

# Contractual Remedies to the Holdup Problem: A Dynamic Perspective

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## Abstract

An important theme of modern contract theory is the role contracts play to protect parties from the risk of holdup and thereby encouraging their relationship specific investments. While this perspective has generated valuable insights about various contracts, the underlying models abstract from realistic investment dynamics. We reexamine the role of contracts in a dynamic model that endogenizes the timing of investments and trade. The resulting interaction between bargaining and investment significantly alters the insights learned from static models. We show that contracts that would exacerbate the parties' vulnerability to holdup — rather than those protecting them from the risk of holdup — can be desirable. For this reason, separate ownership of complementary assets can be optimal, an exclusivity agreement can protect the investments of its recipient, trade contracts can be beneficial with a purely cooperative investment, and the property rule can offer a better legal protection against externalities loss than the liability rule, much in contrast to the existing results (based on static models).

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# 1 Introduction

The holdup problem arises when a business partner makes relationship specific investments that are susceptible to ex post expropriation by his associates. Examples of such investments include acquisition of firm specific skills by workers, subcontractors' efforts to customize their parts to the special needs of manufacturers, and a firm's relocation of its plant adjacent to its partner. These investments create more surplus within a relationship than without. Hence, absent any special safeguard, the fear of holdup may lead the parties to underinvest relative to the efficient level. An important theme of the modern contract literature is how contracts can protect the investors from the risk of holdup.<sup>1</sup>

While this literature has generated valuable insights about various contracts, the underlying models abstract from the realistic investment dynamics present in many business relationships. Specifically, the extant models assume that the partners can invest only once, and only at a certain time. In practice, however, the timing of investment and bargaining is — at least to some extent — chosen endogenously by the parties, and the investment and bargaining stages are often intertwined. For instance, the department of defense may negotiate to order a weapons system from a sole-source defense contractor based on a current R & D knowledge, or it may decide to wait until the latter invests more on R & D and develops a better technology. A similar dynamic interaction of investment and bargaining arises in the development of new building construction, advertising pilots or software projects.<sup>2</sup> We develop a model that allows for the timing of investment and trade to be endogenous. Specifically, in our model, an investing

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<sup>1</sup>A range of organizational and contract forms have been rationalized as safeguards against holdup: Examples include vertical integration (Klein, Crawford and Alchian, 1978; Williamson, 1979), a property rights allocation (Grossman and Hart, 1986; Hart and Moore, 1990), contracting on renegotiation rights (Chung, 1991; Aghion, Dewatripont and Rey, 1994), option contracts (Nöldeke and Schmidt, 1995), trade contracts (Edlin and Reichelstein, 1996), legal rules for assigning property and contract entitlements and the remedies for their breach (Che and Chung, 1999; Rogerson, 1992; Bebchuk, 2001,2002) and injecting market competition (MacLeod and Malcolmson, 1993; Cole, Mailath and Postlewaite, 2001; Felli and Roberts, 2001; Che and Gale, 2003). See Tirole (1999) for the summary of the recent debate on whether inefficiencies can be addressed by contracts and for a general critique on the incomplete contracts paradigm.

<sup>2</sup>Such dynamic interaction is also present in the publishing of academic articles. Consider the editorial procedure at the Berkeley Electronic Journals. Here submission is simultaneous to four vertically differentiated journals. Unless the initial submission is rejected, in principle the author is offered a choice of acceptance at a lower level or following a substantial revision — incremental investment after the negotiation has commenced — acceptance at a higher level.

party accumulates specific investments in each period until either the parties agree to trade or the negotiation breaks down.

The trade decision is negotiated in the shadow of a contract signed at the outset — before investment begins. We adopt a (slight modification of the) bargaining model, first suggested by Binmore, Rubinstein and Wolinsky (1986), in which an exogenous breakdown occurs with some probability,  $1 - \delta \in (0, 1)$  in each period, following the rejection of an offer. If bargaining breaks down, the contract in place takes effect, determining the payoffs the parties collect. An attractive feature of this dynamic, non-cooperative bargaining model is that (for a fixed level of investment) its unique subgame-perfect equilibrium replicates a (generalized) Nash Bargaining Solution (NBS) with the contract serving as its threat point — in the limit as the breakdown probability vanishes.<sup>3</sup> Since the majority of the contract literature adopts the NBS as the solution concept for bargaining and views a contract as determining its threat point,<sup>4</sup> our model serves as a natural dynamic extension of these models that endogenizes the timing of investment and trade decisions. For this reason, the equilibrium results of our model are easily comparable to those in much of the contract literature.

In Che and Sákovics (2004) we studied such a dynamic holdup model, but *without* the possibility of breakdown – and therefore, effectively *in the absence of ex ante contracts*. We showed that the investment dynamics itself can be sufficient to provide incentives for efficient investments — as the parties' common discount factor tends to 1 — provided that a certain individual rationality constraint is satisfied. If the latter condition fails, however, inefficiencies were shown to be unavoidable. Since the condition may fail in reasonable circumstances, the results of Che and Sákovics (2004) need not vitiate the role of contracting in alleviating inefficiencies. The results raise questions, though, as to under what circumstances contracts may be valuable and how they should be designed, if dynamics is to be taken seriously. The current paper focuses on these issues, by studying and comparing the effects of alternative contracts in our dynamic model. Since the causes of inefficiencies differ in our model from those in the static one, the

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<sup>3</sup>Binmore, Rubinstein and Wolinsky (1986), take the limit as the breakdown probability tends to zero, in order to eliminate the first-mover advantage and to show the full equivalence with the NBS. Instead, we assume that in each bargaining round the proposer is chosen randomly, what — for risk-neutral players — yields the same qualitative effect.

<sup>4</sup>Examples include Grossman and Hart (1986), Hart and Moore (1990), Edlin and Reichelstein (1996), Che and Hausch (1999), Hart and Moore (1999), Segal (1999), and Segal and Whinston (2000, 2002). The alternative approach treats contracts as affecting the outside option payoffs of bargaining (MacLeod and Malcomson, 1993; Chiu, 1998; De Meza and Lockwood, 1998).

implications on the effects of contracts will be seen to be different as well.

While the original authors of Transaction Cost Analysis (hereafter, TCA) seem primarily concerned about the inappropriability of *absolute* investment returns due to specificity (see Klein et al. (1978) and Williamson (1979)), the recent contract literature focuses on the inappropriability of returns *at the margin*. More precisely, the latter literature focuses on the effects a contract (or an organizational form) has on the parties' status quo payoffs induced by that contract, in particular in terms of how these payoffs vary with investment. If an investor's contract-induced status quo payoff increases with her specific investment, it effectively reduces the specificity of her investment *at the margin*, thus reducing the exposure of her marginal investment return to expropriation. Such a contract thus improves her incentives to make the specific investment. This view can be used to rank alternative contracts: If a contract reduces the investor's marginal specificity exposure more than another contract, the former contract will protect her investment return better than the latter, thus inducing a higher level of investment. The celebrated prescription of Grossman-Hart-Moore (GHM) that complementary assets should be owned together, can be understood in this light. The common ownership of assets lowers the owner's marginal specificity exposure at the expense of raising the non-owner's. If complementarity of assets were to mean, as was assumed by GHM, that the former's gain exceeds the latter's loss, then common ownership would result in better aggregate incentives than separate ownerships.

By the same token, however, if two contracts entail precisely the same *marginal* specificity, their effect on investment incentives will be precisely the same, even when they differ in terms of the investors' exposure to *absolute* specificity that they induce. In this sense, the absolute degree of specificity investors are exposed to does not matter in the existing models. This is not the case in our dynamic model, however, if  $\delta$  is close to 1 (the probability of breakdown, is sufficiently low). In particular, the larger is the absolute degree of specificity, meaning larger losses a contract were to impose on the parties upon failing to agree, the more severe dynamic threat it makes credible, so the higher investment can be sustained.

This new insight will be shown to generate several surprising implications for contract design. In particular, the contracts that would increase the parties' vulnerability to the risk of holdup, rather than those protecting them from the risk of holdup, can be desirable. Specifically, if the investment specificity is invariant at the margin (which will be the case if no contract or organization can affect the specificity), then separate ownership of strictly complementary assets

is optimal, exclusivity agreement can protect investments of its recipient, trade contracts can be beneficial even when the investment is purely cooperative, and the property rule can offer a better legal protection against externalities loss than the liability rule, all in contrast to the existing results based on static models.

Several papers have developed somewhat similar implications, though in differing modelling contexts. Halonen (2002) shows in a repeated-game model that joint ownership of an asset strictly dominates single ownership for intermediate values of players' discount factor,  $\delta$ , since the former can make the repeated game punishment more severe. Baker et al. (2001, 2002) also demonstrate that the absolute payoff levels can affect the efficiency ranking of different ownership structures in a repeated trade setting. The repeated trade opportunities assumed in these papers make the folk theorem of repeated games applicable, which implies that an efficient outcome is sustainable as  $\delta \rightarrow 1$ , irrespective of the underlying organizational arrangements. In this sense, the organizational issues become irrelevant for a sufficiently large  $\delta$  in these papers. By contrast, the parties have one-shot trading opportunity in our model (just as in the standard holdup problem), which makes the folk theorem inapplicable. Indeed, the organizational issues remain relevant when  $\delta \approx 1$  in our model. Matouschek (2004) studies the effects of ex ante contracts on the ex post trading (in)efficiencies when the parties have two-sided asymmetric information (*à la* Myerson-Satterthwaite), but have no ex ante investments. Contracts inducing low disagreement payoffs can facilitate efficient agreement (but prove more costly when agreement fails).

The rest of the paper is organized as follows. Section 2 describes the model, establishes several benchmarks and reviews the results of the (existing) static contract models. Section 3 analyzes the equilibria in our dynamic contract model and establishes the effects of contracts. Section 4 then applies the results to several well-known contracts/organizations. Section 5 concludes.

## 2 The model

### 2.1 Description of the model

Two risk-neutral parties, 1 and 2, can create a surplus from trade. The joint surplus from trade at a given date increases with the aggregate investment agent 1 (“investor”) has accumulated. If the parties agree on a trade decision,  $q \in Q$ , with the cumulative investment,  $k \in K$ , by agent 1 (measured in the cost of investment), then it creates the joint surplus of  $\phi(q, k)$  (gross of the

investment cost). The set of feasible trades,  $Q$ , is a compact subset of  $\mathbb{R}^m$  for some  $m \in \mathbb{N}$ , which includes a no trade  $q_0$  as an element. We assume that  $K = [0, \bar{k}]$  for some  $\bar{k} > 0$ .<sup>5</sup> We assume that  $\phi(q_0, \cdot) \equiv 0$ ; i.e., if there is no trade, then no surplus is realized. For reasons that will become clear, it is useful to focus on the efficient trade decision conditional on the level of investment,

$$\phi(k) := \max_{q \in Q} \phi(q, k),$$

which we assume is well defined. Further, we assume that the efficient surplus function,  $\phi(\cdot)$ , is strictly increasing, strictly concave and differentiable.

The current model of trading relationship applies to a broad set of circumstances. The trading decision may involve the production and delivery of a good by a seller to a buyer. It could involve joint team production between partners. There may be no direct relationship between the parties; instead, the relationship may involve decisions by a party that could affect the externalities she confers to the other. For instance, party 1 (e.g., coal mine) may be in a position to create pollution that harms party 2 (e.g., laundry). In that case, the surplus,  $\phi(k)$ , would represent the amount of harm reduced by party 1's precaution,  $k$ .

Investments and trade can occur at any discrete period,  $t = 1, 2, \dots$ , in the shadow of a contract,  $m \in \mathcal{M}$ , signed at period  $t = 0$  (i.e., prior to any investments and trade), for some arbitrary set of contracts  $\mathcal{M}$ . The parties are free to negotiate for a trade at any period, and in the process may decide to renegotiate the initial contract,  $m$ . Yet, the contract in place may affect the outcome of negotiation. Just how the contract affects the outcome of negotiation raises a modelling issue. As motivated in the Introduction, we adopt the so-called 'internal option' approach, which applies the NBS to describe the parties' negotiation behavior and views the contract in place as the threat point of the NBS. To capture the dynamic interaction between investment and bargaining, though, it is important that the NBS be modelled via an explicit extensive form that allows for an investment dynamics. This can be done, as suggested by Binmore, Rubinstein and Wolinsky (1986), by explicitly introducing the possibility of bargaining breakdown as part of the extensive form.

Our extensive form is described as follows. A contract  $m \in \mathcal{M}$  signed at  $t = 0$ . Each period  $t \geq 1$  (if it is reached) consists of two stages: first investment and then bargaining. In the investment stage, party 1 makes a non-negative incremental investment, which adds to the existing stock. Once it is made, the investment is sunk and it becomes public (though not

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<sup>5</sup>All of our results remain unchanged, although they become cumbersome to present, if  $K$  is discrete.

verifiable). In the bargaining stage, a party is chosen randomly<sup>6</sup> to offer to his partner a share of the surplus that would result from trade at that point. We assume that party 1 is chosen with probability  $\alpha \in (0, 1)$ , and party 2 is chosen with the remaining probability,  $1 - \alpha$ . If the offer is accepted, then trade takes place, the surplus is split according to the agreed-upon shares between the two parties, and the game ends. If the offer is rejected, then, with probability  $\delta \in [0, 1)$ , the game moves to the next period without trade, and the same process is repeated; i.e., party 1 can make an incremental investment, which is followed by a new bargaining round with a random proposer. With probability  $1 - \delta$ , bargaining breaks down irrevocably, and the two parties collect their *status quo payoffs*,  $\psi_1(k; m)$  and  $\psi_2(k; m)$ , respectively, when the cumulative investment up to that point is  $k$ . As will be seen, these status quo payoffs play the role of the threat point of NBS in the equilibrium that we study.

This model provides a natural dynamic extension of the majority of contract models based on the NBS specification. In particular, it nests those models as a special case that arises when  $\delta = 0$  (i.e., when breakdown is certain following disagreement). In this case, the parties will realize the payoffs of  $\psi_1(k; m)$  and  $\psi_2(k; m)$  given contract  $m$ , unless they agree to trade in the first period. Our model can also accommodate a variety of contracts/organizations, depending on the form that status quo payoffs are allowed to take. In particular, several well-known contract problems are included in our model.<sup>7</sup>

**Example 1 (*Ownership contracts*)** *The asset ownership model of Grossman-Hart-Moore can be represented with  $m$  denoting a partition of total physical assets,  $\mathcal{A}$ , into assets owned by party 1,  $A_1^m$ , and the assets owned by party 2,  $A_2^m$ , and  $\mathcal{M}$  denoting the set of all such partitions. Suppose that, given allocation  $m \in \mathcal{M}$ , the two parties realize the surplus of  $\pi_1(k; A_1^m)$  and  $\pi_2(k; A_2^m)$  from individual production, respectively, when their team production fails. This model is subsumed in our model with  $\psi_1(k; m) := \pi_1(k; A_1^m)$  and  $\psi_2(k; m) := \pi_2(k; A_2^m)$ .*

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<sup>6</sup>This is different from the model of Binmore, Rubinstein and Wolinsky (1986), but it allows for a convenient parametrization of “bargaining power”, without losing the equivalence — for risk-neutral players — with the (asymmetric) NBS.

<sup>7</sup>In principle, a more sophisticated contract, e.g., one requiring exchanges of messages, can be incorporated into our model, with  $\psi_i$  interpreted as the equilibrium payoff of party  $i$  in that contract (sub)game. Of course, there is the issue of how these latter payoffs are determined and what status quo payoffs are feasible. These are difficult questions to address even in the static model (as is well known from the debates on the incomplete contract paradigm). The more complex extensive form makes our dynamic model even less suitable for analyzing, let alone likely to offer any new insight on, these problems. For these reasons, we do not address the question of optimal contracts, limiting attention instead to several well known contracts and organization forms.

**Example 2 (Trade contracts)** Suppose that the parties are the seller (party 1) and the buyer (party 2) of a good. Given investment  $k$ , trading  $q \in [0, \bar{q}] =: Q$  units of the good generates a gross surplus of  $v(q, k)$  to the buyer but requires the seller to incur the production cost of  $c(q, k)$ . Assume  $v(q, k) - c(q, k) > 0$  for some  $(q, k) \in Q \times K$ , and  $v(0, k) = c(0, k) = 0$ .<sup>8</sup> Prior to the investment, the parties sign a contract requiring the exchange of  $\hat{q} \in Q$  units for the price of  $\hat{t} \in \mathbb{R}$ . Edlin and Reichelstein (1996) studied such a contract, which can be indexed by  $m = (\hat{q}, \hat{t})$ . This model conforms to our model with  $\phi(q, k) = v(q, k) - c(q, k)$ ,  $\psi_1(k; \hat{q}, \hat{t}) = \hat{t} - c(\hat{q}, k)$ , and  $\psi_2(k; \hat{q}, \hat{t}) = v(\hat{q}, k) - \hat{t}$ .

**Example 3 (Exclusivity contracts)** Suppose that party  $i = 1, 2$  has the outside opportunity to trade with a third party and realize  $\pi_i^O > 0$ , respectively. Note that these payoffs do not depend on  $k$  when investments are not transferable to the external trades, a case considered by Segal and Whinston (2000). Suppose that at  $t = 0$  the parties can sign an exclusivity agreement that would prohibit such external trades. Segal and Whinston studied whether such a contract can protect returns to specific investments. Our model can be used to reexamine this issue by setting  $\mathcal{M} = \{EX, NEX\}$  and  $\psi_1(k; EX) = \psi_2(k; EX) = 0$  and  $\psi_1(k; NEX) \equiv \pi_1^O, \psi_2(k; NEX) \equiv \pi_2^O$ .

**Example 4 (Legal rules for assigning entitlements and the remedies for their violation)** Suppose a coal mine (party 1) produces soot that causes problems to a laundry (party 2) nearby. The harm inflicted on the laundry depends on both the mine's ex post operation scale  $q \in Q \subset \mathbb{R}_+$ , which is verifiable, and its ex ante preventive investment  $k$ , which is unverifiable. Let  $\pi_1(q)$  be the profit of the coal mine when it operates at the scale of  $q$  and let  $\pi_2$  be the laundry's profit, excluding the harm. The harm to the laundry is  $h(q, k)$  when the coal mine operates. Bebchuk (2001, 2002) studied alternative legal regimes with such a model. The relevant ones are  $\mathcal{M} = \{P1, P2, L\}$ , where  $P1$  is a legal regime in which party 1 enjoys the right to pollute with impunity (and thus chooses  $q$ );  $P2$  is a regime in which party 2 has the right to be free from pollution (and thus chooses  $q$ ); and  $L$  is a regime in which party 1 can pollute but must pay damages of  $h(q, k)$  to party 2. These alternative regimes can be studied in our framework, with  $\phi(q, k) := \pi_1(q) + \pi_2 - h(q, k)$ , and  $\psi_1(k; m)$  and  $\psi_2(k; m)$  for  $\mathcal{M} = \{P1, P2, L\}$  set appropriately (see Subsection 4.4).

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<sup>8</sup>It is also useful for a later purpose to define the investment to be *purely selfish* if  $v_k \equiv 0$  and  $c_k < 0$  (that is, the investor is the direct beneficiary of her investment) and *purely cooperative* if  $c_k \equiv 0$  and  $v_k > 0$  (i.e., the investor's partner is the direct beneficiary of the investment).

As Example 3 illustrates, the absence of a contract is a special case of our model. Clearly, the status quo payoffs are not necessarily lower in such a case, particularly when compared with an exclusivity contract.<sup>9</sup> Throughout, we make the following assumptions, which are natural with the above examples.

**Assumption 1 (*Regularity*)** For each  $m \in \mathcal{M}$ , (a)  $\psi_i(\cdot; m)$  is nonnegative, nondecreasing and twice differentiable; (b)  $\phi(\cdot) - z\psi_2(\cdot; m)$  and  $\psi_1(\cdot; m)$  are strictly concave for all  $z \in [0, 1]$ ;<sup>10</sup> and (c)  $\psi_1(k; m) - k$  is strictly decreasing in  $k$ .

**Assumption 2 (*Specificity*)** For each  $m \in \mathcal{M}$  and  $k \in K$ ,  $\phi(k) - \psi_1(k; m) - \psi_2(k; m)$  is strictly positive and nondecreasing in  $k$ .

Assumption 1 implies that parties' contract-induced disagreement payoffs are nondecreasing in their investments and satisfy other technical conditions. Assumption 1-(c) implies that party 1 will choose no investment unless there is an internal trade between the partners. It simplifies our equilibrium characterization. Assumption 2 means that the parties' investments are specific in the sense that they generate higher total surplus when the parties trade efficiently than when they disagree, both in the *absolute* and *marginal* senses. These assumptions are sensible in the context of the above examples. For instance, the disagreement outcome under any ownership may be inefficient since non-owners may not exert sufficient human capital input (as is often assumed in GHM); a trading contract may not specify the efficient trade level (as with Edlin and Reichelstein (1996) and Che and Hausch (1999)); and external trading does not yield as much surplus as the efficient internal trading both under exclusive and nonexclusive regimes (see Segal and Whinston (2000)). Some applications will require relaxing Assumptions 1(c) and Assumption 2, in which case a remark will be made to note the implication of the relaxation.

Assumption 2 implies that the disagreement outcome can never be efficient, so the social optimum can be characterized independently of the contract in place. The first-best is achieved

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<sup>9</sup>In Che and Sákovics (2004), the status quo payoffs are assumed to be zero. But this is just a normalization, and the reader should not interpret the current paper as assuming that the status quo payoffs will indeed rise as the parties sign some ex ante contract. As the exclusivity example illustrates, a contract may increase or reduce the parties' status quo payoffs.

<sup>10</sup>A sufficient condition is that both  $\phi$  and  $\psi_2$  are concave but the former is more concave; i.e.,  $\phi''(\cdot) \leq \psi_2''(\cdot; m)$ . Existing models often assume that  $\psi_2'(\cdot; m) \equiv 0$ , in which case the assumption follows from the strict concavity of  $\phi$ .

when the seller chooses

$$k^* = \arg \max_{k \in K} \phi(k) - k,$$

and the parties trade immediately to realize  $\phi(k^*)$ . We assume that  $k^* \in (0, \bar{k})$ , which implies that  $k^*$  satisfies  $\phi'(k^*) = 1$ .

## 2.2 Benchmark outcomes

Before proceeding, it is useful to establish the equilibrium outcome of the static game as a benchmark, and replicate the main insight from the traditional contract literature in our model. We do so in a way that will facilitate comparison with the subsequent dynamic results. Consider the following payoff function of party 1 under contract  $m$ :

$$U_\delta(k; m) := \alpha[\phi(k) - (1 - \delta)\psi_2(k; m)] + (1 - \alpha)(1 - \delta)\psi_1(k; m) - (1 - (1 - \alpha)\delta)k, \quad (1)$$

and let  $k_\delta(m) := \arg \max_{k \in K} U_\delta(k; m)$  denote its unique maximizer, which is well defined by Assumption 1(b).

It will be seen later that the highest implementable investment level by party 1 maximizes  $U_\delta(k; m)$ . For now, it is useful to observe that the payoff function corresponds to two important benchmarks when  $\delta$  takes extreme values. First, if  $\delta = 1$ , then  $U_\delta(k; m) = \alpha[\phi(k) - k]$ , the (re-scaled) social objective function. Hence,  $k_1(m) = k^*$  regardless of the contract  $m$  in place. Next, if  $\delta = 0$ , so that the game ends after the first period, then  $U_\delta(k; m) = \alpha[\phi(k) - \psi_2(k; m)] + (1 - \alpha)\psi_1(k; m) - k$ . This is precisely party 1's equilibrium payoff in the (standard) static model, where all the investment is done at once, followed by (asymmetric) Nash bargaining.<sup>11</sup> Hence, its maximizer,  $k_0(m)$ , represents the equilibrium investment induced by contract  $m$  in the static model. Finally, we can observe how  $k_\delta(m)$  varies with  $\delta$ .

**Lemma 1** *Given Assumption 1(a)-(b),  $k_\delta(m) \leq k_{\delta'}(m)$  for  $\delta' > \delta$ ,  $k_\delta(m) \rightarrow k_0(m)$  as  $\delta \rightarrow 0$  and  $k_\delta(m) \rightarrow k^*$  as  $\delta \rightarrow 1$ . If Assumption 1(c) holds as well, then  $k_\delta(m) < k_{\delta'}(m)$  for  $\delta' > \delta$ , whenever  $k_{\delta'} > 0$ .*

*Proof.* See the Appendix.

As mentioned in the Introduction, in the static models, the value of contracts lies in reducing the investor's marginal specificity exposure. For instance, if contract  $m$  causes party 1's status

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<sup>11</sup>In the special case when  $\alpha = \frac{1}{2}$ , it coincides with the NBS.

quo payoff,  $\psi_1(k; m)$ , to increase strictly with  $k$ , then his exposure to holdup at the margin,  $\phi'(k) - \psi'_1(k; m)$ , will be reduced (compared with the no contract case). It is thus useful to characterize how contracts compare in terms of the extent to which they reduce each party's marginal specificity exposure. Let us write  $\psi_i(k; m) = \int_0^k \psi'_i(\tilde{k}; m) d\tilde{k} + \bar{\psi}_i(m)$ , where  $\bar{\psi}_i(m) := \psi_i(0; m)$  for  $i = 1, 2$ .

**Definition 1** (a) A contract  $m'$  is (weakly) **M-specificity reducing on  $m$  for party  $i = 1, 2$** , or  $m' \succ_i^M (\succeq_i^M) m$ , if  $\psi'_i(k; m') > (\geq) \psi'_i(k; m)$  for all  $k \in K$ .

(b) Contracts  $m$  and  $m'$  are **M-specificity invariant for party  $i = 1, 2$** , or  $m \sim_i^M m'$  if  $m \succeq_i^M m'$  and  $m \preceq_i^M m'$ . The contracts are **M-specificity invariant**, or  $m \sim^M m'$ , if  $m \sim_i^M m'$  for  $i = 1, 2$ .

If contracts  $m$  and  $m'$  are M-specificity invariant, then their associated status quo payoffs differ only in absolute magnitude, i.e.,  $\psi_i(k; m') - \psi_i(k; m) = \bar{\psi}_i(m') - \bar{\psi}_i(m)$  for all  $k \in K$ . In the static setting, the investment levels are partially characterized by the order on M-specificity, as is shown next.

**Proposition 1** In the static model ( $\delta = 0$ ), if  $m' \succeq_1^M m$  and  $m' \preceq_2^M m$ , then  $k_0(m') \geq k_0(m)$ . The inequality is strict if (in addition)  $k_0(m') > 0$  and if either  $m' \succ_1^M m$  or  $m \succ_2^M m'$ .

*Proof.* Observe that

$$U'_0(k; m') - U'_0(k; m) = (1 - \alpha)[\psi'_1(k; m') - \psi'_1(k; m)] - \alpha[\psi'_2(k; m') - \psi'_2(k; m)] \geq 0,$$

if  $m' \succeq_1^M m$  and  $m' \preceq_2^M m$ . Since the payoff function has a unique maximizer, given Assumption 1, the inequality implies that  $k_0(m') \geq k_0(m)$ . Further, the inequality is strict if, in addition,  $k_0(m') > 0$  and if  $m' \succ_1^M m$ . The latter condition implies that

$$U'_0(k_0(m'); m) = \alpha[\psi'_2(k_0(m'); m') - \psi'_2(k_0(m'); m)] - (1 - \alpha)[\psi'_1(k_0(m'); m') - \psi'_1(k_0(m'); m)] < 0.$$

Since  $k_0(m') > 0$ , then  $k_0(m) \neq k_0(m')$ , implying that  $k_0(m') > k_0(m)$ . ■

This result shows that a shift from a contract,  $m$ , to another,  $m'$ , improves party 1's incentive for investment, if the shift lowers her marginal specificity exposure (i.e.,  $\phi'(k) - \psi'_1(k; m') \leq \phi'(k) - \psi'_1(k; m)$ ) and raises party 2 (= non-investor)'s. This insight underlies the GHM's asset

ownership prescriptions, as mentioned before.<sup>12</sup> Meanwhile, if two contracts are M-specificity invariant, they yield the same incentive for the investment.

**Corollary 1** *In the static model, M-specificity invariant contracts lead to the same investment: if  $m \sim^M m'$ , then  $k_0(m') = k_0(m)$ .*

In other words, any differences in absolute specificity do not matter. This result underlies Segal and Whinston (2000)'s claim of irrelevance of exclusivity: if investments are not transferable to external trades ( $\psi'_1(k; m) \equiv 0$ ), then any two contracts are M-specificity invariant and therefore whether the parties have an exclusivity agreement does not affect the incentives for specific investment (since it affects the status quo payoffs only in the *absolute* magnitude).

It will be seen later that absolute specificity does matter in our dynamic model. Specifically, the aggregate level of specificity  $\phi(0) - \bar{\psi}_1(m) - \bar{\psi}_2(m)$  may affect the level of sustainable investment. It is thus useful to compare contracts by the associated level of absolute specificity.

**Definition 2** *Contract  $m'$  is (weakly) **A-specificity reducing** on contract  $m$ , or  $m' \succ^A (\succeq^A) m$ , if  $\bar{\psi}_1(m') + \bar{\psi}_2(m') > (\geq) \bar{\psi}_1(m) + \bar{\psi}_2(m)$ .*

### 3 A General Analysis of the Effects of Contracts

#### 3.1 The equilibrium concept

In our model, the stake of bargaining in any given period depends only on the aggregate investment accumulated up to that point. Consequently, the cumulative investment constitutes the only payoff-relevant part of the history. A *Markov strategy* profile specifies for each period party 1's (incremental) investment choice as a function of the cumulative investment up to the *last period*, and, for the bargaining stage, a price offer and a response rule for each party — a function

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<sup>12</sup>To illustrate, suppose as in Hart (1995) that there are two assets,  $A = \{a_1, a_2\}$ . An ownership structure,  $m$ , is then represented by a partition of  $A$ ,  $(A_1^m, A_2^m)$ , where  $A_i^m, i = 1, 2$  refers to the asset(s) party  $i$  owns under ownership structure  $m$ . Suppose as with GHM, that the status quo payoffs of the parties in case of disagreement depend on the assets they own (and possibly, party 1's investment  $k$ ), given by  $\pi_i(k; A_i^m)$ . There are three alternative structures: (1) separate ownership or non-integration:  $m_N := (\{a_1\}, \{a_2\})$ , (2) common ownership by party 1:  $m_1 := (\{a_1, a_2\}, \emptyset)$ , and (3) common ownership by party 2:  $m_2 := (\emptyset, \{a_1, a_2\})$ . Given our one-sided investment model, then the notion of *strictly complementary* assets coincide with an M-specificity order:  $m_1 \succeq_1^M m_N$  (see Hart (1995): conditions (2.2) and (2.3) (p. 37)). Then, Proposition 1 implies that  $k_0(m_1) > k_0(m_N)$ , thus replicating a crucial result of GHM's asset ownership theory.

mapping from an offer to {accept, reject} — both as functions of the *current-period* cumulative investment. We look for subgame perfect equilibria in Markov strategies, called *Markov Perfect equilibria* (MPE).

Formally, we can characterize an equilibrium corresponding to contract  $m \in \mathcal{M}$ , by a pair of *value functions*,  $(\sigma_m, \beta_m) : \mathbb{R}_+ \mapsto \mathbb{R}^2$ , that map from the *current period* cumulative investment into the continuation values for parties 1 and 2, respectively, evaluated immediately before the selection of a proposer in that period; and by an *investment strategy*,  $y_m : K \mapsto K$ , with  $y_m(k) \geq k$ , that maps party 1's previous-period cumulative investment into her current-period cumulative investment.

Given an ex ante contract  $m \in \mathcal{M}$ , a Markov Perfect equilibrium is characterized by a triple,  $\langle \sigma_m, \beta_m, y_m \rangle$ , satisfying that: for each  $k \in K$ ,

$$\begin{aligned} \sigma_m(k) = & \alpha \max\{\phi(k) - \delta\beta_m(y_m(k)) - (1 - \delta)\psi_2(k; m), \\ & \delta[\sigma_m(y_m(k)) - (y_m(k) - k)] + (1 - \delta)\psi_1(k; m)\} \\ & + (1 - \alpha)\{\delta[\sigma_m(y_m(k)) - (y_m(k) - k)] + (1 - \delta)\psi_1(k; m)\}, \end{aligned} \quad (2)$$

$$\begin{aligned} \beta_m(k) = & (1 - \alpha) \max\{\phi(k) - \delta[\sigma_m(y_m(k)) - (y_m(k) - k)] - (1 - \delta)\psi_1(k; m), \\ & \delta\beta_m(y_m(k)) + (1 - \delta)\psi_2(k; m)\} \\ & + \alpha\{\delta\beta_m(y_m(k)) + (1 - \delta)\psi_2(k; m)\}, \end{aligned} \quad (3)$$

$$y_m(k) \in \arg \max_{\tilde{k} \in [k, k]} \{\sigma_m(\tilde{k}) - (\tilde{k} - k)\}. \quad (4)$$

Party 1's value function is explained as follows. Given an existing investment stock of  $k$ , party 1 is chosen with probability  $\alpha$  to make an offer. She can either make an offer which is unacceptable to party 2, in which case she obtains her net continuation value,  $\delta[\sigma_m(y_m(k)) - (y_m(k) - k)] + (1 - \delta)\psi_1(k; m)$ .<sup>13</sup> Or else, she can make the lowest offer acceptable to party 2, which equals his continuation value,  $\delta\beta_m(y_m(k)) + (1 - \delta)\psi_2(k; m)$ . In the latter case, party 1 collects  $\phi(k) - \delta\beta_m(y_m(k)) - (1 - \delta)\psi_2(k; m)$ . She chooses the more profitable of the two alternatives. With probability  $1 - \alpha$ , party 1 becomes the responder. Based on the above reasoning, she will earn her continuation value,  $\delta[\sigma_m(y_m(k)) - (y_m(k) - k)] + (1 - \delta)\psi_1(k; m)$ , no matter how party 2 resolves his trade-off. Party 2's value function has a similar explanation. Clearly, conditions (2) and (3) subsume equilibrium bargaining behavior, given the investment strategy,  $y_m$ . The

<sup>13</sup>The game continues with probability  $\delta$  in which case she collects  $\sigma_m(y_m(k)) - (y_m(k) - k)$  and the bargaining is terminated with probability  $1 - \delta$  in which case she collects her status quo payoff,  $\psi_1(k; m)$ .

last equation, (4), states that party 1 chooses her best incremental investment, taking into account the current stock and the future evolution of the game. Clearly, these conditions are necessary for Markov Perfection. They are also sufficient, since there is no profitable (possibly non-Markovian) deviation, given the single-period deviation principle.

### 3.2 The characterization of implementable investments

We now characterize the set of MPE. Of particular interest is the level of investment sustainable under a given contract. We say that a cumulative investment,  $k \in K$ , is *implementable by contract  $m$*  if there exists an MPE with  $y_m(\cdot)$  in which trade occurs after reaching  $k$ . As was seen by Che and Sákovics (2004), in our dynamic model, typically multiple levels of investments are supportable by different investment dynamics. In particular, an investor (party 1) can be induced to choose a larger investment than she would in the static model, by a strategy that requires her to invest back to the equilibrium target next period if she deviates by falling short of it. The adoption of this latter strategy causes the non-investor (party 2) to anticipate a much bigger cut in the next period (since the investor will raise investment next period) if she rejects an offer, thus causing her to credibly demand more, than if no such investment growth is anticipated. This reduces the investor's deviation payoff, and thus sharpens her incentives, which in turn makes the prescribed investment growth self enforcing.

In order to identify the entire set of implementable investments, we need to characterize party 1's continuation payoff,  $\sigma_m(k)$ , at each  $k$ . This task is not trivial, however, due to the multiplicity of equilibria. Unlike the pure bargaining game, where the continuation payoff can be uniquely pinned down, a range of continuation payoffs corresponding to different investment dynamics can be sustainable, and characterizing the formers requires getting a handle on all implementable investment dynamics. The following lemma provides several observations.

**Lemma 2** *Fix any MPE with contract  $m$  satisfying Assumptions 1 and 2.*

(i) *If  $k' \in K$  is implementable by  $m$ , then party 1 chooses  $k'$  and trade occurs in the first period, and*

$$\sigma_m(k') - k' \leq U_0(k'; m). \quad (5)$$

(ii) *For any  $k \in K$ ,*

$$\sigma_m(k) - k \geq U_\delta(k; m) - \delta[\alpha\psi_2(y_m^\infty(k)) - (1 - \alpha)\psi_1(y_m^\infty(k)) + (1 - \alpha)y_m^\infty(k)], \quad (6)$$

where  $y_m^\infty(k) := \lim_{t \rightarrow \infty} y_m^t(k)$ .

(iii) For any  $k \in K$ ,  $y_m^\infty(k) \leq \max\{k, k_\delta(m)\}$ .

Part (i) shows that party 1's equilibrium payoff under contract  $m$  cannot exceed the payoff she would receive in the static equilibrium. That upper bound is attained if the investment strategy has  $y(k') = k'$ , i.e., no further investment is specified after reaching  $k'$  (in which case a pure bargaining game ensues just as in the static game, and hence the same payoff). Whether party 1 will actually choose  $k'$  depends on what payoff she will receive when deviating from  $k'$ , in particular on the worst sustainable punishment that can be levied against the deviator, which is characterized in part (ii). The RHS of (6) describes the payoff party 1 will receive when the strategy have her jump to the highest investment level sustainable under that strategy,  $y^\infty(k)$ , next period, if no agreement is reached in the current period. It turns out that sending party 1 to the highest implementable investment level (should disagreement occur) constitutes the worst sustainable punishment. One can then use (i) and (ii) to identify the highest implementable investment level: It is the highest level that party 1 wishes to choose given the threat of any downward deviation being punishable by a jump to *that* level next period. The highest sustainable investment level is thus pinned down to be  $k_\delta(m)$ .

We can establish a couple of necessary conditions for MPE based on this lemma. Note that, by definition, once  $k_\delta(m)$  is reached no further investment is possible, so a pure bargaining game will ensue, leading to a unique continuation payoff,  $\sigma_m(k_\delta(m)) = U_0(k_\delta(m); m) + k$ , in any MPE. Since party 1 can guarantee this payoff by choosing  $k_\delta(m)$ , any implementable investment should make party 1 no worse off than that payoff. That is, if  $k \in K$  is implementable, then

$$(IC). \quad U_0(k; m) \geq U_\delta(k_\delta(m); m).$$

Next, suppose that party 1 deviates by not investing and perpetually inducing rejection. Then she will obtain

$$(1 - \delta)[\bar{\psi}_1(m) + \delta\bar{\psi}_1(m) + \delta^2\bar{\psi}_1(m) + \dots] = \bar{\psi}_1(m).$$

If  $k$  is implementable by  $m$ , we must have

$$(IR) \quad U_0(k; m) \geq \bar{\psi}_1(m).$$

for  $k = y_m(0)$ . By Assumption 2, the LHS is no less than the RHS when  $k = 0$ . Since the LHS is maximized at  $k = k_0(m)$  and since  $U_0(\cdot; m)$  is concave (which follows from Assumption 1(b)),

condition (*IR*) holds for all  $k \leq k_{IR}(m) := \sup\{k \leq \bar{k} \mid U_0(k; m) \geq \bar{\psi}_1(m)\}$ . Note that, given the assumptions, (*IR*) is clearly not binding at  $k = k_0(m)$ , which implies that  $k_{IR}(m) > k_0(m)$ .

We have shown that (*IC*) and (*IR*) are necessary for any cumulative investment  $k$  implementable by  $m$ . It turns out that they are sufficient as well.

**Theorem 1** *Given Assumptions 1 and 2, any cumulative investment  $k$  is implementable by a contract  $m$  if and only if*

$$k \in K_\delta(m) := \{k \in K \mid U_0(k; m) \geq \max\{\bar{\psi}_1(m), U_0(k_\delta(m); m)\}\}.$$

*When  $k \in K_\delta(m)$  is implemented, party 1 chooses  $k$  and trade occurs, all in the first period.*

*Proof.* See the Appendix.

As can be seen in the proof, any investment in  $K_\delta$  is sustainable by a strategy that specifies no further investment (off the equilibrium path) once the implemented investment level is reached and specifies the worst punishment of having party 1 go up to  $k_\delta(m)$  for any deviation.

The set of implementable investments,  $K_\delta$ , can be more sharply characterized with the aid of Figure 1(a) and 1(b).

**[Insert Figure 1(a) and 1(b) around here.]**

Since  $U_0(\cdot; m)$  is concave (by Assumption 1), the set  $K_\delta(m)$  is an interval. Its largest element, denoted  $\bar{k}_\delta(m)$ , must satisfy  $\bar{k}_\delta(m) = \min\{k_\delta(m), k_{IR}(m)\} < k^*$ , for all  $\delta < 1$ . As can be seen from Figure 1(a),  $\bar{k}_\delta(m) = k_\delta(m)$  if (*IR*) is not binding; whereas  $\bar{k}_\delta(m) = k_{IR}(m)$  if (*IR*) is binding. The lowest sustainable investment, denoted  $\underline{k}_\delta(m)$ , must also satisfy  $U_0(\underline{k}_\delta; m) = \max\{\bar{\psi}_1(m), U_0(k_\delta(m); m)\}$ . Recall that  $U_0(k; m)$  is maximized at  $k = k_0(m)$ . Clearly, (*IR*) holds strictly at  $k = k_0(m)$  given Assumption 2. Furthermore, if  $k_0(m) \in (0, \bar{k})$  and Assumption 1(a)-(c) holds, then  $k_\delta(m) > k_0(m)$  for any  $\delta > 0$ , so we must have  $U_0(k_0; m) > \max\{\bar{\psi}_1(m), U_0(k_\delta(m); m)\}$ . This means that  $\underline{k}_\delta(m) < k_0(m) < \bar{k}_\delta(m)$  whenever  $\delta > 0$ . In other words, our dynamic game can sustain investments that are strictly higher and those that are strictly lower than the investment level sustainable in the static equilibrium.

Despite the multiplicity, the set of MPE's in our dynamic model has some desirable continuity features. For instance, it attains the static equilibrium outcome as a unique MPE when  $\delta = 0$ : Since  $k_\delta(m) \rightarrow k_0(m)$  as  $\delta \rightarrow 0$ ,  $K_\delta(m)$  collapses to a unique equilibrium investment,  $k_0(m)$ .

Finally, since  $\bar{k}_\delta(m) \leq k^*$  and  $\phi(\cdot)$  is concave, the highest sustainable investment,  $\bar{k}_\delta(m)$ , generates the highest net joint surplus for the parties. If the parties have credible ways to

coordinate on selecting the equilibrium, the highest equilibrium investment will be selected (assuming that any distributional issue can be addressed by the exchange of side payments). Focusing on the best equilibrium also isolates the new predictions made available from the current dynamic model. In particular, we can interpret the existing predictions as coming from our dynamic model but with a particular selection. In this sense, focusing on the best equilibrium serves to effectively complement the existing literature. For this reason, we will focus on the highest implementable investment throughout the rest of the paper.

### 3.3 The effects of a contract shift

To study how the choice of a contract  $m \in \mathcal{M}$  affects the highest implementable investment, we need to examine both  $k_{IR}(m)$  and  $k_\delta(m)$ , since  $\bar{k}_\delta(m) = \min\{k_{IR}(m), k_\delta(m)\}$ . Note, however, that, since  $(IR)$  is not binding at  $k = k_0(m)$ , there exists  $\hat{\delta} > 0$  such that, for all  $\delta \leq \hat{\delta}$ ,  $(IR)$  is not binding so that  $\bar{k}_\delta(m) = k_\delta(m) \leq k_{IR}(m)$ . Hence, for a sufficiently low  $\delta$ , it suffices to examine how  $k_\delta(m)$  varies with contracts. Consider a shift from contract  $m$  to contract  $m'$ . Observe that

$$\begin{aligned} U_\delta(k; m') - U_\delta(k; m) &= (1 - \delta)\{(1 - \alpha)[\psi_1(k; m') - \psi_1(k; m)] - \alpha[\psi_2(k; m') - \psi_2(k; m)]\} \\ &= (1 - \delta)[U_0(k; m') - U_0(k; m)]. \end{aligned}$$

It follows that the effect of a contract shift on  $k_\delta$  is much the same as that on  $k_0$ . This shows that, while the dynamic model allows a higher investment to be sustained than the static model, the comparative static result associated with the effects of a contract shift on the highest sustainable investment remains unchanged, *for a sufficiently low  $\delta$* . In this sense, the results found in the extant contract literature are robust. We thus have the following generalization of Proposition 1:

**Proposition 2** *For any two contracts  $m, m' \in \mathcal{M}$ , there exists  $\hat{\delta} > 0$  such that, for any  $\delta \leq \hat{\delta}$ , if  $m' \succeq_1^M m$  and  $m \succeq_2^M m'$ , then  $\bar{k}_\delta(m') \geq \bar{k}_\delta(m)$ . (The inequality is strict if, in addition,  $\bar{k}_\delta(m') > 0$  and either  $m' \succ_1^M m$  or  $m \succ_2^M m'$ .) In particular, if  $m \sim^M m'$ , then  $\bar{k}_\delta(m') = \bar{k}_\delta(m)$ .*

What happens for a large  $\delta$ ? We next show that our dynamic contract model does make a difference in this case. In such a case, the  $(IR)$  constraint may be binding, which introduces a different type of consideration in terms of contract design. In fact, there is a sense in which comparison of alternative contracts is meaningful only if  $(IR)$  is binding. To see this, fix any contract  $m \in \mathcal{M}$ , and suppose that  $k_{IR}(m) \geq k^*$  so that  $(IR)$  can never be binding. Then, since

$k_\delta(m) \leq k^* \leq k_{IR}(m)$ , we have  $\bar{k}_\delta(m) \rightarrow k^*$  as  $\delta \rightarrow 1$ , so the first-best outcome is attainable arbitrarily closely. This result holds regardless of the contract (or its absence<sup>14</sup>) as long as the  $(IR)$  holds at  $k^*$  with that contract. In that sense, contracts become irrelevant for  $\delta \approx 1$ , unless  $(IR)$  binds with some contracts.

**Proposition 3 (Irrelevance of IR Contracts in the Limit)** *For any  $m \in \mathcal{M}$  with  $k_{IR}(m) \geq k^*$ ,  $\bar{k}_\delta(m) \rightarrow k^*$  as  $\delta \rightarrow 1$ .*

To the extent that one cares about the implementability of the best equilibrium, this proposition underscores the significance of  $(IR)$  in comparative contract research when  $\delta \approx 1$ . To see whether the  $(IR)$  constraint introduces a new insight absent in the existing literature, again fix a contract  $m$  and write the  $(IR)$  condition as

$$\alpha[\phi(k) - \psi_2(k; m)] + (1 - \alpha)\psi_1(k; m) - k \geq \bar{\psi}_1(m), \quad (7)$$

which can be rewritten as:

$$\alpha\phi(k) + (1 - \alpha) \int_0^k \psi'_1(\tilde{k}; m) d\tilde{k} - \alpha \int_0^k \psi'_2(\tilde{k}; m) d\tilde{k} - \alpha[\bar{\psi}_1(m) + \bar{\psi}_2(m)] \geq 0. \quad (8)$$

Clearly, the same change in M-specificity that would increase  $k_\delta$  also appears to relax the  $(IR)$  constraint: An increase in  $\psi'_1$  and a decrease in  $\psi'_2$  relaxes the constraint. There is a new effect, however: *absolute* specificity matters for  $(IR)$ . In particular, the higher the absolute specificity, the more relaxed the constraint is; i.e., a decrease in  $\bar{\psi}_2 + \bar{\psi}_1$  relaxes the constraint. The effect of a contract shift can be clearly seen by how it affects the LHS of (8). Consider as before a shift from contract  $m$  to contract  $m'$ . The resulting increase in the LHS is written as:

$$(1 - \alpha) \int_0^k [\psi'_1(\tilde{k}; m') - \psi'_1(\tilde{k}; m)] d\tilde{k} - \alpha \int_0^k [\psi'_2(\tilde{k}; m') - \psi'_2(\tilde{k}; m)] d\tilde{k} - \alpha[\bar{\psi}_1(m') + \bar{\psi}_2(m') - \bar{\psi}_1(m) - \bar{\psi}_2(m)]. \quad (9)$$

Inspecting (9) yields our main general result.

**Proposition 4** *For any two contracts  $m, m'$ , if  $m' \succeq_1^M m$ ,  $m' \preceq_2^M m$ , and  $m' \preceq^A m$ , then  $\bar{k}_\delta(m') \geq \bar{k}_\delta(m)$ . The latter inequality is strict if one of the first two orders is strict and  $(IR)$  is not binding for both contracts, or the last order is strict and  $(IR)$  is binding for either contract.*

<sup>14</sup>In Che and Sákovics (2004), we show asymptotic efficiency in a model without breakdown (with discounting), corresponding to the absence of a contract.

*Proof.* As noted, if  $m' \succeq_1^M m$  and  $m' \preceq_2^M m$ , then  $k_\delta(m') \geq k_\delta(m)$ . Also, if  $m' \succeq_1^M m$  and  $m' \preceq_2^M m$ , and  $m' \preceq^A m$ , then  $k_{IR}(m') \geq k_{IR}(m)$ . The last statement follows from that fact that  $\bar{k}_\delta(\tilde{m}) = \min\{k_{IR}(\tilde{m}), k_\delta(\tilde{m})\}$ , for  $\tilde{m} = m, m'$ . ■

The new effect becomes most clear when two contracts are M-specificity invariant.

**Corollary 2 (*Status Quo Minimization Principle*)** *Suppose that  $m' \sim^M m$ . Then,  $\bar{k}_\delta(m') \geq \bar{k}_\delta(m)$  if and only if  $m' \preceq^A m$ , or equivalently,*

$$\bar{\psi}_1(m') + \bar{\psi}_2(m') \leq \bar{\psi}_1(m) + \bar{\psi}_2(m). \quad (10)$$

*If (10) is strict and (IR) is binding for either  $m$  or  $m'$ , then  $\bar{k}_\delta(m') > \bar{k}_\delta(m)$ .*

This corollary has some striking implications for several contracts, as will be seen in the next section.

## 4 Applications

This section applies Proposition 4 and Corollary 2 to various contracting/organizational settings. In some cases, we will focus on M-specificity invariant contracts, i.e., the ones that expose the investor to the same degree of marginal specificity. As was summarized in Corollary 1, the traditional contract literature predicts the same outcome for all such contracts. As was seen in Corollary 2, such contracts will entail different outcomes if the investor is exposed to different degrees of *absolute* specificity. The primary reason for focusing on the M-specificity invariant contracts is that they bring out our novel contributions in the clearest way. In fact, any effects through the changes in M-specificity conform to the insight developed in the traditional contract literature (as was established in Proposition 4). At the same time, the studied class of contracts is realistic in many settings. For instance, if the investments are completely specific and their values are not realized without the participation of both parties' human capital, then the status quo payoffs arising from different ownership structures will not vary with the parties' specific investment, so they will differ only in the absolute magnitude. Likewise, if the investments are not transferable to the external trades, whether the internal partners have exclusivity agreements or not will affect the status quo payoffs only in absolute magnitude. In these cases, we derive some surprising new results.

## 4.1 Ownership contracts

To illustrate, suppose that there are two assets,  $A = \{a_1, a_2\}$ . An ownership structure,  $m$ , is then represented by a partition of  $A$ ,  $(A_1^m, A_2^m)$ , where  $A_i^m, i = 1, 2$  refers to the asset(s) party  $i$  owns under ownership structure  $m$ . Suppose, as with GHM, that the status quo payoffs of the parties in case of disagreement depend on the assets they own (and possibly, party 1's investment  $k$ ), given by  $\pi_i(k; A_i^m)$ . There are three alternative structures: (1) separate ownership or nonintegration:  $m_N := (\{a_1\}, \{a_2\})$ , (2) common ownership (or integration) by party 1:  $m_1 := (\{a_1, a_2\}, \emptyset)$ , and (3) common ownership (or integration) by party 2:  $m_2 := (\emptyset, \{a_1, a_2\})$ . As mentioned earlier, this setup is accommodated in our model with  $\psi_i(k; m) := \pi_i(k; A_i^m)$  for  $i = 1, 2$ . Suppose that these alternative ownership structures are M-specificity invariant. As mentioned before, specificity invariance may arise because the investment is totally specific, so the entire return to the investment is lost when there is disagreement: i.e.,  $\pi_i(k; A_i^m) = \pi_i(0; A_i^m)$  for all  $k$ .

Of special interest in the GHM model is the case in which the assets are strictly complementary. Complementarity of assets is defined there as the common ownership of assets lowering the owner's marginal specificity of investment more than raising the non-owner's marginal specificity of his investment, relative to separate ownership. Since we want to be able to talk about complementarity even under M-specificity invariance, we define an alternative, perhaps more natural, notion of complementarity:

**Definition 3** *Assets are **absolutely complementary** if*

$$\bar{\psi}_2(m_N) + \bar{\psi}_1(m_N) < \min\{\bar{\psi}_1(m_1) + \bar{\psi}_2(m_1), \bar{\psi}_1(m_2) + \bar{\psi}_2(m_2)\}.$$

In words, this simply means that — in the absence of investment effects — the total asset values are higher when they are owned together than when they are owned separately. Clearly, given absolutely complementary assets, the status quo minimization principle (Corollary 2) yields the following implication.

**Result 1** *If the alternative ownership contracts are M-specificity invariant and the assets are absolutely complementary, then separate ownership implements a [strictly] higher investment than either form of common ownership [if individual rationality were binding at the common ownership].*

This result contrasts with the important GHM prescription that (marginally) complementary assets should always be owned together. Clearly, the difference stems from the fact that the IR constraint plays an important role in our dynamic setting. As mentioned above, the GHM result is based on, and driven by, the effects ownership structures have on how the status quo payoffs depend on the investment. While the above result assumes that the status quo payoffs do not vary with investment, it will continue to hold when some degree of such co-movement is allowed.

**Remark 1 (*Joint ownership*)** *A similar argument can be used to justify joint ownership of assets, which is often viewed as suboptimal in the static literature. Suppose, as is often argued by authors, that a joint ownership of assets entails even less efficient utilization of assets than a common ownership of the assets, for instance because conflicting interests result in ineffective decision making under joint ownership.*<sup>15</sup> *This simply means that*

$$\bar{\psi}_2(m_J) + \bar{\psi}_1(m_J) < \min\{\bar{\psi}_1(m_1) + \bar{\psi}_2(m_1), \bar{\psi}_1(m_2) + \bar{\psi}_2(m_2)\},$$

*where  $m_J$  refers to the joint ownership regime. Then, our status quo minimization principle will imply that the joint ownership can support higher investment than any common ownership, provided that the joint ownership is  $M$ -specificity invariant to the other ownership structures. A similar implication can be drawn on the value of joint ownership relative to separate ownership, if the former entails less efficient use of assets than the latter.*

**Remark 2 (*Incentivizing effect of asset ownership*)** *The above analysis casts doubt on the robustness of some incentivizing effects of asset ownership found in the existing literature. The GHM view on this was that transferring an asset ownership to an agent unambiguously motivates his specific investments. Meanwhile, the static models based on the outside option specification of bargaining hold that, given some plausible condition,<sup>16</sup> the ownership transfer unambiguously demotivates the asset gainer's investment. Our dynamic model affirms neither view. According to Result 1, given absolute complementarity of assets, transfer of an asset to party 1 motivates his investment, starting from a common ownership by party 2, but demotivates his investment when transfer occurs starting from separate ownership.*

<sup>15</sup>For instance, Hart (1995, pp48) describes joint ownership as a regime in which both parties have veto power over the use of assets. According to this view, the joint ownership will be similar in effect to a mutual exclusivity agreement, discussed below.

<sup>16</sup>The condition assumed is that the status quo payoffs under alternative ownership structures do not depend sensitively on the investments.

## 4.2 Exclusivity contracts

A long-held belief was that a right to prohibit one's partner from trading with a third party — the so-called exclusivity agreement — protects the return of specific investments made by the recipient of that right, thus promoting such an investment. A recent paper by Segal and Whinston (2000) challenges the wisdom of this belief. They observe that, if investments are specific and non-transferable to external trade, such an agreement can affect the parties' status quo payoffs only in absolute magnitude but not in the way investments affect those payoffs. Hence, exclusivity and non-exclusivity are M-specificity invariant. As is clear from Corollary 1, then an exclusivity agreement would have no effect on the outcome in such a circumstance according to the traditional models (see Segal and Whinston (2000), Proposition 1).

In our dynamic model, however, a change in the absolute level of the status quo payoffs matters, since it affects the individual rationality constraints. To see this, suppose as in Example 3 that the parties' status quo payoffs are  $\pi_1^O > 0$  and  $\pi_2^O > 0$  if there is no exclusivity agreement, whereas their status quo payoffs are zero under the exclusivity agreement. It thus follows that

$$\bar{\psi}_1(EX) + \bar{\psi}_2(EX) = 0 < \pi_1 + \pi_2 \equiv \bar{\psi}_1(NEX) + \bar{\psi}_2(NEX).$$

Given the M-specificity invariance between the two regimes, the next result follows immediately from the status quo minimization principle:

**Result 2** *An exclusivity agreement implements a [strictly] higher investment [if the individual rationality were binding under non-exclusivity].*

This result shows that, if the individual rationality constraint were binding for a party, awarding him the exclusivity right can protect his specific investment, which contrasts with the irrelevance result by Segal and Whinston (2000). It is worth noting that exclusive dealing relaxes the individual rationality constraints by making the status quo outcome (= failure of internal trade) less attractive for the parties. It is also important to note that this effect arises not only for the party who offers the exclusivity protection but also for the recipient of such a protection (who may not offer the same protection to the other party in return).

In the middle of writing this paper, we became aware of another paper making this point. de Meza and Selvaggi (2003) obtain the same result as ours, using a three-way bargaining game with the possibility of reallocating/subcontracting the production assignment (in case of non-exclusivity). These results complement each other toward a positive role exclusivity may play in promoting specific investments.

### 4.3 Trade contracts

Trade contracts specify the terms of ex post trade between the parties. Whether such trade contracts — renegotiable if the parties so choose — can solve the holdup problem has been the focus of much of the recent contract literature (see Chung 1991; Edlin and Reichelstein, 1996; Che and Hausch, 1999; and Segal and Whinston, 2002). The answer to this question depends on the nature of the investment. If the investment is selfish in the sense that the investor directly benefits from her investment ( $c_k < 0$  and  $v_k = 0$  in our case), then Edlin and Reichelstein (1996) showed that a simple trade contract specifying a fixed quantity of trade,  $\hat{q}$ , and a transfer payment  $\hat{t}$  (to be made by the buyer [party 2] to the seller [party 1]) can implement the first best outcome. Rephrased in the conceptual framework of our model, the main idea behind the efficiency result is that the trade contract, with an appropriately chosen  $\hat{q}$ , can eliminate the investor's marginal specificity exposure entirely if the investment is *selfish*. By contrast, if the investment is cooperative in the sense that the direct beneficiary of the investment is the investor's partner ( $c_k = 0$  and  $v_k > 0$  in our case), then no such trade contract can reduce the investor's exposure to marginal specificity. Rather it reduces her partner's marginal specificity, so the contract can only worsen underinvestment, as shown in Che and Hausch (1999).

The effects of trade contracts can be reexamined in our dynamic model. As will be seen, some flavor of the standard results carries over, but there are some new elements. To begin, consider again the simple trade contract indexed by  $(\hat{q}, \hat{t})$ . The status quo payoffs associated with this contract are  $\psi_1(k; \hat{q}, \hat{t}) = \hat{t} - c(\hat{q}, k)$  and  $\psi_2(k; \hat{q}, \hat{t}) = v(\hat{q}, k) - \hat{t}$ . Clearly, trade contracts with different  $\hat{q}$  will not be M-specificity invariant. It is useful to consider the cases of selfish and cooperative investments separately.

#### 4.3.1 Selfish investments

If the investments are purely selfish, then we have  $v_k \equiv 0$  while  $c_k < 0$ . Suppose, as with Edlin and Reichelstein, that the parties sign a contract specifying  $\hat{q} = q^*(k^*) := \arg \max_{q \in Q} v(q, k^*) - c(q, k^*)$ . ( $\hat{t}$  does not influence the outcome and thus can be arbitrary.) Note that, with this contract,

$$\begin{aligned} U_\delta(k; \hat{q}, \hat{t}) &= \alpha[\phi(k) - (1 - \delta)\psi_2(k; \hat{q}, \hat{t})] + (1 - \alpha)(1 - \delta)\psi_1(k; \hat{q}, \hat{t}) - (1 - (1 - \alpha)\delta)k \\ &= \alpha[\phi(k) - (1 - \delta)(v(q^*, k) - \hat{t})] + (1 - \alpha)(1 - \delta)(\hat{t} - c(q^*, k)) - (1 - (1 - \alpha)\delta)k. \end{aligned}$$

Differentiating this function with respect to  $k$ , using  $v_k \equiv 0$ , yields

$$U'_\delta(k; \hat{q}, \hat{t}) = [1 - (1 - \alpha)\delta](\phi'(k) - 1).$$

It then follows that  $k_\delta(\hat{q}, \hat{t}) = k^*$ , with  $\hat{q} = q^*(k^*)$ . Since  $U_\delta(k; q^*, \hat{t})$  is maximized at  $k = k^*$ , regardless of  $\delta$ ,  $U_0(k; q^*, \hat{t})$  is also maximized at  $k = k^*$ . Since  $U_0(\cdot; q^*, \hat{t})$  is strictly concave, this means that only  $k = k^*$  satisfies

$$U_0(k; \hat{q}, \hat{t}) \geq U_0(k_\delta(\hat{q}, \hat{t}); \hat{q}, \hat{t}).$$

Hence, the only investment level implementable by a contract with  $\hat{q} = q^*$ , if there is any, would be  $k = k^*$ . Indeed, it is straightforward to show that  $k^*$  can be implemented in equilibrium by the investment strategy<sup>17</sup>

$$y(k) = \begin{cases} k^* & \text{if } k \in [0, k^*], \\ k & \text{if } k \in (k^*, \bar{k}] \end{cases}$$

It follows that the set of MPE investments  $K_\delta(\hat{q}, \hat{t})$  is a singleton  $\{k^*\}$ . In other words, the chosen trade contract implements the first-best outcome as a unique equilibrium.

**Result 3** *With selfish investments, a trade contract  $(\hat{q}, \hat{t})$  with  $\hat{q} = q^*(k^*)$  implements the first-best outcome as a unique MPE.*

This result reaffirms the efficiency result obtained by Edlin and Reichelstein (1996) in the static setting. This efficiency result has an added dimension in our dynamic setting though, as it suggests the role of the trade contract as *eliminating* bad equilibria as much as attaining the efficient one: While the efficient outcome may be *an* equilibrium under different contracts or even without any contract if  $\delta$  is close to 1 (as suggested in the previous section), it admits less efficient outcomes as well, which the current contract eliminates.

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<sup>17</sup>The payoff at  $k$  given the strategies is:

$$\sigma(k) - k = \begin{cases} \max\{U_\delta(k) - \delta[\alpha\psi_2(k^*)] - (1 - \alpha)\psi_1(k^*) + (1 - \alpha)k^*, \delta[U_0(k^*) + k] - k\} & \text{if } k \in [0, k^*], \\ U_0(k) & \text{if } k \in (k^*, \bar{k}]. \end{cases}$$

Ignoring the payoff term associated with delay (which is always less than  $U_0(k^*)$ ), the payoff function is quasi-concave and attains a maximum at  $k^*$ . Hence the described strategies are an equilibrium.

### 4.3.2 Cooperative investments

If the investments are purely cooperative, then  $c_k \equiv 0$  while  $v_k > 0$ . We examine how trade contracts with different  $\hat{q}$  will affect the outcome in this case. Since  $\bar{k}_\delta(m) = \min\{k_\delta(m), k_{IR}(m)\}$ , we investigate the effects on each of the two terms. First, the effect on  $k_\delta$  can be studied by examining

$$U_\delta(k; \hat{q}, \hat{t}) = \alpha[\phi(k) - (1 - \delta)(v(\hat{q}, k) - \hat{t})] + (1 - \alpha)(1 - \delta)(\hat{t} - c(\hat{q}, k)) - (1 - (1 - \alpha)\delta)k.$$

Differentiating this function with respect to  $k$ , using  $c_k \equiv 0$ , gives

$$U'_\delta(k; \hat{q}, \hat{t}) = \alpha\phi'(k) - \alpha(1 - \delta)v_k(\hat{q}, k) - [1 - (1 - \alpha)\delta]. \quad (11)$$

Since  $v_k(q, k) > 0$  for all  $k > 0$  and  $v_k(0, k) \equiv 0$ , this derivative is maximized when  $\hat{q} = 0$ . It follows that  $k_\delta(\hat{q}, \hat{t})$  is maximized when  $\hat{q} = 0$ . This observation is consistent with that of Che and Hausch (1999) in that no non-trivial contract can improve the incentives for specific investment. However, this does not prove that trade contracts are worthless. Even with  $\hat{q} > 0$ ,  $k_\delta(\hat{q}, \hat{t}) \rightarrow q^*$  as  $\delta \rightarrow 1$ , just as observed in Proposition 3. Hence, whether a contract is valuable for  $\delta \approx 1$  depends crucially on whether the contract can relax the IR constraint that would bind without it.

With a trade contract and cooperative investment, the (*IR*) constraint can be rewritten as follows:

$$\begin{aligned} \alpha[\phi(k) - \psi_2(k; \hat{q}, \hat{t})] + (1 - \alpha)\psi_1(k; \hat{q}, \hat{t}) - k &\geq \bar{\psi}_1(\hat{q}, \hat{t}) \\ \iff \alpha\phi(k) - k &\geq \alpha[v(\hat{q}, k) - c(\hat{q}, k)]. \end{aligned} \quad (12)$$

Clearly, the RHS of (12) vanishes with  $\hat{q} = 0$ . If  $k_{IR}(0, 0) \geq k^*$ , so that the IR constraint is never binding in the absence of contracts, then clearly any trade contract would be worthless, since  $\bar{k}_\delta(0, 0) = k_\delta(0, 0) > k_\delta(\hat{q}, \hat{t}) \geq \bar{k}_\delta(\hat{q}, \hat{t})$  for any  $(\hat{q}, \hat{t})$  with  $\hat{q} > 0$ . Alternatively, if  $v(\hat{q}, k) - c(\hat{q}, k) \geq 0$  for all  $\hat{q} \in Q$ , then contracts are worthless again since they cannot relax the IR constraint.

Suppose now that  $k_{IR}(0, 0) < k^*$ , so that the IR constraint would be binding in the absence of contracts for some large  $\delta$ . In this case, a contract specifying a quantity that would create a negative surplus, i.e., quantity in the set,

$$Q_\delta^- := \{q' \in Q \mid v(q', k_{IR}(0, 0)) - c(q', k_{IR}(0, 0)) < 0\},$$

can relax the (*IR*) constraint. This set will be nonempty if  $Q$  includes an excessive large level of quantity so that any trade will impose net loss, regardless of  $k$ . Of course, even when a

contract with  $\hat{q} \in Q_\delta^-$  exists, it may not implement a higher investment than no contract, since  $k_\delta(0,0) > k_\delta(\hat{q}, \hat{t})$  for  $\hat{q} > 0$ . As mentioned earlier, the latter inequality becomes unimportant as  $\delta \rightarrow 1$ , since the second term in (11) vanishes. That is, as  $\delta \rightarrow 1$ ,  $(IR)$  is all that matters. If  $k_{IR}(0,0) < k^*$ , then  $(IR)$  is binding in the absence of contracts. In this case, a trade contract specifying  $\hat{q} \in Q_1^-$  relaxes the constraint *and* it can implement a higher investment than no contract.

**Result 4** *Any trade contract (specifying a fixed trade level) is worthless if  $k^* \leq k_{IR}(0,0)$  or if  $Q_1^-$  is empty. If  $k_{IR}(0,0) < k^*$  (or equivalently  $\alpha\phi(k^*) - k^* < 0$ ) and  $Q_1^-$  is nonempty, then a trade contract specifying  $\hat{q} \in Q_1^-$  is strictly welfare improving for  $\delta$  sufficiently close to 1.*

This result modifies the conclusion of Che and Hausch (1999) in an important way: A trade contract generating a negative surplus can be strictly welfare improving in the dynamic setting. While such a contract may appear unappealing, it can improve the incentives by making credible a larger punishment than would be feasible without that contract.

#### 4.4 Legal rules for externalities

A long-standing issue in legal scholarship concerns the optimal legal rule for assigning entitlements and compensating for their violations. Following the pioneering work by Calabresi and Melamed (1972), many authors have debated over the relative merits of property rules and liability rules. A liability rule gives a potential injurer an option to harm others upon compensating for their loss, whereas the property rule either completely prohibits the harm-causing activity (in case victims have the property rights) or permits it without any sanction.

This difference would not matter if the parties can bargain costlessly in the Coasian fashion (see Coase (1960)). Such a perfect bargain is often more fabled than real, however. Calabresi and Melamed postulated a tradeoff between the two rules, in case bargaining costs are not negligible. On the one hand, if parties cannot negotiate to overturn incorrect assignment of property rights, an inefficient outcome is inevitable under the property rule, but it can be avoided in the liability rule, making the latter preferable when the bargaining cost is prohibitively high. On the other hand, the property rule is subject to less enforcement uncertainty than the liability rule<sup>18</sup>, so

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<sup>18</sup>The liability rule is more susceptible to enforcement errors since the court may not establish causality accurately or may not assess the harm accurately. In particular, compensatory damages typically do not involve “intangible loss” such as pain and suffering, so they tend to underestimate the actual harm. No such issue, and thus accompanying enforcement errors, arise under the property rule.

the parties are able to bargain more efficiently in the former rule. Hence, the property rule would be preferable when bargaining is not prohibitively costly. However, this simple maxim — prescribing the liability rule for a high bargaining cost and the property rule for a low bargaining cost — has been challenged by recent authors, who argue that a liability rule tends to perform uniformly better than the property rule, once the bargaining costs are endogenized rather than assumed in an ad hoc fashion to favor the property rule.<sup>19</sup> The recent difficulty with justifying the property rule does not accord well with the historical survival and thriving of that rule (see Ayres (2003, pp??)). In our view, the recent favoring of the liability rule appears to result from the failure to capture the alleged bargaining difficulties caused by the enforcement errors associated with the liability rule.

As will be seen, our simple model below endogenizes the bargaining cost, just like the recent literature, but it also explicitly introduces the enforcement errors associated with the liability rule. While its static version will reaffirm the recent finding, its dynamic version will be seen to capture the alleged bargaining difficulty under the liability rule, and thus reestablish the classical “trade off” between the two rules.

Our model is adapted from that of Bebchuk (2001, 2002), in which a victim [a.k.a. party 2] receives harm,  $h(q, k)$ , that depends on the *ex post* activity,  $q \in \{0, 1\}$ , chosen by an injurer [a.k.a. party 1] and her *ex ante* precaution  $k \in K := [0, \bar{k}]$ . Let  $\pi_1(q)$  be party 1’s profit when she picks  $q$  and let  $\pi_2$  be party 2’s profit, excluding the harm. We assume that  $\pi_1(1) > \pi_1(0)$  and  $h(1, k) > h(0, k) \equiv 0$ , so party 1 prefers to be active but this causes harm to party 2. We also assume that, for all  $k \in K$ ,

$$\phi(1, k) := \pi_1(1) + \pi_2 - h(1, k) > \pi_1(0) + \pi_2 =: \phi(0, k), \quad (13)$$

so it is efficient for party 1 to be active. Moreover, we assume that  $h(1, \cdot)$  is decreasing, strictly convex, and differentiable, and that the first-best precaution level,  $k^* := \arg \min_{k \in K} \{h(1, k) + k\}$  is in  $(0, \bar{k})$ .

In our context, the relevant legal regimes consist of  $\mathcal{M} = \{P1, P2, L\}$ , where  $P1$  is a regime in which party 1 enjoys the right to pollute with impunity (and thus chooses  $q$ );  $P2$  is a regime in which party 2 has the right to be free from pollution (and thus chooses  $q$ ); and  $L$  is a regime in

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<sup>19</sup>Ayres and Talley (1995) endogenizes the bargaining cost as arising from informational asymmetry. Bebchuk (2001, 2002) endogenizes the bargaining cost as arising from noncontractability of ex ante investment, in a holdup problem setting. See also Kaplow and Shavell (1996). An exception is Ayres and Talley (199?), which obtains the opposite result, given a reasonable restriction on the liability rule.

which party 1 can pollute but must pay some “compensatory damages” to party 2. To explicitly accommodate the enforcement errors under the liability rule mentioned above, we assume that party 2 can recover only  $(1 - \epsilon)h(q, k)$  in expected value from a lawsuit, for some  $\epsilon \in (0, 1)$ . The enforcement error could represent a plaintiff’s inability to establish liability (with probability  $\epsilon$ ) or the court’s reluctance to compensate the “intangible losses” (in the amount of  $\epsilon h(q, k)$ ). The role of this assumption will be made clear later.

As mentioned above, the comparison of these rules is nontrivial with a bargaining cost. Just as in Bebchuk, in our model, the bargaining costs arise endogenously from the non-verifiability of the ex ante precaution. That is, the ex post decision  $q$  is verifiable but not the ex ante decision  $k$ , so the parties can only bargain over the former.

The parties’ status quo payoffs are determined as follows. In regime P1, party 1 has the right to choose  $q$ . Given our assumptions, she will choose  $q = 1$ .<sup>20</sup> In regime P2, party 2 has the right to be free from pollution. He will choose  $q = 0$ . In regime L, party 1 can pollute but must pay  $(1 - \epsilon)h(q, k)$  on average. Hence, party 1 chooses  $q_L = 1$ .<sup>21</sup> Then, the status quo payoffs under the alternative regimes are described as follows.

$$(\psi_1(k; m), \psi_2(k; m)) = \begin{cases} (\pi_1(1), \pi_2 - h(1, k)) & \text{if } m = P1 \\ (\pi_1(0), \pi_2), & \text{if } m = P2 \\ (\pi_1(1) - (1 - \epsilon)h(1, k), \pi_2 - \epsilon h(1, k)) & \text{if } m = L. \end{cases}$$

Substituting these into (1) with  $\delta = 0$ , we have

$$U_0(k; m) = \begin{cases} \pi_1(1) - k & \text{if } m = P1 \\ \alpha(\pi_1(1) - h(1, k)) + (1 - \alpha)\pi_1(0) - k, & \text{if } m = P2 \\ \pi_1(1) - (1 - \epsilon)h(1, k) - k, & \text{if } m = L. \end{cases}$$

• **Static benchmark** ( $\delta = 0$ ): Suppose first the static model ( $\delta = 0$ ), in which party 1 chooses  $k$  to maximize  $U_0(k; m)$  in regime  $m = P1, P2, L$ . In regime P1, party 1 has no incentive for precaution, so she chooses  $k_0(P1) = 0$ . In regime P2, the precaution choice is given by  $k_0(P2) = \arg \min_{k \in K} \{\alpha h(1, k) + k\}$ . Since  $\alpha < 1$ , this precaution level is lower than the first

<sup>20</sup>Recall that, in our model, the default regime arises only when the bargaining fails. This contrasts with the models of Che and Hausch (1999) in which the action is chosen prior to bargaining, so the action is chosen to influence the bargaining. No such consideration arises in the current model.

<sup>21</sup>By (13)  $\pi_1(1) - h(1, k) > \pi_1(0)$ , which implies that  $\pi_1(1) - (1 - \epsilon)h(1, k) > \pi_1(0)$ .

best,  $k^*$ . This result parallels the standard under-investment found in the static holdup problem. In regime L, the precaution choice is given by  $k_0(L, \epsilon) = \arg \min_{k \in K} \{(1 - \epsilon)h(1, k) + k\}$ . Again,  $k_0(L, \epsilon) < k^*$  whenever there is an enforcement error  $\epsilon > 0$ . Without any enforcement error,  $k_0(L, 0) = k^*$ . Bebchuk (2001, 2002) focuses on this latter case. In fact, as long as  $1 - \epsilon > \alpha$ , our static predictions conform to those found in Bebchuk (2001, 2002): *Regime L dominates regime P2, which in turn dominates regime P1*. This result is in a sense not surprising since the liability rule uses more information than the property rule: the sanction on P1 depends on the actual harm under the liability rule, whereas the property rule does not use that information. In practice, the assessment of harm is unlikely to be accurate, but the effects of assessment errors will be similar to the enforcement errors. Consequently, with the small errors in assessment, the result will be in line with the recent findings.

- **Dynamic model** ( $\delta > 0$ ): The comparison of the alternative legal regimes is quite different for a large  $\delta$ . To see this, fix any  $\delta > 0$ , and consider the IR constraint in each case. In particular, in regime P1, the constraint becomes

$$\pi_1(1) - k \geq \pi_1(1), \quad (14)$$

which can only hold at  $k = 0$ . Hence, it follows that  $K_\delta(P1) = \{0\}$ , regardless of  $\delta$ .

In regime P2, the constraint simplifies to:

$$\alpha[\pi_1(1) - h(1, k) - \pi_1(0)] - k \geq 0. \quad (15)$$

Given (13), this inequality holds strictly at  $k = k_0(P2)$ .<sup>22</sup> Hence,  $\bar{k}_\delta(P2) > k_0(P2)$ . It is in fact possible that (15) holds at  $k^*$ , in which case the first-best would be attainable in the limit as  $\delta \rightarrow 1$ .

The (IR) constraint for regime L is a little different from its usual form, since Assumption 1(c) does not hold. That is, the “autarchy” outcome involves positive investment since  $\psi_1(k; L, \epsilon) - k$

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<sup>22</sup>By definition,

$$U_0(k_0(P2); P2) - \pi_1(0) > U_0(0; P2) - \pi_1(0).$$

Since

$$U_0(0; P2) - \pi_1(0) = \alpha[\pi_1(1) - h(1, 0) - \pi_1(0)] > 0,$$

where the inequality follows from (13),

$$U_0(k_0(P2); P2) - \pi_1(0) = \alpha[\pi_1(1) - h(1, k_0(P2)) - \pi_1(0)] - k_0(P2) > 0.$$

is not maximized at  $k = 0$ . Rather its maximum is attained at  $k_0(L, \epsilon)$ . Hence, (IR) at arbitrary  $k$  can be written as:

$$\pi_1(1) - (1 - \epsilon)h(1, k) - k \geq \pi_1(1) - (1 - \epsilon)h(1, k_0(L, \epsilon)) - k_0(L, \epsilon). \quad (16)$$

Observe that the LHS attains its maximum at  $k = k_0(L, \epsilon)$  and it equals the RHS at the maximum. This means that the constraint can only be satisfied at  $k = k_0(L, \epsilon)$ . Hence, the equilibrium set collapses to a singleton:  $K_\delta(L; \epsilon) = \{k_0(L, \epsilon)\}$ . This level equals  $k^*$  if  $\epsilon = 0$  but is strictly less than  $k^*$  if  $\epsilon > 0$ . Combining the above observations lead to the following conclusion.

**Result 5** (a) If  $\epsilon = 0$ , then  $L$  dominates  $P2$ , which in turn dominates  $P1$ , regardless of  $\delta \in [0, 1]$  and of equilibrium selection. (b) If  $\epsilon > 0$  and  $\alpha[\pi_1(1) - h(1, k^*) - \pi_1(0)] - k^* \geq 0$ , then there exists  $\hat{\delta}(\epsilon) < 1$  such that for all  $\delta \geq \hat{\delta}(\epsilon)$ ,  $\bar{k}_\delta(P2) > \bar{k}_\delta(L; \epsilon) = k_0(L, \epsilon) > \bar{k}_\delta(P1) = 0$ . (c)  $\hat{\delta}(\epsilon)$  is strictly decreasing in  $\epsilon$ .

Clearly, enforcement errors under the liability rule play an important role for the comparison. Without any enforcement errors, the liability rule implements the first-best outcome in a unique equilibrium, so it dominates the property rules, just as in the static benchmark. If there are enforcement errors associated with the liability rule, however, then any equilibrium outcome achievable under the liability rule is bounded away from the first-best, whereas the victim's right property rule can arbitrarily closely approximate the first-best if  $\delta$  is sufficiently large. To the extent that the probability of bargaining breakdown,  $1 - \delta$ , captures the magnitude of implicit bargaining cost, the result in (b) can be seen as reaffirming the Calabresi-Melamed maxim prescribing "the property rule for low bargaining cost and the liability rule for high bargaining cost." Again the basic intuition behind this result can be traced to the status quo minimization principle. The victim's right property rule imposes a bigger loss in the event of a bargaining breakdown than the liability rule, so the former rule can discipline the parties more credibly than the latter, toward a desirable outcome.

**Remark 3** Considering enforcement errors for the liability rule, but not for the property rule, may be construed as stacking the decks in favor of the latter, although as argued earlier differing degrees of enforcement uncertainty across the regimes are a key ingredient of the Calabresi-Melamed theory (and of the accurate description of reality). One may wonder, for instance, what happens if the property rule is also subject to some errors, say in terms of establishing the

*proof of violation. It turns out that the above result remains unchanged if the property rule faces similar, but small, errors. For instance, if party 1's violation is detected with probability  $1 - \epsilon$ , then the default regime is P2 with probability  $1 - \epsilon$  and P1 with probability  $\epsilon$ . Given a stronger IR condition,*

$$(1 - \epsilon)\alpha[\pi_1(1) - h(1, k^*) - \pi_1(0)] - k^* \geq 0,$$

*$\bar{k}_\delta(P2) \rightarrow k^*$  as  $\delta \rightarrow 1$ , thus Result 5-(b) continues to hold. Moreover, our main result does not require a particular direction of the enforcement errors: for instance  $\epsilon < 0$  will yield the same result. Finally, enforcement errors are not needed for the result, if the victim also has an *ex ante* precaution decision to make. It is well known that the liability rule entails inefficiencies if both parties need to invest in precaution. Meanwhile, the main argument for the property rule continues to hold, even though the two-sided investments are beyond the scope of the current model.*

## 5 Conclusion

We have shown that allowing for a simple and plausible investment dynamics in a holdup model produces much different implications on the design of important contracts and organizations than have been suggested in the literature. The novel theme in our prediction is that the incentives for specific investments depend not just on how a contract affects the investor's exposure to the specificity *at the margin* — the focus on the recent contract/organization literature — but more importantly on how the contract affects the investor's exposure to *absolute* specificity. The absolute specificity was never a concern in the static models since the individual rationality constraint is never binding there, but it is an important consideration in our dynamic model since the steeper incentives provided by investment dynamics causes the latter constraint to be binding.

A shift of emphasis from marginal specificity to absolute specificity in a sense takes us back to the original TCA authors (Klein et al., 1978; Williamson 1983,1985), who were largely concerned about the absolute level of specificities parties are subject to, as the source of inefficiencies and the rationale for organizational interventions. While we agree that the absolute specificity matters, our specific predictions differ from these authors as well. Our theory predicts that contracts that would exacerbate the parties' vulnerability to holdup — rather than those protecting them from holdup (as proposed by the TCA authors) — can be desirable. As discussed

in the paper, this view throws a more positive light on a variety of “hostage taking” or “hands-tying” arrangements such as exclusivity agreements, joint ownership of assets, trade contracts compelling parties to trade excessive amounts, and legal rules that assign an extreme measure (e.g., property rule prohibiting harm causing activity even when the benefit from the activity outweighs its harm). These contracts/organization forms can perform well in our dynamic model since they can create a strong equilibrium punishment for deviation.

The fact that our predictions are largely based on the absolute level of quasi-rents could also make them more readily confrontable to empirical testing. As Whinston (2002) points out, the GHM theory is more difficult to test, since the (marginal) effects of investment on the disagreement payoffs are harder to estimate (especially, since most feasible levels of investment are not made in equilibrium). In particular, whether the absolute level of specificity matters and, if so, in what way should be testable, if one can isolate an environment (and data) in which the relevant investments are so specific that no marginal values of investments are realizable in alternative status quo regimes.

## 6 Appendix: Proofs

**Proof of Lemma 1:** Given Assumption 1(a)-(b),  $U_\delta(k; m)$  has increasing differences in  $(k; \delta)$  and has a unique maximizer. Hence, we have  $k_\delta(m) \leq k_{\delta'}(m)$  for  $\delta' > \delta$ . The limiting properties hold by Berge's theorem of maxima since  $U_\delta$  is continuous and has a unique maximizer. Given Assumption 1(a)-(c), for any  $k \in K$

$$\begin{aligned} U'_\delta(k; m) &= \alpha[\phi'(k) - (1 - \delta)\psi'_2(k; m)] + (1 - \alpha)(1 - \delta)\psi'_1(k; m) - [1 - (1 - \alpha)\delta] \\ &< \alpha[\phi'(k) - (1 - \delta')\psi'_2(k; m)] + (1 - \alpha)(1 - \delta')\psi'_1(k; m) - [1 - (1 - \alpha)\delta'] = U'_{\delta'}(k; m), \end{aligned}$$

where the strict inequality holds since  $\psi'_1(k; m) < 1$  by Assumption 1(c) and since  $\psi'_2(k; m) \geq 0$ . Since  $U'_1(k^*; m) = 0$ , this inequality means, with  $\delta' = 1$ ,  $U'_\delta(k^*; m) < 0$ , which means that  $k_\delta(m) < k^* < \bar{k}$  for all  $\delta$ . Now consider any  $\delta' > \delta$  such that  $k_{\delta'}(m) > 0$ . Since  $k_{\delta'}(m) < \bar{k}$  from the above argument, we must have  $0 = U'_1(k_{\delta'}(m); m) > U'_\delta(k_{\delta'}(m); m)$ , which implies that  $k_\delta(m) \neq k_{\delta'}(m)$ . Given the first statement of the lemma, we then have  $k_\delta(m) < k_{\delta'}(m)$ , as needed. ■

We start by proving necessity. This will involve proving three lemmas on the way. Let  $y_m^\infty$  denote the limiting investment level given  $y_m(\cdot)$  if there is neither trade nor a breakdown in the negotiation.

**Proof of Lemma 2:** The proof consists of several steps:

**Step 1** In any MPE with contract  $m$  satisfying Assumptions 1 and 2, we have

$$\sigma_m(k) \geq U_\delta(k; m) + k - \delta[\alpha\psi_2(y_m^\infty(k)) - (1 - \alpha)\psi_1(y_m^\infty(k)) + (1 - \alpha)y_m^\infty(k)],$$

for any  $k \in K$ . If the parties agree to trade after reaching  $k$  in MPE, then

$$\sigma_m(k) \leq U_0(k; m) + k.$$

*Proof.* For notational simplicity, we suppress the dependence on  $m$ . Observe first that

$$\begin{aligned}
\sigma(k) &\geq \alpha\phi(k) - \alpha\delta\beta(y(k)) - \alpha(1-\delta)\psi_2(k) + (1-\alpha)\delta[\sigma(y(k)) - (y(k) - k)] + (1-\alpha)(1-\delta)\psi_1(k) \\
&= \alpha\phi(k) - (1-\delta)[\alpha\psi_2(k) - (1-\alpha)\psi_1(k)] - (1-\alpha)\delta(y(k) - k) \\
&\quad - \alpha\delta[\delta\beta(y^2(k)) + (1-\delta)\psi_2(y(k))] + (1-\alpha)\delta[\delta[\sigma(y^2(k)) - (y^2(k) - y(k))] + (1-\delta)\psi_1(y^2(k))] \\
&= \alpha\phi(k) - (1-\delta)\sum_{t=1}^{\infty}\delta^{t-1}[\alpha\psi_2(y^{t-1}(k)) - (1-\alpha)\psi_1(y^{t-1}(k))] - (1-\alpha)\sum_{t=1}^{\infty}\delta^t(y^t(k) - y^{t-1}(k)) \\
&\quad + \lim_{t \rightarrow \infty}\delta^t[(1-\alpha)\sigma(y^t(k)) - \alpha\beta(y^t(k))] \\
&= \alpha[\phi(k) - (1-\delta)\psi_2(k)] + (1-\alpha)(1-\delta)\psi_1(k) + (1-\alpha)\delta k \\
&\quad - (1-\delta)\sum_{t=1}^{\infty}\delta^t[\alpha\psi_2(y^t(k)) + (1-\alpha)(y^t(k) - \psi_1(y^t(k)))], \\
&= U_\delta(k) + k - (1-\delta)\sum_{t=1}^{\infty}\delta^t[\alpha\psi_2(y^t(k)) + (1-\alpha)(y^t(k) - \psi_1(y^t(k)))],
\end{aligned}$$

where the first inequality follows since the trade may not be optimal, the first equality holds by substituting the value functions (whether trade occurs or not at  $y(k)$ ), the second equality is obtained by following the procedure recursively and collecting terms, and the penultimate equality follows by rearranging the summation series and since  $\beta(\cdot)$  and  $\sigma(\cdot)$  are both finite (each lying in  $[0, \phi(\bar{k})]$ ).

To prove the first claim, note that the bracketed term in the last line is nondecreasing in  $y^t(k)$ , by Assumption 1(a) and (c). Thus, since  $y^t(k) \leq y^\infty(k)$ , the last line cannot be less than

$$\begin{aligned}
&U_\delta(k) + k - (1-\delta)\sum_{t=1}^{\infty}\delta^t[\alpha\psi_2(y^\infty(k)) + (1-\alpha)(y^\infty(k) - \psi_1(y^\infty(k)))] \\
&= U_\delta(k) + k - \delta[\alpha\psi_2(y^\infty(k)) + (1-\alpha)(y^\infty(k) - \psi_1(y^\infty(k)))],
\end{aligned}$$

as needed. To prove the second claim, notice that the first inequality (of the above string) holds with equality if trade occurs immediately at  $k$ . Next, recall that the bracketed term in the last line is nondecreasing in  $y^t(k)$ . Thus, since  $y^t(k) \geq k$ , the last line — and therefore  $\sigma(k)$  — is no greater than

$$\begin{aligned}
&U_\delta(k) + k - (1-\delta)\sum_{t=1}^{\infty}\delta^t[\alpha\psi_2(k) + (1-\alpha)(k - \psi_1(k))] \\
&= U_\delta(k) + k - \delta[\alpha\psi_2(k) + (1-\alpha)(k - \psi_1(k))] \\
&= U_0(k) + k.
\end{aligned}$$

■

**Step 2:** *From any initial level of investment, in an MPE with contract  $m$  satisfying Assumptions 1 and 2, trade occurs in the first period.*

*Proof.* Suppose trade does not occur immediately following  $k = y(0) \geq 0$ . (The dependence on the contract in place,  $m$ , will be suppressed throughout.) Then, we have from (2) that

$$\sigma(k) = \delta[\sigma(y(k)) - (y(k) - k)] + (1 - \delta)\psi_1(k). \quad (17)$$

Next, we show that  $\sigma(k) = \sigma(y(k)) - (y(k) - k) = \psi_1(k)$ . On the one hand, incentive compatibility implies that

$$\sigma(y(k)) - (y(k) - k) \geq \sigma(k)$$

(since  $y(k)$  is an equilibrium investment path from  $k$ ), on the other hand, it also implies that

$$\sigma(k) - k \geq \sigma(y(k)) - y(k)$$

(since  $k = y(0)$  and  $y(k)$  is also feasible starting from zero investment). These two inequalities give

$$\sigma(k) = \sigma(y(k)) - (y(k) - k),$$

which implies, when substituted into (17), that  $\sigma(k) = \sigma(y(k)) - (y(k) - k) = \psi_1(k)$ , as was to be shown.

We consider two cases. Suppose first  $k = y(0) > 0$ . If party 1 deviates from  $k = y(0)$  by not investing at all and perpetually inducing rejection then she will obtain

$$(1 - \delta) [\psi_1(0) + \delta\psi_1(0) + \delta^2\psi_1(0) + \dots] = \psi_1(0).$$

Since  $\sigma(k) - k = \psi_1(k) - k < \psi_1(0)$  by Assumption 1(c), the deviation is profitable. Hence, we have a contradiction.

Suppose now  $y(0) = 0$ . Since  $y(y(0)) = 0$ , if no trade occurs at  $y(0) = 0$ , then  $\beta(0) = \delta\beta(0) + (1 - \delta)\psi_2(0)$ , implying that  $\beta(0) = \psi_2(0)$ . Hence,

$$\sigma(0) + \beta(0) = \psi_1(0) + \psi_2(0).$$

Meanwhile, it follows from (2) and (3) that

$$\sigma(0) + \beta(0) \geq \phi(0).$$

We then have a contradiction since, by Assumption 2,  $\phi(0) > \psi_1(0) + \psi_2(0)$ . ■

**Step 3** In any MPE with contract  $m$  satisfying Assumptions 1 and 2,  $y_m^\infty(k) \leq \max\{k, k_\delta(m)\}$ .

*Proof.* Suppose not. Then, there must exist  $k \in K$  such that  $\hat{k} := y(k) > \max\{k, k_\delta\}$ . (The dependence on  $m$  is suppressed to economize on notation.) Since  $\hat{k}$  is an equilibrium choice starting from  $k$ , we must have

$$\sigma(\hat{k}) - \hat{k} \geq \sigma(k') - k', \forall k' \geq k.$$

Let

$$k^o := \sup\{k' | \sigma(k') - k' = \sigma(\hat{k}) - \hat{k}\}.$$

By definition, for each  $\epsilon > 0$ , there exists an  $k_\epsilon \leq k^o$  within  $\epsilon$  distance from  $k^o$ , such that  $\sigma(k_\epsilon) - k_\epsilon = \sigma(\hat{k}) - \hat{k}$ . Since  $\sigma(k'') - k'' < \sigma(\hat{k}) - \hat{k}$  for all  $k'' > k^o$ ,  $y^\infty(k_\epsilon) \leq k^o$ . Further, this must hold for any  $\epsilon > 0$ , so we must have  $y^\infty(k') \leq k^o$  for any  $k' < k^o$ .

Meanwhile, for  $\epsilon > 0$  sufficiently small, trade will occur after reaching  $k_\epsilon$  (since  $k_\epsilon \rightarrow k^o$  as  $\epsilon \rightarrow 0$  and  $y(k_\epsilon) \leq k^o$ , as noted above). Hence, by Lemma Step 1,

$$U_0(k_\epsilon) \geq \sigma(k_\epsilon) - k_\epsilon \geq U_\delta(k_\epsilon) - \delta[\alpha\psi_2(y^\infty(k_\epsilon)) + (1 - \alpha)(y^\infty(k_\epsilon) - \psi_1(y^\infty(k_\epsilon)))].$$

Now, take the limit of all sides as  $\epsilon \downarrow 0$ , using the fact that  $\lim_{\epsilon \downarrow 0} k_\epsilon = k^o$  and that  $y^\infty(k_\epsilon) \leq k^o$ , which implies that  $\lim_{\epsilon \downarrow 0} y^\infty(k_\epsilon) = k^o$ . Then, the left-most and the right-most sides converge to  $U_0(k^o)$ , implying that  $\lim_{\epsilon \downarrow 0} \{\sigma(k_\epsilon) - k_\epsilon\} = U_0(k^o)$ . Since, by the definition of  $k_\epsilon$ ,  $\lim_{\epsilon \downarrow 0} \{\sigma(k_\epsilon) - k_\epsilon\} = \sigma(\hat{k}) - \hat{k}$ , we must have  $\sigma(\hat{k}) - \hat{k} = U_0(k^o)$ .

Suppose now that, starting at  $k$ , party 1 deviates to  $k' \in [\max\{k, k_\delta\}, \hat{k}]$  instead of investing up to  $\hat{k} = y(k)$ . We have

$$\begin{aligned} \sigma(k') - k' &\geq U_\delta(k') - \delta[\alpha\psi_2(y^\infty(k')) + (1 - \alpha)(y^\infty(k') - \psi_1(y^\infty(k')))] \\ &\geq U_\delta(k') - \delta[\alpha\psi_2(k^o) + (1 - \alpha)(k^o - \psi_1(k^o))] \\ &> U_0(k^o) \\ &= \sigma(\hat{k}) - \hat{k}, \end{aligned}$$

where the first inequality follows from Step 1, the second inequality follows since  $y^\infty(k') \leq k^o$  as noted above, the strict inequality follows since  $k^o \geq \hat{k} > k' \geq k_\delta$ ,<sup>23</sup> and the last equality

<sup>23</sup>Note that

$$U_0(k^o) - \delta[\alpha\psi_2(k^o) + (1 - \alpha)(k^o - \psi_1(k^o))] = U_\delta(k^o) < U_\delta(k'),$$

since  $k^o > k' \geq k_\delta$ .

follows from the argument established above. The inequality proves that the deviation to  $k' < \hat{k}$  is profitable, contradicting the initial hypothesis that  $y(k) > \max\{k, k_\delta\}$  for some  $k \in K$ . ■

Steps 1 and 2 yield parts (i) and (ii), whereas Step 3 has proven part (iii). ■

**Proof of Theorem 1:** We have already shown in the text that  $k \in K_\delta$  if  $k$  is implementable by  $m$ . Hence, it suffices to show that any  $\hat{k} \in K_\delta(m)$  can be implemented as an MPE. Let  $\bar{k}_\delta(m) := \max K_\delta(m)$ , and consider the following investment strategy:

$$\hat{y}_m(k) = \begin{cases} \hat{k} & \text{if } k \in [0, \hat{k}], \\ \bar{k}_\delta(m) & \text{if } k \in (\hat{k}, \bar{k}_\delta(m)] \\ k & \text{if } k \in (\bar{k}_\delta(m), \bar{k}]. \end{cases}$$

With this strategy, party 1 initially invests to  $\hat{k} = y_m(0)$  and never increases further (i.e.,  $y_m(y_m(0)) = \hat{k}$ ).

Substituting this investment strategy into (2) and (3) and solving for party 1's payoff, we obtain

$$\hat{\sigma}(k) - k = \begin{cases} \hat{\sigma}(k) - k & \\ \left\{ \begin{array}{ll} \max\{U_\delta(k) - \delta[\alpha\psi_2(\hat{k})] - (1-\alpha)\psi_1(\hat{k}) + (1-\alpha)\hat{k}, \delta[U_0(\hat{k}) + k] + (1-\delta)\psi_1(k) - k\} & \text{if } k \in [0, \hat{k}], \\ \max\{U_\delta(k) - \delta[\alpha\psi_2(\bar{k}_\delta)] - (1-\alpha)\psi_1(\bar{k}_\delta) + (1-\alpha)\bar{k}_\delta, \delta[U_0(\bar{k}_\delta) + k] + (1-\delta)\psi_1(k) - k\} & \text{if } k \in (\hat{k}, \bar{k}_\delta] \\ U_0(k) & \text{if } k \in (\bar{k}_\delta(m), \bar{k}]. \end{array} \right. & \end{cases}$$

(The dependence on  $m$  is suppressed to economize on space.)

Note first that trade follows immediately after  $\hat{k}$  is reached, as claimed. That is,  $\hat{\sigma}(\hat{k}) = U_0(\hat{k}) + \hat{k} \geq \delta[U_0(\hat{k}) + \hat{k}] + (1-\delta)\psi_1(\hat{k})$  since  $\phi(k) \geq \psi_1(k) + \psi_2(k)$  by Assumption 2. Observe next that likewise,  $\hat{\sigma}(k) - k = U_0(k; m)$  at  $k = \bar{k}_\delta(m)$ . In particular, these two values constitute local maxima, with the former being the global maximum. In other words, party 1's payoff at  $k = \hat{k}$  dominates her payoff at any  $k \in K$ , and her payoff at  $k = \bar{k}_\delta(m)$  dominates her payoff at any  $k \in (\hat{k}, \bar{k}]$ . It then follows that  $\hat{k} \in \arg \max_{\tilde{k} \in [k, \bar{k}]} \{\hat{\sigma}(\tilde{k}) - \tilde{k}\}$  for any  $k \leq \hat{k}$ , and  $\max\{\bar{k}_\delta(m), k\} \in \arg \max_{\tilde{k} \in [k, \bar{k}]} \{\hat{\sigma}(\tilde{k}) - \tilde{k}\}$  for any  $k > \hat{k}$ . Hence, the aforementioned strategy,  $\hat{y}_m(\cdot)$ , is optimal, given  $\hat{\sigma}(\cdot)$ .

The last statement follows from Lemma 2-(i). ■

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