

Partnership*

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Abstract

Individuals are called partners when it is in their best interest to help each other, when by doing so they increase the probability of being together in the future when, for similar reasons, they will continue to help each other. Kinsmen or individuals who often face (hedonic) situations in which helping is the dominating strategy are committed to help each other. Partnership may develop among them since the loss of the other means the loss of a guaranteed helper. Thus, they may be willing to take additional risks to help the other. Partnership may occur among unrelated individuals and with no hedonic situations. Partnership creates bonds between partners which may be much stronger than between kinsmen, an individual may take more risks for his partner than he will ever take for a kin. Partnership may evolve without the sophistication and memory required for reciprocation altruism.

Although kin selection, partnership and reciprocation are likely to appear fused as the causes for altruism, we argue that it may be possible to distinguish between them in some situations.

We show that as the partners get older partnership may become less important to them. We also show that like cooperation, and for analogous reasons, malice may evolve among partners so that each will be willing to take additional risks in order to eliminate the other.

key words: Cooperation, Altruism, Partnership, Survival.

Partnership

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1 Introduction

Individuals are selected to behave in a way that increases the expected longevity of their reproductive life. In most studies of population game theory, individuals, when they maximize their survival probability in a given event do so under the assumption that their actions in this event do not influence their or their partners' survival in subsequent events. This assumption is not valid when social networks exist. In a social network, the survival of an individual in future events may depend on the presence and actions of other individuals in the network. At the same time the future behavior of others in the network, their welfare and in particular their very survival may depend on the individual's current actions. Thus an individual's current actions influence his survival in future events.

The case in which one's present actions affect the future *behavior* of his partners has been often considered in the literature. It has been shown how reciprocal altruism (cooperation) may evolve in such situations (Trivers, 1972, Axelrod [1]).

The case in which an individual's action affects the welfare of others in the network and thus their potential ability to help in the future, has been rarely discussed in the literature, with the exception of Eshel & Cohen [4] and Eshel & Motro [5] who show that kinship altruism amplifies itself. If an individual knows (as in kinship) that the other will help him in future dangers, then his loss means the loss of a reliable helper, and it is in the individual's interest to help the other survive. Thus mutual altruistic behavior among such individuals will strengthen itself. We term this amplifying factor *partnership*. Partnership differs from reciprocation altruism in that it is the *ability* to help, not the willingness to help, which is conditional on past behavior. Thus, partnership is the relation established between two or more individuals when it is in the interest of one to help the other, since by doing so he increases the probability of the other to survive and be present in future situations where for similar reasons it will be in the other's best interest to help the first individual. In this work we concentrate on partnership between non related individuals.

Where one finds altruistic cooperation its reasons are likely to be mixed: kinship, partnership and reciprocation¹. Hunting dogs, inbreeding within the pack, are likely to develop altruism due to kin selection. But their cooperation leads to a strong mutual dependence in their hunting and in skirmishes with rival packs or with other predators. The loss of a few members may be lethal to the pack and lead to its extinction, cooperation will therefore amplify itself and

¹Altruism may also evolve by natural selection in structured population (see Wright [15]). In a recent paper Eshel et al. [6] have shown that altruism developing due to a neighborhood structure resembles altruism among kinsfolk. For a more general treatment of neighbors as kin see Hamilton [10].

partnership will develop (van Lawick-Goodall [14]). Elaborate alliances and social bonds develop in the pack and memory of past behavior leads to cooperation based on reciprocation. Although intense partnership may be established between young unrelated male lions who form a lifelong close bond based on mutual dependence (Schaller, [12]), it is more common for a sibling, when present, to be the chosen companion, amplifying dependency and creating a partnership. The common courtship-like behavior among kin-partners suggests that reciprocation plays an important role in maintaining and intensifying the relationship, perhaps even initiating it.

Although the three factors, kinship, partnership and reciprocation, are interwoven in establishing altruistic behavior, we demonstrate that they may each have a different effect on the resulting behavior. We suggest that each of the above factors leads to qualitatively different, if related, predictions that may be conceivably distinguished on the basis of field observations on a sufficiently wide range of situations.

In contrast to kinship and reciprocation, the effect of partnership on animal and human behavior has not been studied. In order to distinguish between partnership altruism on one side and kinship and reciprocation altruism on the other, we assume that the potential partners are unrelated, so that each aims to increase only his own expected life-span. To isolate the effects of partnership from those of reciprocation we describe a situation in which the actions of one individual can affect only the survival probability of the others, not their future behavior, thus precluding the possibility of punishment. This assumption, which is formally equivalent to the lack of memory, fits a situation in which an organism developed physiological factors for symbiosis and partnership. Among humans, this may describe a case of help without the receiver being aware of it, a case which cannot be explained by reciprocal altruism (although it may be caused by kinship).

Under these assumptions, we demonstrate that partnership altruism may evolve, beginning at a level in which the cost of helping the other is low, and amplifying itself to higher levels of mutual dependence. We show that, like in kinship altruism, an individual chooses his action to maximize his 'inclusive survival' consisting of the sum of his own survival probability and the product of a *partnership coefficient* and the probability that *both* he and his partner survive. Unlike the kinship coefficient, the partnership coefficient may assume arbitrarily high values or indeed negative values. High values of the coefficient correspond to situations of extreme mutual dependence when the probability of survival without the partner is low. Negative values of the partnership coefficient correspond to malice, when the individuals are bound together but the presence of one is detrimental to the survival of the other. In partnership altruism, unlike in kinship altruism, the altruistic behavior is aimed at increasing the partner's survival probability *conditional* on the altruist's own survival.

In later sections of the paper we discuss how the age, mortality and ecological factors affect partnership altruism differently than kinship or reciprocal altruism.

2 The Model

A large population, in which the individuals are either paired or single, faces a stream of dangerous events. The events, when they occur, may kill one or both of individuals. The events differ in their nature but the individuals recognize the type of danger and can take some specific actions which affect their survival. In addition, individuals whether single or paired may die a natural death. Death in one of the events is distinguished from natural death in that the individuals have no effect on the probability of the natural death. Natural death comes with intensity λ , i.e. at any infinitesimal time interval dt an individual (single or paired) may die naturally with probability λdt . A dangerous event occurs with intensity μ , and a single individual is matched with intensity ν . We refer to paired individuals as *partners*. Thus a paired individual may die a natural death or be killed by an event. He remains with his partner for as long as both live, if the partner dies he remains single for a while and he may die a natural death or by one of the dangerous events, he may also find another partner and continue his life in a pair.

At each encounter, the actions taken by the partners determine the survival probabilities of each of them and of both jointly. Thus, the effect of the actions can be described by 3 probabilities: the survival probability of each partner and their joint survival probability (the probability of both dying is the residual probability). We assume that there are 2 strategies available at each encounter: C and D . The dependence of the survival probabilities on the actions taken by the two players can be written as a symmetric game, in which each cell contains two probabilities: the survival probability of the player taking the action and the joint survival rate. Thus an encounter \underline{V} can be described by:

$$\underline{V} = \left\{ \begin{array}{c} C \\ D \end{array} \begin{array}{|cc|} \hline C & D \\ \hline v_{11}, w_{11} & v_{12}, w_{12} \\ \hline v_{21}, w_{12} & v_{22}, w_{22} \\ \hline \end{array}, \theta \right\}$$

Here v_{ij} is the first partner's probability of survival when he takes action i and the other partner has taken action j . The survival probability of the other partner is then v_{ji} and the probability of *both* surviving the event is w_{ij} . We assume that² $w_{ij} = w_{ji}$. Finally, when a single individual faces this encounter his probability of surviving it is θ .

An encounter is therefore characterized by 8 probabilities (v_{ij}, w_{ij}, θ) . The distribution of future encounters $\underline{V} \in \Omega = [0, 1]^8$ is assumed to be time independent and is given by $F(\underline{V})$.

Throughout most of the paper we assume that \underline{V} has a Prisoners' Dilemma structure, with the first strategy representing cooperation and the second defection, i.e. $v_{21} > v_{11} > v_{22} > v_{12}$, a player can always increase his survival probability by defecting. However, we will in most examples assume that

²For a mathematically oriented analysis of a special case of this model, with the restriction $w_{ij} = v_{ij}v_{ji}$, see Eshel and Weinshall [7].

$w_{11} > w_{12} = w_{21} \geq w_{22}$, so that defecting means a lower probability of surviving together. This introduces a conflict between wishing to increase one's own survival probability and wishing to have the other player alongside if, indeed, he is supposed to help in some forthcoming encounters.

In order to distinguish between partnership and reciprocation we assume that an individual always takes the same action in a particular event irrespective of the past. A *global strategy* for an individual is therefore a plan of how to act at each possible encounter, i.e. a measurable function \mathbf{x} from the set Ω of all possible encounters to the unit interval, such that for all $\underline{V} \in \Omega$, $\mathbf{x}(\underline{V})$ is a (mixed) *local strategy*, determining the probability of playing C in the encounter \underline{V} . Thus, at each moment in time when both partners are present, their future (which depends only on their global strategies) is independent of time.

Stability We assume that the individuals in the population are selected to behave in a way that increases their expected life span. In this model, the longevity of an individual, either single or paired, depends on his own strategy and on the distribution of strategies in the entire population.

We shall be interested in strategies that when played by the whole population are in some sense evolutionarily stable. Denote by $S(\mathbf{x}, \mathbf{y})$ and $U(\mathbf{x}, \mathbf{y})$ the expected life span of (respectively) a single and paired \mathbf{x} -player in a population fixed on \mathbf{y} -players. A strategy \mathbf{y} , which is fixed in the population, is an ESS if it is strictly advantageous against any small group of mutants playing \mathbf{x} , i.e. $S(\mathbf{x}, \mathbf{y}) < S(\mathbf{y}, \mathbf{y})$, or equivalently $U(\mathbf{x}, \mathbf{y}) < U(\mathbf{y}, \mathbf{y})$. In most of the paper we use a weaker stability concept, an *agent equilibrium* or *agent ESS*, in which only a single local deviation from the global strategy \mathbf{y} will be considered. In this equilibrium the only mutants considered are those that deviate from \mathbf{y} in a single event. This concept is similar to Selten's agent strategic form (see [13], or section 12.5.2. in [11]).

A comment on the Modeling We have assumed a continuum of events, each occurring with probability 0, in order to achieve two aims:

- Our modeling guarantees that changing the action in a single event has no effect on the long run survival probability, since this event will re-occur with probability 0. Thus, when considering what to play in a single event an individual need not take into account the long run effects of this action. If, on the other hand, there is a finite number of events, or one of the events occurs with a positive probability then changing the action in that event will substantially change the global strategy and the long run survival probabilities.
- A continuum of events enables us to assume, as we do later, that the events are densely ranked by a single parameter and each event is surrounded by close and similar events. Thus it may be possible to deduce the behavior in one event from the behavior in the neighboring ones. In a discrete set

of events, we would need an additional assumption to ensure that the distances between ‘neighboring’ events are sufficiently small.

The continuum model can be seen as a limit case of a single parameter discrete set of events each occurring with small probability and where neighboring events are close to each other.

Some Notations When both partners face an encounter \underline{V} together and their (mixed) actions are $\mathbf{x}(\underline{V}) = (x_1, x_2)$, $\mathbf{y}(\underline{V}) = (y_1, y_2)$, the survival probability of partner 1 *to the end of the encounter* is given by

$$\pi^1(\underline{V}, \mathbf{x}(\underline{V}), \mathbf{y}(\underline{V})) = \sum_{i,j=1}^2 v_{ij} \mathbf{x}_i(\underline{V}) \mathbf{y}_j(\underline{V}).$$

The probability of both partners surviving the encounter \underline{V} is:

$$\pi^{12}(\underline{V}, \mathbf{x}(\underline{V}), \mathbf{y}(\underline{V})) = \sum_{i,j=1}^2 w_{ij} \mathbf{x}_i(\underline{V}) \mathbf{y}_j(\underline{V}).$$

The probability of partner 1 surviving a random forthcoming encounter, given the global strategies \mathbf{x}, \mathbf{y} of the two players and the distribution of events $F(\underline{V})$, is therefore

$$\tilde{\pi}^1 = \tilde{\pi}^1(\mathbf{x}, \mathbf{y}) = \int_{\Omega} \pi^1(\underline{V}, \mathbf{x}(\underline{V}), \mathbf{y}(\underline{V})) dF(\underline{V}).$$

Similarly the probability of both players to survive a forthcoming encounter is:

$$\tilde{\pi}^{12} = \tilde{\pi}^{12}(\mathbf{x}, \mathbf{y}) = \int_{\Omega} \pi^{12}(\underline{V}, \mathbf{x}(\underline{V}), \mathbf{y}(\underline{V})) dF(\underline{V}).$$

Finally, the probability of a single individual to survive such a (potential) encounter is:

$$\tilde{\theta} = \int_{\Omega} \theta(\underline{V}) dF(\underline{V}).$$

2.1 Partnership Coefficient and Inclusive Survival

Let a mutant \mathbf{x} player be paired with an individual taken from a population of \mathbf{y} players and let the two face an encounter \underline{V} . Let the mutant consider his strategy choice in the event \underline{V} , assuming that his partner plays the global strategy \mathbf{y} (in particular his partner will play $y = \mathbf{y}(\underline{V})$ in this event), and he himself plays the global strategy \mathbf{x} in all other events and the strategy x in the event \underline{V} . Let his conditional life span, given this behavior, be is $U(x, y|\underline{V}, \mathbf{x}, \mathbf{y})$.

- Definition 1**
1. A global strategy \mathbf{x} is a local best response to \mathbf{y} if for any encounter \underline{V} , except perhaps for a set of measure zero, the strategy $x = \mathbf{x}(\underline{V})$ maximizes the conditional expected life span $U(x, \mathbf{y}(\underline{V}) | \underline{V}, \mathbf{x}, \mathbf{y})$.
 2. A global strategy \mathbf{y} is an agent equilibrium if it is a Local Best Response to itself.

Agent equilibrium is weaker than a Nash equilibrium: clearly any best response is an agent best response and hence any equilibrium is an agent equilibrium but not necessarily vice versa.

To calculate the conditional expected life span, note that if the two players play x, y in the encounter \underline{V} , then the probability that both survive the encounter is $\pi^{12}(\underline{V}, x, y)$ and the probability that only the mutant will survive it is $\pi^1(\underline{V}, x, y) - \pi^{12}(\underline{V}, x, y)$. In the first case the expected life span of the mutant is $U(\mathbf{x}, \mathbf{y})$, and when he is left on his own it is $S(\mathbf{x}, \mathbf{y})$, hence

$$\begin{aligned} U(x, y | \underline{V}, \mathbf{x}, \mathbf{y}) &= S(\mathbf{x}, \mathbf{y}) [\pi^1(\underline{V}, x, y) - \pi^{12}(\underline{V}, x, y)] + U(\mathbf{x}, \mathbf{y}) \pi^{12}(\underline{V}, x, y) = \\ &= S(\mathbf{x}, \mathbf{y}) \pi^1(\underline{V}, x, y) + [U(\mathbf{x}, \mathbf{y}) - S(\mathbf{x}, \mathbf{y})] \pi^{12}(\underline{V}, x, y) = \\ &= S(\mathbf{x}, \mathbf{y}) [\pi^1(\underline{V}, x, y) + K(\mathbf{x}, \mathbf{y}) \pi^{12}(\underline{V}, x, y)] \end{aligned}$$

where

$$K(\mathbf{x}, \mathbf{y}) = \frac{U(\mathbf{x}, \mathbf{y}) - S(\mathbf{x}, \mathbf{y})}{S(\mathbf{x}, \mathbf{y})}. \quad (1)$$

The values $K(\mathbf{x}, \mathbf{y})$, $S(\mathbf{x}, \mathbf{y})$ are determined by the two global strategies and do not depend on any particular encounter. When an individual chooses his action in an event \underline{V} , he leaves his global strategy unchanged and so $S(\mathbf{x}, \mathbf{y})$, $U(\mathbf{x}, \mathbf{y})$ and therefore $K(\mathbf{x}, \mathbf{y})$ can be taken as constants. Thus:

$$U(\mathbf{x}, \mathbf{y}) \propto \pi^1 + K \pi^{12}.$$

We have shown that the conditional life span of an individual in an encounter \underline{V} is proportional to his own survival probability in this event plus K times the joint survival of the two partners in this event. The constant K measures the affinity between the two partners and the extent to which one partner will want to sacrifice some of his survival probability in order to increase the joint survival probability.

- Definition 2**
1. The constant $K(\mathbf{x}, \mathbf{y})$ is the partnership coefficient of the two players.
 2. The function $I(x, y | \underline{V}, \mathbf{x}, \mathbf{y}) = \pi^1(\underline{V}, x, y) + K(\mathbf{x}, \mathbf{y}) \pi^{12}(\underline{V}, x, y)$ is the Partnership Inclusive Survival of player 1 when playing x against y in the encounter \underline{V} .

The conditional life span of an individual in an encounter \underline{V} is proportional to his partnership inclusive survival in this encounter. Of particular interest to us is the symmetric case of a homogeneous population in which all individuals play \mathbf{y} . In that case \mathbf{y} is an agent equilibrium if and only if for almost all encounters \underline{V} the pair of (local) strategies $(\mathbf{y}(\underline{V}), \mathbf{y}(\underline{V}))$ is a symmetric Nash equilibrium of the symmetric partnership inclusive survival game $I(x, y|\underline{V}, \mathbf{x}, \mathbf{y})$.

Definition 3 *The global strategy \mathbf{y} is an Agent-ESS if and only if for almost all encounters \underline{V} , $\mathbf{y}(\underline{V})$ is an ESS of the symmetric partnership inclusive-survival game $I(x, y|\underline{V}, \mathbf{x}, \mathbf{y})$.*

2.2 Calculation of the Partnership Coefficient

We begin by calculating the expected life span of an individual (single and paired) who plays the strategy \mathbf{x} in a population of \mathbf{y} players. We denoted the expected life span of a paired individual by $U(\mathbf{x}, \mathbf{y})$ and by $S(\mathbf{x}, \mathbf{y})$ when he is single.

We first calculate $U(\mathbf{x}, \mathbf{y})$. In a short time interval τ , there is probability $\lambda\tau$ that he will die, with probability $\lambda\tau$ his partner will die then he will be single and his expected life span is $S(\mathbf{x}, \mathbf{y})$. With probability $\mu\tau$ the two partners will face a random encounter, in that case with probability $\tilde{\pi}^{12} = \tilde{\pi}^{12}(\mathbf{x}, \mathbf{y})$ both will survive it and with probability $\tilde{\pi}^1 - \tilde{\pi}^{12}$ only the first will survive it. With the residual probability $1 - (2\lambda + \mu)\tau$ none of the above will happen, our player remains with his partner and his conditional life expectancy is $U(\mathbf{x}, \mathbf{y})$. Hence (ignoring terms with higher order of τ) :

$$U = \tau + [1 - (2\lambda + \mu)\tau]U + \lambda\tau S + \mu\tau[(\tilde{\pi}^1 - \tilde{\pi}^{12})S + \tilde{\pi}^{12}U]$$

by letting τ approach to 0 we obtain:

$$[2\lambda + \mu(1 - \tilde{\pi}^{12})]U = [\lambda + \mu(\tilde{\pi}^1 - \tilde{\pi}^{12})]S + 1. \quad (2)$$

Similarly, we obtain an equation for $S(\mathbf{x}, \mathbf{y})$:

$$\nu U = [\lambda + \mu(1 - \tilde{\theta}) + \nu]S - 1. \quad (3)$$

Solving the equations for U, S and by equation [1] we find that:

$$K(\mathbf{x}, \mathbf{y}) = \frac{\mu(\tilde{\pi}^1 - \tilde{\theta})}{2\lambda + \mu(1 - \tilde{\pi}^{12}) + \nu}.$$

Denote

$$p = \frac{2\lambda + \nu}{\mu}, \quad (4)$$

then the partnership coefficient K can be written as:

$$K(\mathbf{x}, \mathbf{y}) = \frac{\tilde{\pi}^1 - \tilde{\theta}}{1 + p - \tilde{\pi}^{12}}. \quad (5)$$

Note that for given global strategies, a positive partnership coefficient K decreases with p , note also that K increases with μ and decreases with λ, ν . Hence it becomes less beneficial to take a risk for a partner who is likely to die (a high λ), when it is easy to find a new partner (a high ν), or when the encounters are less likely to happen (a low μ).

The partnership coefficient K depends on the global strategies (through $\tilde{\pi}^1, \tilde{\pi}^{12}$). On the other hand, the local stability of the strategies \mathbf{x}, \mathbf{y} and therefore the (equilibrium) strategies themselves depend on K . We investigate this interrelation between the two in the following sections.

3 An Example: Mutual Help Among Partners

Assume that each event has the prisoners' dilemma form and that the survival probability in mutual defection equals that of being single, then defection in all events is an ESS. If no one ever cooperates then the individuals are doing equally well when single or paired. If a small group of mutants begins to cooperate, they will be worse off since they sacrifice some of their survival probability for cooperation but receive nothing in return. However, non cooperation is not always stable against small deviations of the *entire* population. If some small degree of cooperation has been established in the whole group, or in a sufficiently large group, then cooperation may amplify itself and reach higher degree of cooperation. For if the whole population cooperates in events in which some help can be offered for a small cost, and if such events occur often enough then each individual is no longer indifferent to the existence of the others. He may now help the others in situations in which help demands a higher sacrifice in order to secure the other's existence and thereby his help in situations for which cooperation has already been established. We demonstrate this argument in the following example. An ESS is said to be continuously stable (CSS) if in addition to its stability against deviations of small groups it is also stable against small deviations of large groups (see Eshel and Motro 1981[5], Eshel 1982 [3]).

Consider a family of events each of which exposes an unaided individual (whether single or paired) to a fixed probability of death ε . A paired individual can help his partner survive by reducing his own survival probability by h . The events differ only in the cost h of helping the other. The events (parametrized by h) are distributed according to the function $F(h)$ on $(0, 1 - \varepsilon]$. Mutual cooperation makes the partners survive or die together. An event of this type

can be described by the following game matrix:

$$\begin{array}{c} C \\ D \end{array} \begin{array}{cc} C & D \\ \hline 1-h, 1-h & 1-h-\varepsilon, 1-h-\varepsilon \\ \hline 1, 1-h-\varepsilon & 1-\varepsilon, (1-\varepsilon)^2 \end{array}, \theta = 1-\varepsilon$$

Note that the part of the matrix which describes the player's own survival probabilities corresponds to the situation of additive costs and benefits, analyzed by Hamilton (1964) in his classic argument for kin-selection.

To defect in all events is an ESS. The partnership coefficient is 0, reflecting the fact that each partner is indifferent to the survival of the other.

We show that under certain conditions the partners will cooperate in some of the events. In particular we show that

- Under certain conditions there exist agent ESS's in which the population cooperates in a wide range of events. These agent ESS are also evolutionarily and continuously stable, i.e. they are stable against small groups of mutants and against small deviations of the whole population.
- When such cooperative ESS exist then the totally non cooperative ESS is continuously unstable, that is, if the whole population cooperates in events with very low costs of cooperation (h close to 0), then increasing rather than decreasing the range of cooperation is advantageous. In fact, when there is a high concentration of events around $h = 0$, and the entire population has, for some reason, begun to cooperate then it is beneficial for any individual to further increase his range of cooperation. Thus, little cooperation can amplify itself and reach high levels of cooperation.
- The partnership coefficient $K(\mathbf{x}, \mathbf{x})$ may assume arbitrarily large values for evolutionarily and continuously stable strategies \mathbf{x} . This ensures the partners will cooperate in a wide range of events, including events in which the cost of cooperation is large

We will consider only *simple strategies* in which there is cooperation in all events up to a certain h and defection for all other events:

$$\mathbf{x}_\alpha(h) = \begin{cases} 1 & \text{if } h \leq \alpha \\ 0 & \text{if } h > \alpha \end{cases}.$$

Let $K(\alpha) = K(\mathbf{x}_\alpha, \mathbf{x}_\alpha)$ be the partnership coefficient when the population is fixed on a simple strategy \mathbf{x}_α . By (5) :

$$K(\alpha) = \frac{\int_0^\alpha (\varepsilon - h) dF(h)}{1 + p - \left[\int_0^\alpha (1-h) dF(h) + \int_\alpha^\infty (1-\varepsilon)^2 dF(h) \right]}. \quad (6)$$

The partnership coefficient depends on the global strategy played, on the other hand, a given partnership coefficient determines the local best response to any strategy played by the partner. A strategy \mathbf{x}_α is an agent equilibrium if the partnership coefficient it induces makes cooperation the best response to cooperation for events below α and defection the best response to defection above α .

An agent equilibrium is stable against local mutations, i.e. against mutations occurring in the action of a single event. However, for this family of events local stability implies that no other mutation, however elaborate, can invade the population. An agent equilibrium in simple strategies is an ESS. The proof can be found in the appendix.

To find the agent equilibria we first investigate the best responses for a given coefficient K .

Cooperation is the best response to cooperation in an encounter h , when:

$$(1 - h) - 1 + K [(1 - h) - (1 - h - \varepsilon)] \geq 0$$

or:

$$h \leq K\varepsilon.$$

Similarly, cooperation is the best response to defection in an event h if:

$$(1 - h - \varepsilon) - (1 - \varepsilon) + K [(1 - h - \varepsilon) - (1 - \varepsilon)^2] \geq 0$$

or:

$$h \leq \frac{K}{1 + K} \varepsilon (1 - \varepsilon).$$

Denote:

$$H_1(K) = K\varepsilon, \quad H_2(K) = \frac{K}{1 + K} \varepsilon (1 - \varepsilon). \quad (7)$$

Note that for any $K > 0$:

$$H_2(K) \leq H_1(K)$$

hence C is the dominant strategy for events $h < H_2(K)$, defection D is the dominant strategy for $H_1(K) < h$, and for intermediate events $H_2(K) < h < H_1(K)$ cooperation is the best response to itself and defection the best response to itself.

We have thus proved:

- Lemma 4**
1. If $h < H_2(K)$ then C is the local dominant strategy in event h , i.e. it is the unique best response to any strategy of the partner.
 2. If $h > H_1(K)$ then D is the local dominant strategy in event h , i.e. it is the unique best response to any strategy of the partner.

3. If $H_2(K) < h < H_1(K)$ then C is the unique best response to C in event h and D is the unique best response to D .

The following proposition follows directly from the lemma:

Proposition 5 *A simple strategy \mathbf{x}_α is an agent ESS if and only if $H_2(K(\alpha)) \leq \alpha \leq H_1(K(\alpha))$.*

Proof. If $\alpha < H_2(K(\alpha))$ then by lemma 4 the local best response to \mathbf{x}_α in events $h : \alpha < h < H_2(K(\alpha))$ is to cooperate contrary to what \mathbf{x}_α prescribes, which cannot therefore be a local ESS. If $\alpha > H_1(K(\alpha))$ then the local best response in events $\alpha > h > H_1(K(\alpha))$ is to defect, hence \mathbf{x}_α cannot be a local ESS.

If, however, $H_2(K(\alpha)) \leq \alpha \leq H_1(K(\alpha))$ then for all events h , except $h = H_1(K(\alpha)), H_2(K(\alpha))$, the strategy $\mathbf{x}_\alpha(h)$ is the strict best response to itself. ■

If events with low cost of cooperation rarely happen, then it is not worthwhile for any of the partners to sacrifice anything for a partner who, even if he were so inclined, would rarely have the opportunity to help. Indeed, if $K(\alpha)$ (equation 6) can be made small relative to α , $\alpha > \varepsilon K(\alpha) = H_1(K(\alpha))$, then (by proposition 5) the strategy of defecting in all events is continuously stable. If the whole population cooperates up to a small α , then each individual will benefit by cooperating less. To obtain $\alpha > \varepsilon K(\alpha)$ around $\alpha = 0$, the probability weight of events in this neighborhood should be small.

If, on the other hand, there is a high concentration of events around $h = 0$ then there exist agent ESS's in simple strategies in which there is cooperation, and in any of these ESS's there is a minimal level of cooperation. This is shown in the following proposition.

Proposition 6 *Let the events be distributed according to a continuous function $F(h)$, with a density function $f(h)$, and let*

$$f(0) > \frac{1+p-(1-\varepsilon)^2}{\varepsilon^2(1-\varepsilon)},$$

then

1. *There are two events $\alpha_1 > \alpha_2 > 0$ such that for all events $\alpha : \alpha_1 > \alpha > \alpha_2$ the simple strategy \mathbf{x}_α is an agent ESS.*
2. *For $\alpha < \alpha_2$ the simple strategy \mathbf{x}_α is not an agent ESS.*
3. *The strategy \mathbf{x}_0 (defection in all events) is an ESS but is not continuously stable.*

Proof. Note first that $H_2(K(0)) = 0$ and that for any $K \geq 0$ $H_2(K) < \varepsilon(1 - \varepsilon)$, hence for $\alpha > \varepsilon(1 - \varepsilon)$, $H_2(K) < \alpha$. If the derivative of $H_2(K(\alpha))$ is greater than 1 at $\alpha = 0$ then $H_2(K(\alpha))$ is above the diagonal around $\alpha = 0$. The derivative of $K(\alpha)$ at $\alpha = 0$ is (see equation 6)

$$K'(0) = \frac{\varepsilon f(0)}{1 + p - (1 - \varepsilon)^2},$$

hence by the definition of $H_2(K)$:

$$\frac{d}{d\alpha} H_2(K(0)) = \frac{\varepsilon^2(1 - \varepsilon)f(0)}{1 + p - (1 - \varepsilon)^2}$$

and this is greater than 1 by assumption. Thus $H_2(K(\alpha))$ is above the diagonal around $\alpha = 0$ and will (strictly) cross the diagonal for the first time at some $\alpha_2 < \varepsilon(1 - \varepsilon)$.

Since $H_2 < H_1$ it follows that $H_1(K(\alpha)) > \alpha$ for $0 < \alpha \leq \alpha_2$, hence there exists $\alpha_1 > \alpha_2$ (possibly $\alpha_1 = 1 - \varepsilon$) such that for all $\alpha \in [\alpha_2, \alpha_1]$: $H_2(K(\alpha)) \leq \alpha \leq H_1(K(\alpha))$.

1. By proposition 5 the strategy \mathbf{x}_α , $\alpha \in [\alpha_2, \alpha_1]$ is an agent ESS.
2. For $\alpha < \alpha_2$: $\alpha < H_2(K(\alpha))$, hence by proposition 5, the strategy \mathbf{x}_α is not an agent ESS.
3. The strategy \mathbf{x}_0 is not continuously stable since when the entire population deviates from defection and cooperates up to an event α , for some $\alpha < \alpha_1$, then given this degree of cooperation each individual would want to increase his cooperation up to α_1 . Hence a small deviation of the entire population leads to even more deviations bringing the population further away from total non cooperation.

This completes the proof of the proposition. ■

The last proposition demonstrates that under certain conditions (a high concentration of events around $h = 0$) if some cooperation has been established, the population will cooperate at least up to the event α_2 , and it is possible for the population to cooperate up to α_1 .

The next lemma shows that by choosing an appropriate distribution of events the partnership coefficient can be made arbitrarily large . A large partnership coefficient makes α_1, α_2 large and gurantees that if htere is cooperation it will necessarily be in a wide range, at least up to α_2 , and that cooperation may be achieved even in events with h close to $1 - \varepsilon$.

Lemma 7 *The partnership coefficient K can be made arbitrarily large by choosing a high concentration of events around $h = 0$ in which the population cooperates and by letting p be close to 0. For sufficiently large K , the point α_2 , at which $H_2(K(\alpha))$ crosses the diagonal for the first time, can be made arbitrarily close to $\varepsilon(1 - \varepsilon)$ and the point α_1 can be made arbitrarily close to $1 - \varepsilon$.*

Proof. From the definition of $K(\alpha)$ (equation 6) it is straightforward to see that $K(\alpha)$ can be made arbitrarily large for α 's close to 0, by making p close to 0 and by choosing a distribution of events which puts nearly all weight on events with h close to 0. Moreover, where the density of events is low, the derivative of K w.r.t. α is close to 0. Thus K can be made arbitrarily large around 0 after which it changes very little.

This ensures that $H_2(K(\alpha))$ will start close to $\varepsilon(1 - \varepsilon)$ around 0, and will cross the diagonal for the first and last time arbitrarily close to $\varepsilon(1 - \varepsilon)$.

For similar reasons $H_1(K(\alpha))$ will be large around $\alpha = 0$ and will be above the diagonal for all $\alpha < 1 - \varepsilon$. ■

Thus if events occur with a high probability and if most of them have small values of h , then partners will cooperate in nearly all the events.

Figure 1 illustrates proposition 6 and lemma 7:

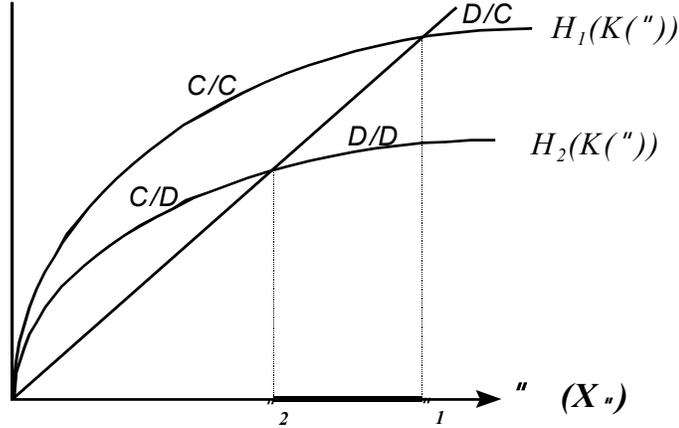


Figure 1: Agent equilibria with cooperation

Where $H_2(K(\alpha))$ is above the diagonal cooperation is the best response to defection (C/D), while defection is the best response to itself (D/D) when $H_2(K(\alpha))$ is below the diagonal. Similarly, cooperation is the best response to itself (C/C) when $H_1(K(\alpha))$ is above the diagonal. Thus all simple strategies \mathbf{x}_α with $\alpha \in [\alpha_2, \alpha_1]$ are agent equilibria. The conditions of lemma 7 ensure that both functions $H_i(K(\alpha))$ start above the diagonal and cross it for high values of α_2, α_1 .

The events in our example have the property that the willingness to take high risks for a defecting partner is limited. When $h > \varepsilon(1 - \varepsilon)$ an individual will not cooperate against a defecting partner, for by cooperating he lowers both his own and the joint survival probabilities. One can easily think of families of

events for which this is not the case. In such families, the partners will be willing to cooperate even against a defecting partner in order to increase the joint survival probability.

4 Malice and Destructive Dependence

4.1 Malice

The partnership coefficient need not always be positive as assumed until now. It can be negative and induce the individuals to take risks to their own life in order to *decrease* rather than increase the joint survival probability. Such behavior, in which an individual endangers himself in order to lower the probability of being with his partner in the future, we call *malice*.

Not any harm done to a partner is malice. Rivalry and direct competition between individuals is common in nature and in such situations it is often to the advantage of an individual to injure his partner-opponent. In these cases, harming the other is due to egoism, not malice. In this sense, competitive behavior is analogous to hedonic altruism which is also derived from selfish behavior rather than goodwill.

A negative partnership coefficient can be established when the partners often meet in competitive situations. If then an event occurs in which both mutual cooperation and mutual defection are possible, the negative coefficient may tilt the balance in favor of malicious behavior. But, as the following example shows, a negative partnership coefficient may also develop without the presence of rivalry or competition, in situations which accommodate both competition and cooperation.

Consider a variation of the additive help example of section 3:

$$\begin{array}{c}
 C \\
 D
 \end{array}
 \begin{array}{cc}
 C & D \\
 \hline
 1-h, 1-h & 1-h-\varepsilon, 1-h-\varepsilon \\
 \hline
 1, 1-h-\varepsilon & 1-\varepsilon, (1-\varepsilon)^2
 \end{array}, \quad \theta = 1 - \varphi$$

with $\varphi < \varepsilon$, $0 < h < \min\{\varphi, \varepsilon(1-\varepsilon)\}$ and with h distributed according to some function $F(h)$. The only difference between this event and the events of section 3, is that the survival probability of a single individual is higher than in mutual defection: $1-\varphi > 1-\varepsilon$. Defection creates a competitive situation: When the other defects he becomes a nuisance, since the survival probability is higher when single. Indeed, when both partners defect in all events the partnership coefficient is negative:

$$K(\mathbf{0}, \mathbf{0}) = \frac{(\varphi - \varepsilon)}{1 + p - (1 - \varepsilon)^2} < 0,$$

A negative value of K ensures that defection is the dominant strategy in the partnership inclusive survival game defined by this K , since:

$$h + K [h - \varepsilon(1 - \varepsilon)] > 0.$$

In this example there is no malice per se, confronted with a partner's defection an individual can only gain by defecting, he increases his own survival probability and reduces the joint one. However, total defection is an agent ESS for these events and it establishes a negative partnership coefficient. This negative coefficient prepares the ground for malice to appear when a suitable event occurs, then an individual will be willing to reduce his own survival probability in order to reduce their joint survival.

Note that in this example, like in section 3, there can be a cooperative agent ESS when there is a high concentration of events around $h = 0$. Thus in this family of events it is possible to establish cooperation with a positive partnership coefficient, leading to altruistic behavior in other events, or alternatively have no cooperation and a negative coefficient which may lead to malice in other events.

The following example shows that malice, like cooperation, can amplify itself to extreme levels, even in situations where cooperation may also be established. Consider the following situation in which two neighbors live independently and peacefully next to each other and may continue to do so, except that occasionally the opportunity arises for each to harm the other. In event $h \leq 1/2$, the aggressor survives a one sided attack with probability $1 - h$, while the peaceful neighbor survives it with probability h , the peaceful neighbor is never the sole survivor of such an attack, i.e. the joint survival probability is h . When both attack, one of them will die and each survives with a small probability σ . The risk h is distributed in $(0, 1/2]$ according to a function $F(h)$. The situation is described by the matrix:

$$\begin{array}{c} C \quad D \\ D \end{array} \begin{array}{|c|c|} \hline 1, 1 & h, h \\ \hline 1 - h, h & \sigma, 0 \\ \hline \end{array}, \quad \theta = 1.$$

Let \mathbf{x}_α be the simple strategy of attacking in all events in which the cost is lower than α : $h < \alpha$. The partnership coefficient when the entire population plays \mathbf{x}_α is:

$$K(\alpha) = K(\mathbf{x}_\alpha, \mathbf{x}_\alpha) = -\frac{(1 - \sigma) F(\alpha)}{1 + p - [1 - F(\alpha)]}$$

for $\alpha = 0$ (complete cooperation) the coefficient is 0, for $\alpha > 0$ it is negative, also $-1 < K(\alpha) \leq 0$.

Analogous to the discussion in section 3 we show the following:

Proposition 8 1. *The simple strategy of full cooperation \mathbf{x}_0 , is an agent ESS.*

2. If events are concentrated around $h = 0$ and events occur with high probability ($p \rightsquigarrow 0$) then there exist $\alpha^* < 1/2$, such that for all $\alpha \in [\alpha^*, 1/2]$ the simple strategy \mathbf{x}_α is an agent ESS. In that case no strategy \mathbf{x}_α with $0 < \alpha < \alpha^*$ is an agent ESS, and the strategy of complete cooperation \mathbf{x}_0 is not continuously stable³. Moreover, when σ is small then α^* can be made close to $1/2$.

Proof. Following the propositions and lemmas of section 3:

- Cooperation is the best response to itself in the partnership inclusive game of event h if and only if $h > -K(1 - K) = H_1(K)$.
- Defection is the best response to itself in the partnership inclusive game of event h if and only if $h < \sigma/(1 + K) = H_2(K)$.
- If events occur with high probability ($p \rightsquigarrow 0$), and if most events are concentrated around $h \rightsquigarrow 0$, then the value $K(0)$ can be made arbitrarily close to $-(1 - \sigma)$, thereby making $H_1(K(0))$ arbitrarily close to $(1 - \sigma)/(2 - \sigma)$ and $H_2(K(0))$ to 1.
- Both functions $H_i(K(\alpha))$ increase with α but very slowly since most events are around $h = 0$. Hence $H_1(K(\alpha))$ does not cross the diagonal in $(0, 1/2]$, and $H_2(K(\alpha))$ crosses the diagonal in the interval $[(1 - \sigma)/(2 - \sigma), 1/2]$.
- For all $\alpha \in [(1 - \sigma)/(2 - \sigma), 1/2]$ the simple strategy \mathbf{x}_α is an agent ESS.
- For small values of σ the expression $(1 - \sigma)/(2 - \sigma)$ is close to $1/2$. This ensures that there are agent ESS in which the partners defect in events with h close to $1/2$. In this case an individual is willing to forgo a survival probability of $1/2$ for a low survival probability of σ in order not to be with his partner.

Part 1 follows immediately from the first two points and part 2 from the rest. ■

The proposition demonstrates the symmetry between cooperation and malice, both can start at a low level and be amplified. Here, full cooperation is an ESS, but (in some circumstances) when a small level of aggression establishes itself in the population then each individual will become more aggressive until a level of aggression α^* , or higher, will be achieved.

Malice between partners may begin by competition while altruistic cooperation may start by hedonic cooperation. Although both competition and hedonic situations abound in nature, the first is probably more common. This does not necessarily indicate that malice is to be found more often than positive partnership. In situations of malice it is best for the partners to break the partnership, seek a new partner or new pastures, and generally avoid each other. Although it may not always be possible for the partners to move away from each other, some

³Like in section 3, it can be shown that an agent equilibrium in simple strategies is an ESS (see appendix).

situations of malice will therefore not manifest themselves and will be avoided. On the other hand, in cooperation the partners' bond will be strengthened and they will be inclined to stay together.

4.2 Destructive Dependence

Cooperation may turn out not to be so good for the partners. The partners may encounter an event in which they will cooperate, due to the positive partnership coefficient they established elsewhere, but cooperation may not be the efficient outcome of that event. Their mutual dependence may prove to be destructive by trapping them in action which lowers their expected life span.

Imagine that two partners have established a positive partnership coefficient and they now face a new dangerous encounter. In this encounter they can hide together from the danger (cooperate), in that case they live and die together. In all other cases at most one of them will survive the event. When only one of them escapes (defects) the survival probability of each partner will increase compared to hiding but that of the one remaining in hiding will increase more. The event can be described by the game matrix:

$$\begin{array}{c}
 C \quad D \\
 \begin{array}{|c|c|}
 \hline
 C & v_{11}, v_{11} \quad v_{12}, 0 \\
 \hline
 D & v_{21}, 0 \quad v_{22}, 0 \\
 \hline
 \end{array}
 \end{array}
 \theta$$

where $v_{12} > v_{22} > v_{21} > v_{11} > 0$.

In the partnership inclusive survival game of this event, cooperation is the best response to defection, irrespective of the value of K . Cooperation is the best response to itself when $v_{11} + Kv_{11} > v_{21}$, or:

$$K > \frac{v_{21} - v_{11}}{v_{11}}. \quad (8)$$

Thus when K is sufficiently large cooperation is the only agent equilibrium in this event. Cooperating in this event seems irrational when detached from their long term relationship. They hold on to each other while each could increase his and his partner's chances of surviving this event by escaping, however, since hiding is the only way to survive *together*, they prefer to forgo some survival probability. It is instructive to learn that, for a range of K values, cooperating in this event lowers not only the survival chances in this event but also reduces the expected life span of each of the partners. The conditional life span of a partner when both remaining hiding is: $v_{11}U$, while if they both escape it is $v_{22}S$ (where U, S are the expected longevity of an individual who has a partner or a single individual, respectively, see section 2.1), since $K = (U - S)/S$, the latter is higher the former when:

$$K < \frac{v_{22} - v_{11}}{v_{11}}.$$

Thus when the established partnership coefficient K is between the two values (the left hand side is smaller than the right hand side by our assumption: $v_{21} < v_{22}$)

$$\frac{v_{21} - v_{11}}{v_{11}} < K < \frac{v_{22} - v_{11}}{v_{11}}$$

the partners will cooperate in this event but defecting would have increased their expected life span. Note that for values of K outside this interval, this odd situation is avoided. For lower values of K , cooperation is no longer an equilibrium, while for higher values of K cooperating is a stable equilibrium which also guarantees the highest expected life span.

This situation demonstrates the difference between kinship and partnership, this type of apparently ruinous behavior cannot be caused by kinship alone, kinfolk faced by this event will not hide together since by escaping they can increase their inclusive fitness.

5 Ageing

So far we have taken the basic parameters of the model $\lambda, \mu, \nu, F(\cdot)$ to be constant. This, in general, need not be the case, one obvious situation to consider is ageing, when λ the natural death rate increases with the age of an individual. When two individuals are old and their death is imminent they will provide little help to each other since it is doubtful whether they will both survive to reap the benefits of their sacrifice. When they are both young and aware that old age gradually approaches they will help each other but less than in the absence of ageing, since they know that in the future they will help each other less.

A model taking the change in λ into account is very difficult to solve, particularly since we expect an individual to be familiar with his and his partner's ages. Thus a strategy should depend not only on the individual's own age but also on the age of his current partner. To simplify matters we present here a special case of this model, in which the events consist mainly of hedonic situations with a very small chance of another event occurring and where a single individual cannot find a new partner, and therefore, once single will remain single for the rest of his life.

Let two partners be of the same age and have the same increasing natural death rate $\lambda(t) = \lambda_0 e^{\delta t}$. If left alone because of the partner's death an individual cannot find another partner ($\nu = 0$). Each moment an event may occur with intensity μ . The event is one in which mutual cooperation ensures survival and anything else ensures the death of both:

$$\begin{array}{c} C \quad D \\ C \begin{array}{|c|c|} \hline 1, 1 & 0, 0 \\ \hline 0, 0 & 0, 0 \\ \hline \end{array} \\ D \end{array}, \theta = 0$$

The partners will always cooperate in this event irrespective of their age, here altruism is hedonic, one can only gain by cooperating, cooperation is the unique ESS of this game. This mass of hedonic events creates a positive partnership coefficient, so that each individual is prepared to forgo some of his own survival in order to increase the joint survival. The ratio of sacrifice to gain depends on the current value of K . Assume that, very rarely, an opportunity for such a sacrifice arises in the form of a non-hedonic event in which there is a possibility of making a sacrifice for the other, e.g. an event of the type discussed in section 3. The higher their current partnership coefficient K , the wider the range of such events in which they will cooperate. We show that K approaches 0 with time, so that when young the partners will cooperate in a range of events which shrinks as time goes on.

At each moment in time $K = (U - S) / S$, where U is the expected life span (at time t) when with a partner and S the expected life span when single. The intensity of death is higher when an individual is single, since in the absence of a partner he will certainly die when a hedonic event occurs, thus $U > S$. However, as the intensity of natural death λ increases both U, S approach 0, and since the occurrence of an event becomes insignificant relative to the looming (natural) death, the ratio U/S approaches 1.⁴ Thus the partnership coefficient approaches 0 and the range of cooperation decreases with time.

Ageing has an effect on individuals only if their expected life span is long enough to enable them to reach old age. If individuals die young because of predators or events other than natural death, their strategy will not take into account the decrease of cooperation due to old age (see Hamilton [9]). To show that the ageing effect plays a role in the actions taken by the young we need to show that they continue to live sufficiently long even after introducing ageing. Beginning with a world in which there is no ageing, i.e. the natural death rate λ is a constant ($\delta = 0$), we gradually introduce ageing: δ becomes positive. It can be easily shown that the expected life span of a young individual gradually decreases as δ increases. In formal terms: the derivative with respect to δ of the functions $U, S, U/S$ taken at $\delta = t = 0$, is finite, so that they live sufficiently long for ageing to affect the partners' behavior when they are young.⁵

6 Partnership, Kin Selection and Reciprocation.

6.1 Partnership and Kin Selection.

Differentiating between behavior which is motivated by kinship and one derived from partnership is not an easy task. One reason is that they are likely to appear intermingled, socially interacting kinsmen are likely to develop some degree of mutual dependence, becoming partners ([4], [5]), while partners are often chosen

⁴The differential equations for S, U are: $S' = (\lambda + \mu)S - 1, U' = 2\lambda U - (1 + \lambda S)$. It is straightforward (but somewhat cumbersome) to solve the equations and prove the claimed properties. We omit the calculations.

⁵The simple but lengthy calculations are omitted.

among kinsmen. Another reason is that, as we have shown in this work both types of behavior, kinship and partnership can be similarly described each with the help of a suitable coefficient by inclusive fitness or by partnership inclusive survival.

Partnership cannot be determined by observing only one interaction between the partners, even if it is of a repeated nature. The existence of other interactions and the behavior of the partners in those events is essential in determining the partners behavior in a particular interaction.

Both kinship and partnership may lead an individual to exhibit altruistic behavior by helping another individual at a cost to himself, provided the cost is not too high. However, partnership-altruism qualitatively differs from kin-altruism in three factors:

- While the kinship-coefficient r can take values between 0 and 1 only, the partnership-coefficient K can assume arbitrarily high values or negative values.
- Kin altruism can be described by assigning a fixed weight r to the survival of the kin. Partnership altruism attaches weight to the survival of the other *conditioned* on one's own survival.
- While the kinship-coefficient r of two individuals is fixed for life, the partnership-coefficient K depends on the environmental parameters (λ , μ and ν in our model). For example, K decreases when the exogenously given intensity of death increases, as a special case, K decreases with age.

The following examples demonstrate how kinship and partnership may lead to different behavior in seemingly similar situations.

Consider two individuals interacting while facing a danger of a prisoners' dilemma type:

$$\begin{array}{cc}
 & \begin{array}{cc} C & D \end{array} \\
 \begin{array}{c} C \\ D \end{array} & \begin{array}{|cc|} \hline v_{11} & v_{12} \\ \hline v_{21} & v_{22} \\ \hline \end{array}
 \end{array} \tag{9}$$

where $v_{21} > v_{11} > v_{22} > v_{12}$, and $v_{21} \leq \frac{1}{2}$, and where the payoff v_{ij} describes the survival probability of an individual when he has taken action i and the other action j .

If the two individuals are kinsmen with a kinship coefficient r , cooperation (C) is evolutionarily stable (see Hamilton [8]) when the expected advantage of defecting is smaller than r times the expected harm inflicted on the other, i.e.

$$r > \frac{v_{21} - v_{11}}{v_{11} - v_{12}}. \tag{10}$$

It is crucial for the theory of kin selection that the above condition ensures cooperation in this game matrix irrespective of the interpretation of the game,

the age of the participants and other past or future interactions they have been or will be involved in.

We now consider the same situation between non related *partners*, for this case we need to know their past and future interactions (summarized in the coefficient K) and also the joint survival probabilities. We first take the joint survival probability to be 0, i.e. whatever the partners do, at most one will survive the danger. The partnership matrix is:

$$\begin{array}{c}
 \\
 C \quad \quad D \\
 \begin{array}{|c|c|}
 \hline
 C & v_{11}, 0 \\
 \hline
 D & v_{21}, 0 \\
 \hline
 \end{array}
 \end{array}
 \quad (11)$$

Regardless of the value of the coefficient K , the only globally stable strategy here is defection D . So, whereas kinsmen may cooperate in this situation, pure partners never will. Unlike kinsmen partners are not interested in the other's existence after their own death.

Now consider a different way of adding the joint survival probabilities to the game:

$$\begin{array}{c}
 \\
 C \quad \quad D \\
 \begin{array}{|c|c|}
 \hline
 C & v_{11}, v_{11} \\
 \hline
 D & v_{21}, v_{12} \\
 \hline
 \end{array}
 \end{array}
 \quad (12)$$

Here if both cooperate they will survive and die together, if only one cooperates he has a lower chance of surviving and the defector has a higher chance, if both defect at most one will survive the danger. To illustrate the differences between kinship and partnership assume, in addition, that:

$$\frac{v_{21} - v_{11}}{v_{11} - v_{12}} > 1,$$

this ensures that condition (10) of stability of cooperation among kinsmen cannot be satisfied ($r < 1$).

Given a partnership coefficient K , the condition for evolutionary stability of cooperation is:

$$K > \frac{v_{21} - v_{11}}{v_{11} - v_{12}}, \quad (13)$$

This condition is identical to the condition (10), however, the partnership coefficient K can take any positive value. Thus if the partnership bonds are sufficiently strong the partners will cooperate in this situation as opposed to kinsmen who will not.

The difference between the behavior of kinsmen and partners, particularly since the strength of partnership changes with time, may explain many observed phenomena in human behavior. For example, the well known human tendency to remember in their last will kinsmen rather than life long friends.⁶

⁶ A model incorporating this phenomenon would have to allow individuals to have different relations with different persons. Some would be relatives, others partners. The degree of cooperation will vary with the person and with age.

6.2 Partnership and Reciprocation.

Like kinship altruism, altruism that relies on reciprocation leads to partnership, because once the other individual will provide help in the future (for whatever reason) he becomes important, his mere existence is valuable and cooperation will amplify itself. However, partnership can develop without complex memory which is essential for reciprocation. Partnership may begin with hedonic situations so one needs only identify the individuals with whom one encounters sufficiently many hedonic situations. It therefore becomes important for a potential partner to signal his willingness to cooperate. Unlike in reciprocation, the signal required for partnership is relatively primitive, there is no need to memorize how each individual behaved last period and to reward or punish them accordingly. For partnership it is enough to be aware that the other is a helper (or a menace) and to know the degree of the partnership (K). Thus, partnership may develop among primitive organisms as self restraint in exploiting a limited resource (see Cohen & Eshel [2]) For partnership all that is required is a signal that can be imprinted rather a signal that needs to be continuously checked, it is therefore more primitive and cheap. Once such a signal evolves it is in the interest of the recipient to identify and accept it (Axelrod & Hamilton, [1]). The sender and the receiver of the signal would be able to reach higher levels of cooperation to their mutual benefit. Thus partnership may enhance the evolution of simple signalling which may later expand to more complex signals and lead to reciprocation.

7 Conclusion

Partnership has been defined as the relation established between two or more individuals when it is in their best interest to help each other, since by doing so they increase the probability of being together in the future when, for similar reasons they will continue to help each other. Partnership is inevitable among kinsmen, and it may start among non-relatives when they face hedonic situations. In both cases it is worthwhile helping the other because it will be in the other's best interest, if both survive, to provide help. We have shown that partnership may evolve even without hedonic cooperation.

Kin selection, partnership and reciprocation are likely to appear fused as the causes for altruism. However, we have shown that it may be possible to distinguish between kin altruism and partnership altruism by their different effects in some situations. We have also argued that partnership requires a lesser degree of sophistication than reciprocation. Our work may provide some theoretical ground for field biologists, anthropologists or sociologists in their observations of altruism.

In this work we have concentrated on the *survival* aspect of partnership, since survival and life expectancy are significant factors in biology. However, the concept of partnership can be extended to include general payoffs like welfare, assets, income and power. Helping a partner to increase his income ensures a

more powerful helper in the future. Such a generalization may provide tools for applying partnership to economics and the social sciences.

8 Appendix

In the section 3 we presented an example of a family of encounters for which we have considered, as candidates for equilibrium, only simple strategies in which one cooperates up to a certain encounter and defects for all others. We have also considered only agent equilibria. Here we show that all agent equilibria in simple strategies are necessarily ESS

Consider a family of encounters, parametrized by h , with a given distribution function. The first lemma applies to all such situations, the subsequent lemmas hold for the situations generalizing the example in section 3.

First we observe that if a strategy is a best response to another then it must also be a local best response to it.

Lemma 9 *Let $\mathbf{x}(h)$ be a best response global strategy to $\mathbf{y}(h)$ then $\mathbf{x}(h)$ is a local best response to $\mathbf{y}(h)$.*

Proof. The proof is straightforward, and we give here only a sketch of it. If there is an interval of events for which $\mathbf{x}(h)$ was not the local best response to $\mathbf{y}(h)$, then \mathbf{x} can be corrected on a small set of events to the local best response. This will improve the expected lifetime of the player on this set of events and therefore in all other events. ■

We define a family of events which generalizes in an intuitive way our additive example in section 3. The encounters of the family are ordered according to a parameter h , so that as h increases the benefits of cooperating are become smaller both for the player's survival and for the joint survival of the partners.

We consider families of events ordered by a single parameter h , an event is described by the following survival probabilities (which are functions of h) :

$$\left. \begin{array}{|c|c|} \hline v_{11}, w_{11} & v_{12}, w_{12} \\ \hline v_{21}, w_{12} & v_{22}, w_{22} \\ \hline \end{array} \right\} \theta.$$

Assumptions:

1. The gains of cooperating decrease with h , i.e. the following differences *decrease* with h :

$$v_{11} - v_{21}, \quad v_{12} - v_{22}, \quad w_{11} - w_{12}, \quad w_{12} - w_{22}$$

2. For each h , the gains to the player and the gains to the joint survival from cooperating (rather than defecting) against a cooperator

are higher than cooperating against a defector, i.e.

$$\begin{aligned} v_{11} - v_{21} &> v_{12} - v_{22} \\ w_{11} - w_{12} &> w_{12} - w_{22}. \end{aligned}$$

It follows immediately from these assumptions that if for some positive K and some event h cooperating is the the best response (in the inclusive survival game) against a pure strategy X then it is also the best response to X in events with lower h . Also, if for some h , the strategy C is a best response to D , then for all smaller h 's the strategy C is a best response to C .

It is straightforward to see that the additive example of section 3 satisfies the assumptions and that these ensure that $H_2(K) < H_1(K)$, and that C is the best response to D for events h below $H_2(K)$ and C is the best response to C for $h < H_1(K)$.

Lemma 10 *Let \mathbf{x}_δ be the simple strategy of cooperating up to the event δ . Let \mathbf{y} be a best response to \mathbf{x}_δ , then \mathbf{y} is a simple strategy.*

Proof. From lemma 9, \mathbf{y} is a local best response. If \mathbf{y} is not a simple strategy then there exist $h_1 < h_2$ such that $\mathbf{y}(h_1) = 0, \mathbf{y}(h_2) = 1$. There could be 3 cases concerning the location of δ relative to h_1, h_2 . If $\delta < h_1 < h_2$ then the local best response to defection at h_2 is to cooperate, hence by our assumption the best response to defection at h_1 should also be cooperation and it cannot be the case that $\mathbf{y}(h_1) = 0$. A similar argument holds for the case $h_1 < h_2 < \delta$. When δ is between the two values $h_1 < \delta < h_2$, the best response to defect at h_2 is to cooperate and hence by our assumption C is the best response to C at a lower event h_1 , so that $\mathbf{y}(h_1)$ should be 1. ■

The next lemma proves that if \mathbf{x}_δ is an agent equilibrium then it is a strict equilibrium and hence an ESS.

Lemma 11 *Let \mathbf{x}_δ be an agent equilibrium then \mathbf{x}_δ is the best response to itself.*

Proof. Let \mathbf{y} be a best response to \mathbf{x}_δ , then by lemma 10 it is a simple strategy $\mathbf{y} = \mathbf{x}_\alpha$. We now show that $\alpha = \delta$.

Let $\alpha < \delta$, since \mathbf{x}_α is a best response it is (by lemma 9) a local best response hence at $h = \alpha$ the best response to \mathbf{x}_δ 's cooperation is both to defect and cooperate, i.e.

$$v_{11} + K(x_\alpha, x_\delta)w_{11} - v_{21} - K(x_\alpha, x_\delta)w_{21} = 0 \quad (14)$$

At $h = \delta$ the best response to cooperation is to cooperate and the best response to defection is to defect, hence:

$$v_{11} + K(x_\delta, x_\delta)w_{11} - v_{21} - K(x_\delta, x_\delta)w_{21} \geq 0. \quad (15)$$

Recall that v_{11}, w_{11} etc. are functions of h .

We show that this leads to a contradiction, since as a function of h between α, δ the left hand side cannot increase from equality to inequality. To show it we need to write $K(x_h, x_\delta)$ explicitly. The numerator of $K(x_h, x_\delta)$, for $h \leq \delta$, is:

$$\begin{aligned} & \left[\int_0^h v_{11} dF(v) + \int_h^\delta v_{21} dF(v) + \int_\delta^\infty v_{22} dF(v) - \int_0^\infty \theta dF(v) \right] \\ = & \left[\int_0^\delta v_{11} dF(v) + \int_\delta^\infty v_{22} dF(v) - \int_0^\infty \theta dF(v) + \int_h^\delta (v_{21} - v_{11}) dF(v) \right] \\ = & \left[T + \int_h^\delta (v_{21} - v_{11}) dF(v) \right] \end{aligned}$$

where

$$T = \int_0^\delta v_{11} dF(v) + \int_\delta^\infty v_{22} dF(v) - \int_0^\infty \theta dF(v).$$

The denominator of $K(x_h, x_\delta)$ is:

$$\begin{aligned} & 1 + p - \left[\int_0^h w_{11} dF(v) + \int_h^\delta w_{21} dF(v) + \int_\delta^\infty w_{22} dF(v) \right] \\ = & 1 + p - \left[\int_0^\delta w_{11} dF(v) + \int_\delta^\infty w_{22} dF(v) \right] - p \int_h^\delta (w_{21} - w_{11}) dF(v) \\ = & 1 + p - B - \int_h^\delta (w_{21} - w_{11}) dF(v). \end{aligned}$$

where

$$B = \int_0^\delta w_{11} dF(v) + \int_\delta^\infty w_{22} dF(v).$$

Substituting the explicit value of K in the left hand side of 14 and multiplying by the denominator of K , the following expression should not decrease as a function of h :

$$\begin{aligned} & (v_{11} - v_{21}) \left[1 + p - B - \int_h^\delta (w_{21} - w_{11}) dF(v) \right] \\ & - (w_{21} - w_{11}) \left[T + \int_h^\delta (v_{21} - v_{11}) dF(v) \right]. \end{aligned}$$

However, we show that its derivative is negative and hence obtaining a contradic-

tion. The derivative is

$$\begin{aligned} & (v_{11} - v_{21})'_h \left[1 + p - B - \int_h^\delta (w_{21} - w_{11}) dF(v) \right] \\ & - (w_{21} - w_{11})'_h \left[T + \int_h^\delta (v_{21} - v_{11}) dF(v) \right] \\ & + (v_{11} - v_{21})(w_{21} - w_{11})f(h) \\ & + (w_{21} - w_{11})(v_{21} - v_{11})f(h) \end{aligned}$$

the last two terms cancel, and we are left with:

$$\begin{aligned} & (v_{11} - v_{21})'_h \left[1 + p - B - \int_h^\delta (w_{21} - w_{11}) dF(v) \right] \\ & - (w_{21} - w_{11})'_h \left[T + \int_h^\delta (v_{21} - v_{11}) dF(v) \right]. \end{aligned}$$

By our assumption on the family of games this expression is negative and hence α cannot be less than δ .

By a completely analogue method it can be shown that α cannot be greater than δ . Hence $\alpha = \delta$. ■

We have shown that when \mathbf{x}_δ is an agent equilibrium then the *only* best response to \mathbf{x}_δ is \mathbf{x}_δ itself. Hence \mathbf{x}_δ is a strict equilibrium and hence an ESS.

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