Consensus reaching in committees*

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Abstract. In this paper, we apply a consensus model to decision-making in committees that have to choose one or more alternatives from a set of alternatives. Decision makers may be advised to adjust their preferences in order to obtain a better consensus. A simple example is presented.

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1 The consensus model

The problem of consensus reaching appears very frequently. In particular, a committee has to solve this problem when choosing one or more alternatives from a set of applicants, proposals, etc. Using concepts introduced in Carlsson et al. (1992), we formulate a consensus model for decision-making in committees.

The formal model is as follows. Let N be the set of all decision makers who try to reach consensus on some alternatives. Let A denote the set of all alternatives. Let C denote the set of all criteria. Each decision maker is supposed to have an evaluation of the importance of the criteria. Hence, for each $i \in N$, we assume $h_i : C \to [0,1]$, such that

$$\forall i \in N \ [\sum_{c \in C} h_i(c) = 1], \tag{1}$$

where $h_i(c)$ is i's evaluation (or weight) of criterion c. Moreover, for each $i \in N$, we also assume $g_i : C \times A \to [0,1]$ such that

$$\forall c \in C \ [\sum_{a \in A} g_i(c,a) = 1], \tag{2}$$

where $g_i(c,a)$ is the value of alternative a to decision maker i with respect to criterion c. In order to determine their evaluations $h_i(c)$ of the importance of the criteria c and their values $g_i(c,a)$ of alternative a with respect to criterion c, decision makers i might use Saaty's Analytical Hierarchy Process (Saaty, 1977, 1980) or the MACBETH software (Bana e Costa and Vansnick, 1999; Bana e Costa et al., 2003).

Let $(h_i(c))_{c \in C}$ denote the $1 \times |C|$ matrix representing the evaluation (comparison) of the criteria by decision maker i, and let $(g_i(c,a))_{c \in C, a \in A}$ denote the $|C| \times |A|$ matrix containing i's evaluation (comparison) of all alternatives with respect to each criterion in C. For each $i \in N$, we define $f_i : A \to [0,1]$ such that

$$(f_i(a))_{a \in A} = (h_i(c))_{c \in C} \cdot (g_i(c,a))_{c \in C, a \in A}, \tag{3}$$

where $f_i(a)$ is i's evaluation of alternative a, and $(f_i(a))_{a \in A}$ is the $1 \times |A|$ matrix containing i's evaluation of each alternative.

The 'distances' between decision makers i and j are calculated as follows:

$$d(f_i, f_j) = \sqrt{\frac{1}{|A|} \sum_{a \in A} (f_i(a) - f_j(a))^2}. \tag{4}$$

By (1), (2), and (3), $0 \leq d(f_i, f_j) \leq 1$. A generalized consensus degree δ^* for a committee is defined as

$$\delta^* = 1 - d^* = 1 - \max\{d(f_i, f_j) \mid i, j \in N\}. \tag{5}$$

In the model a certain consensus degree $\widetilde{\delta}$ is required in advance. We say that a committee reaches consensus if $\delta^* \geq \widetilde{\delta}$.

We assume a kind of mediator, called the chairman. If the decision makers reach a consensus degree $\delta^* < \widetilde{\delta}$, it is the chairman who decides who should adjust his/her preferences in order to reach a better consensus. Let i^* denote a decision maker who will be advised to adjust his/her preferences. Let

$$D^* = \{i \in N \mid \exists j \in N \; [d(f_i, f_j) = d^*]\}. \tag{6}$$

$i^* \in D^*$ is a decision maker from D^* who satisfies the following condition

$$i^* = \arg\min_{k \in D^*} d^*_{-k}, \tag{7}$$

where d^*_{-k} is defined for $k \in D^*$ as

$$d^*_{-k} = \max\{d(f_i, f_j) \mid i, j \in N \setminus \{k\}\}. \tag{8}$$

If there are two members satisfying condition (7), the chairman chooses one of them. We assume a majority degree $\widetilde{m}$, that is, the minimal number of decision makers necessary to make a decision. The chairman's advice always leads to a increase of δ^*. If i^* refuses to change his/her preferences, he/she is 'excluded' from further discussion. If $|N \setminus \{i^*\}| \geq \widetilde{m}$, the remaining decision makers try to reach consensus. If $|N \setminus \{i^*\}| < \widetilde{m}$, the committee does not reach consensus. If i^* follows the chairman's advice, then the new generalized consensus degree δ'^* is calculated, and if $\delta'^* \geq \widetilde{\delta}$, the committee reaches consensus. If $\delta'^* < \widetilde{\delta}$, then a new decision maker, i'^*, is appointed by the chairman for adjusting his/her preferences, etc.

If consensus is reached by the committee, that is, if the generalized (final) consensus degree is not smaller than $\widetilde{\delta}$, a mean consensus decision is calculated. Let $N^* \subseteq N$ denote the set of the decision makers who succeeded in reaching consensus. Assuming that the decision makers might be unequally 'important', we add up the weighted (final) values of the alternatives to all decision makers from N^*. For each $a \in A$, the weighted value $f(a)$ of alternative a is defined as

$$f(a) = \sum_{i \in N^*} w'_i \cdot f_i(a), \tag{9}$$

where for each $i \in N^*$

$$w'_i = \frac{w_i}{\sum_{j \in N^*} w_j}, \tag{10}$$

and w_i means the 'weight' of decision maker $i \in N^*$. The committee chooses the alternatives with the greatest value of $f(a)$. In particular, if only one alternative may be chosen, the committee chooses the alternative a^* such that

$$a^* = \arg\max_{a \in A} f(a). \tag{11}$$

If there are (at least) two alternatives satisfying condition (11), and only one alternative may be chosen, the chairman decides for one of them.

2 The example

In this section, we present an application of the consensus model to a COST Action activity. Suppose that the Management Committee (MC) of the COST Action has to choose candidates for a grant for a 'Short Term Scientific Mission' (STSM). We consider a small MC, consisting of only five members, i.e., $N = \{1, 2, 3, 4, 5\}$. Suppose there are two grants to be assigned, but four applicants, i.e., $A = \{a_1, a_2, a_3, a_4\}$. Each MC member will evaluate each applicant with respect to three criteria. We have $C = \{c_1, c_2, c_3\}$, where c_1 is the quality of the detailed work plan, c_2 is the relevance of the STSM proposal to the COST Action and to the research conducted by the host group, and c_3 is the quality of the CV of the applicant.

The majority degree is assumed to be $\widetilde{m} = 3$. Since the STSM grants are very attractive to junior researchers, the MC wants to be rather unanimous in choosing the candidates, and the required consensus degree is $\widetilde{\delta} = 0.95$. If more than two applicants are chosen by our consensus method, then we propose the chairman to choose only two of them.

The relative weights that the different MC members attach to the criteria c_1, c_2, c_3 are as follows: $h_1 = (0.4, 0.3, 0.3)$, $h_2 = (0.4, 0.4, 0.2)$, $h_3 = (0.1, 0.1, 0.8)$, $h_4 = (0.4, 0.2, 0.4)$, $h_5 = (0.2, 0.6, 0.2)$. Moreover, we assume that

$$(g_1(c,a))_{c\in C, a\in A} = \begin{pmatrix} 0.2 & 0.3 & 0.3 & 0.2 \\ 0.1 & 0.2 & 0.4 & 0.3 \\ 0.5 & 0.2 & 0.2 & 0.1 \end{pmatrix}$$

$$(g_2(c,a))_{c\in C, a\in A} = \begin{pmatrix} 0.35 & 0.25 & 0.2 & 0.2 \\ 0.25 & 0.25 & 0.25 & 0.25 \\ 0.2 & 0.5 & 0.2 & 0.1 \end{pmatrix}$$

$$(g_3(c,a))_{c\in C, a\in A} = \begin{pmatrix} 0.3 & 0.4 & 0.1 & 0.2 \\ 0.2 & 0.1 & 0.2 & 0.5 \\ 0.3 & 0.4 & 0.2 & 0.1 \end{pmatrix}$$

$$(g_4(c,a))_{c\in C, a\in A} = \begin{pmatrix} 0.25 & 0.25 & 0.25 & 0.25 \\ 0.1 & 0.2 & 0.5 & 0.2 \\ 0.25 & 0.35 & 0.2 & 0.2 \end{pmatrix}$$

$$(g_5(c,a))_{c\in C, a\in A} = \begin{pmatrix} 0.3 & 0.2 & 0.3 & 0.2 \\ 0.5 & 0.1 & 0.2 & 0.2 \\ 0.5 & 0.2 & 0.1 & 0.2 \end{pmatrix}.$$

Hence, the MC members' evaluations of the applicants are as follows:
$(f_1(a))_{a\in A} = (0.26, 0.24, 0.3, 0.2)$, $(f_2(a))_{a\in A} = (0.28, 0.3, 0.22, 0.2)$,
$(f_3(a))_{a\in A} = (0.29, 0.37, 0.19, 0.15)$, $(f_4(a))_{a\in A} = (0.22, 0.28, 0.28, 0.22)$,
$(f_5(a))_{a\in A} = (0.46, 0.14, 0.2, 0.2)$.
Table 1 presents the distances between the MC members.

ABOUT HERE TABLE 1

Hence, we get $d^* = d(f_3, f_5) = 0.1453$, $D^* = \{3, 5\}$. One of the MC members from D^* will be asked to change his/her preferences. In order to find which one, we calculate d^* when 3 is removed, and d^* when 5 is removed. Table 2 presents the distances between the MC members when 3 is removed, and Table 3 presents the distances when 5 is removed.

ABOUT HERE TABLE 2

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Hence, $d^*_{-3} = d(f_4, f_5) = 0.1449$, and $d^*_{-5} = d(f_1, f_3) = 0.09$. Therefore, it is decision maker 5 who is asked to change his preferences, i.e., $i^* = 5$. Suppose that 5 announces that he will not change his preferences at all, and hence, the MC decides to exclude him from further discussion. Now, only decision makers 1, 2, 3, and 4 try to reach consensus. We have then $d'^* = 0.09$, $D'^* = \{1, 3\}$, and moreover, $d'^*_{-1} = 0.0806$, $d'^*_{-3} = 0.051$, which gives $i'^* = 3$. Hence, 3 is asked to change his preferences. Let us suppose that he is very cooperative, and agrees to follow the chairman's advice. Assume his new preferences are as follows: $h'_3 = (0.3, 0.3, 0.4)$,

$$(g'_3(c, a))_{c \in C, a \in A} = \begin{pmatrix} 0.3 & 0.3 & 0.2 & 0.2 \\ 0.2 & 0.1 & 0.3 & 0.4 \\ 0.3 & 0.4 & 0.2 & 0.1 \end{pmatrix},$$

and consequently, 3's new evaluations are $(f'_3(a))_{a \in A} = (0.27, 0.28, 0.23, 0.22)$. Table 4 shows the distances between decision makers from the set $\{1, 2, 3, 4\}$ after 3 adjusted his preferences.

ABOUT HERE TABLE 4

We have then $d''^* = 0.051$, $D''^* = \{1, 2\}$, $d''^*_{-1} = 0.0447$, $d''^*_{-2} = 0.0418$, and hence 2 is asked to change his preferences, i.e., $i''^* = 2$. Decision maker 2 would be willing, in principle, to adjust his preferences. Nevertheless, since he can see that even without him a decision will be made, he refuses to change his preferences. In this situation, that is, with 2 removed, we have $m = 3 \geq \widetilde{m}$, $d'''^* = 0.0418$, $\delta'''^* = 0.9582 > 0.95 = \widetilde{\delta}$, and therefore MC members 1, 3, and 4 reach consensus. In order to choose two candidates, they calculate the total values of the candidates. The MC decides to treat all the members equally, that is, for $k = 1, 2, 3, 4$ we have $f(a_k) = f_1(a_k) + f'_3(a_k) + f_4(a_k)$.
Hence, $f(a_1) = 0.75$, $f(a_2) = 0.8$, $f(a_3) = 0.81$, $f(a_4) = 0.64$, and choosing two candidates with the greatest total value, the MC members propose to give the STSM grants to candidates a_2 and a_3.

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Table 1: Distances $d(f_i, f_j)$, where $i, j \in N = \{1, 2, 3, 4, 5\}$.

	1	2	3	4	5
1		0.051	0.09	0.0316	0.1225
2	0.051		0.0458	0.0447	0.1208
3	0.09	0.0458		0.0806	0.1453
4	0.0316	0.0447	0.0806		0.1449
5	0.1225	0.1208	0.1453	0.1449	

Table 2: Distances $d(f_i, f_j)$, where $i, j \in N = \{1, 2, 4, 5\}$.

	1	2	4	5
1		0.051	0.0316	0.1225
2	0.051		0.0447	0.1208
4	0.0316	0.0447		0.1449
5	0.1225	0.1208	0.1449	

Table 3: Distances $d(f_i, f_j)$, where $i, j \in N = \{1, 2, 3, 4\}$.

	1	2	3	4
1		0.051	0.09	0.0316
2	0.051		0.0458	0.0447
3	0.09	0.0458		0.0806
4	0.0316	0.0447	0.0806	

Table 4: Distances $d(f'_i, f'_j)$, where $i, j \in \{1, 2, 3, 4\}$, and $f'_i = f_i$ for $i \in \{1, 2, 4\}$.

	1	2	3	4
1		0.051	0.0418	0.0316
2	0.051		0.0158	0.0447
3	0.0418	0.0158		0.0354
4	0.0316	0.0447	0.0354	