher borrowing cost, comparing to the first issue of given security. The dummy variable indicating whether bond is issued for the first time is found to have no significant effect on the overall bids level μ.

Similar results are obtained for 52W Treasury bills. Parameter β depends heavily on the amount supplied, increasing by zl. 1.06 million for additional zl. 1 million of face value offered. Secondary market volatility decreases the amount sued. What regards the inflection point μ, it is by far mostly determined by the secondary market rate before the auction. The coefficient is not significantly different from 1. The bids are also less aggressive if more amount is on offer. Again, secondary market volatility does not significantly affect this measure of average price level. We note that stock market index is still significant but has opposite sign. This need not be considered surprising however, as difference in time to maturity can plausibly affect the extent to which security prices exhibit similar pattern of responses to exogenous shocks as the stock market does. The dispersion parameter σ is positively affected by the supply offered by the Treasury and negatively by the value of the WIG index. Thus large supply of the bills offered on auction and bad general economic climate contribute to increased uncertainty regarding optimal bidding level.

5.1 Out-of-sample prediction of the parameters of bidding functions and resulting profits

To assess the strength of the model, I perform an out-of-sample prediction of the three parameters of aggregate demand functions and corresponding underpricing and primary dealers' profits. To this end I first estimate the SUR model on the sample of first 50 observations for 52W bills auctions (the sample of 2Y auctions is rather small, rendering tests of predictive power almost infeasible). Basing on actual values of explanatory economic variables and estimated coefficients, I compute the predicted values of β, μ and λ Next, plugging the predicted values into equation (12) I construct theoretical demand functions for auctions 51-74 on a 0.1 basis points yield rate grid (corresponding to the minimal admissible "tick"). This allows computation of the expected underpricing (difference between expected average yield paid in the auction and yield on the secondary market at t-1) and profits of primary dealers (defined here as the product of underpricing and amount purchased). Table 5 reports the results.

We conclude that the model generates on average roughly correct predictions for all of the three parameters of the bidding functions. The same applies to the aggregate profits of primary dealers. Table 5 also displays validity of the predictions at given level, i.e. the fraction of predicted values that fell into certain interval around the actual value (in the case of profits that can attain also negative value I also report fraction of predicted values within range of $+/-1$ st. dev. computed for the prediction subsample). Prediction of the inflection point μ is nearly perfect, 96% of the out-of-sample predictions being between 99% and 101% of the real value. Forecasts of other parameters are somewhat less precise, particularly we note the poorly predicted standard deviation of the scale parameter β, which is largely due to two outlier observations (one with very high demand, the other with very low one). On the whole however, the model can be said to deliver unbiased and reasonably accurate forecasts of crucial characteristics of bidding behavior and auction results.

Table 5. Out-of-sample prediction of dealers' profits and demand function parameters.

beta/1000	**observed**	**predicted**
mean	2.277	2.624
std. dev.	0.725	0.126
validity at 20%	x	33%
mu	**observed**	**predicted**
mean	0.071	0.071
std. dev.	0.004	0.005
validity at 1%	x	96%
sigma*1000	**observed**	**predicted**
mean	0.376	0.431
std. dev.	0.136	0.094
validity at 20%	x	37.5%
profit	**observed**	**predicted**
mean	1.659	2.152
std. dev.	0.515	0.562
validity at 20%	x	29%
validity at 1 s. d.	x	38%

6 Conclusion

This paper shows the power of the modified approach to modeling aggregate demand functions in Treasury auctions, based on the normal cdf rather than the standard logistic formulation. The former appears to outperform the latter slightly in terms of goodness of fit. The reasonably accurate prediction (despite a rather small sample at hand) of the parameters of normal cdf specification based on economic variables known before the auction makes forecasts of auction results possible. This enables the economists (and the Treasury alike) to monitor performance of the auction design used. Any substantial and systematic deviation from the predicted shape of the demand functions and corresponding profit obtained by the primary dealers should induce an in-depth investigation and consideration of possible institutional changes. In particular in case of a collusive agreement, early detection of the resulting underpricing is essential to avoid huge losses on part of the Treasury. Finally, the model delivers a powerful tool for the analysis of results of any possible change in the auction design of auctions on behavior of the dealers and the corresponding cost of public debt servicing.

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