PREFERENCE UNCERTAINTY, VOTER PARTICIPATION AND ELECTORAL EFFICIENCY: AN EXPERIMENTAL STUDY*

by

Jens Großer[i,ii]

Tamar Kugler[iii]

Arthur Schram[ii]

ABSTRACT

We experimentally study the impact of preference uncertainty on voter turnout and electoral efficiency in a participation game. We find higher participation rates when the electorate is informed about the level of support for various candidates than when group sizes remain uncertain. Moreover, despite higher free riding incentives majorities win more often than minorities. In aggregate, whether or not group sizes are known hardly affects welfare. We also distinguish between allied and floating voters and our data show that the lower turnout under uncertainty can be attributed to floating voters participating less. Finally, our results match better the predictions by quantal response (logit) equilibria than by (Bayesian-) Nash equilibria.

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[i] Department of Economics, University of Cologne; Albertus-Magnus-Platz, 50923 Cologne, Germany.
[ii] Center for Research in Experimental Economics and political Decision making, University of Amsterdam, Roeterstraat 11, 1018 WB Amsterdam, the Netherlands.
[iii] Department of Management and Policy, University of Arizona, McClelland Hall, P.O. Box 210108, Tucson, Arizona 85721-0108, US.

email: j.w.grosser@uva.nl; tkugler@eller.arizona.edu; schram@uva.nl

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1. INTRODUCTION

For many people the goal of elections is to determine the electorate's preferences and to allocate political power accordingly. To achieve this, high participation is often considered to be desirable. For example, Lijphart (1997) argues that *low* turnout is a serious democratic problem because it usually involves *unequal* turnout, causing unequal political influence. Indeed, if voter turnout is voluntary it is not obvious that the result of an election will reflect the electorate's preferences, even if every vote is cast sincerely for the most preferred candidate. In fact, some models of costly voluntary voting yield equilibria where turnout in the majority is lower than in the minority (Palfrey and Rosenthal, 1983). One reason is that the incentive to free ride on the costly participation of other group members is higher for voters in the majority. As a consequence, the minority's preferences are over-represented, which may result in welfare losses for society as a whole.

In order to properly evaluate the 'high turnout is better' claim, a welfare analysis is needed. Given that there are costs related to casting a vote, changes in turnout affect welfare directly. One needs to analyze the trade-off between this effect and the welfare consequences of a change in the majority's probability of winning the election (Palfrey and Rosenthal, 1983; Ledyard. 1984).[1] The result of such a welfare analysis may well be that higher participation results in lower welfare. In a model of simple majority voting with uncertainty about costs and other voters' preferences, Börgers (2004) shows that voluntary participation dominates compulsory voting in terms of welfare. Hence, there may indeed be circumstances in which it is in a society's interest to sacrifice benefits derived from the election outcome in favor of saving voting costs.

In this paper, we study the welfare implications of endogenous voter participation. We do so in an environment of preference uncertainty, *i.e.*, voters do not know the exact levels of support for the candidates. This is caused by a group of 'floating voters', who remain uncertain about the candidate to support until the election takes place. Note that this kind of incomplete information is quite realistic for most elections.[2] In fact, if preferences were known there would be no need for an election, one could simply apply a decision rule. Uncertainty about the electorate's level of disagreement, defined as the size of the minority (Feddersen and Sandroni, 2004), has indeed long been recognized as an important feature of

[1] More generally, costs involved in the provision of a public good should be taken into account when evaluating the welfare consequences of its provision (*e.g.*, Ledyard and Palfrey, 1994, 1999, 2002).

[2] *E.g.*, in 2004 the pollster Populus categorized 35% of the UK electorate as 'floating voters' (Times Online, 07.09.2004, "Boost for Kennedy as Blair and Howard slip")

many elections (Daudt, 1961). Empirical studies document the large impact that floating voters can have on an electoral outcome (Campbell, 1960; Kramer, 1971; DeNardo, 1980).

A possible consequence of uncertainty in elections is that costly participation may be low. For example, uncertainty about the size of the electorate yields low equilibrium participation levels (Myerson, 1998), as does uncertainty about voting costs and other voters' preferences (Palfrey and Rosenthal, 1985). Moreover, given preference uncertainty expected equilibrium participation decreases in voting costs, the size of the electorate, and the expected level of disagreement (Goeree and Großer, 2004). The efficiency trade-off between the lower turnout costs on the one hand and the effect of lower turnout on the majority's probability of victory on the other, remains a topic to be investigated. There is theoretical evidence for certain environments, however, that pre-election information aimed at decreasing uncertainty, *e.g.* through polls, can increase turnout and stimulate the majority (minority) to participate less (more), with negative welfare implications (Goeree and Großer, 2004).[3]

Our goal is to investigate the effects of preference uncertainty on turnout levels and welfare. This investigation will take place at two complementary levels: theoretically and experimentally. Both utilize the framework of a participation game (Palfrey and Rosenthal, 1983, 1985; hereafter PR83, PR85). Laboratory experiments are also a useful complement to empirical studies on voter turnout (Matsusaka and Pada, 1993), because they allow one to control the environment in which the decision is made. In particular, we will take advantage of this method by exerting control over distinct 'types' of voters and 'closeness' of elections.

With regard to voter types, we will be able to distinguish between 'allied' and 'floating' voters. The former have a stable alliance to a candidate. Floating voters, on the other hand, are often described as having a weaker party identification, because they (expect to) benefit less from a preferred candidate winning than allied voters, or are less interested in politics (Campbell, 1960). A second difference between voter types is that a floating voter does not decide on whom to support until shortly before the elections and often switches from one candidate to the other. As a consequence, all voters are uncertain about the level of support for the two candidates when deciding whether or not to vote. It is this uncertainty (and the way it differs between allied and floating voters) that we are interested in. Laboratory control allows us to let allied and floating voters derive the same benefits from winning the election

[3] Lohmann (1994) shows how (costly) pre-election political action may give biased aggregate information in equilibrium, with negative effects for welfare.

(*i.e.*, correct for possible differences in benefits between the two types) and therefore to focus purely on the uncertainty effect.[4]

The second focus of this research is closeness. It is important to start by clearly defining what closeness means in this context. In particular, empirical studies measure *ex post* closeness: the difference in the number of actual votes cast by supporter groups (Matsusaka and Pada, 1993). In contrast, our experiment allows us to impose *ex ante* closeness. We determine the 'level of disagreement' in the electorate by controlling group size differences. Information about this level is typically not fully available outside the laboratory, and can only be approximated by polls and election outcomes. Moreover, we control the information regarding closeness given to the voters before the election. In this way, we are able to directly isolate the impact of uncertainty about other voters' preferences on turnout and welfare for various realizations of a preference distribution in the electorate.

Our experimental results show that participation is 20% lower when preferences are uncertain than when the electorate's level of disagreement is known. We can attribute this completely to floating voters, who vote less when preferences are uncertain. Allied voters are not affected by this information. Moreover, participation of informed voters increases with the electorate's level of disagreement: turnout is high when groups are of equal size and low when minorities are smallest. When voters are uncertain about other voters' preferences, turnout is in between these two extremes. The welfare effects that we observe depend on the ex ante closeness. Preference uncertainty decreases electoral efficiency when the level of disagreement is low and increases efficiency when disagreement is high. In aggregate, welfare is barely affected by whether or not the electorate is informed about the level of disagreement.

The participation game used in our experimental design underlies many theoretical studies of turnout. In the original version of this game (PR83), there are two (or more) groups with sizes known to all voters. Everyone decides privately whether or not to 'participate' in an action, where participation is costly. Participation is beneficial to every member of one's own group and harmful to members of other groups: the (higher) reward is given to the group with the higher number of 'participants'.[5] The participation game simultaneously combines a between-group conflict for the higher reward with a within-group conflict, where each group

[4] Note that it may be in the interest of both types, allied and floating voters, not to reveal their preferences before elections. Such behavior could also create preference uncertainty but is not the focus of our study.

[5] The participation game assumes private values. For examples of voting games involving common values see *e.g*. Feddersen and Pesendorfer (1996, 1997, 1999).

member has an incentive to free ride on costly participation by her co-members. Typically, multiple Nash equilibria occur. In some equilibria the minority has a higher probability of victory than the majority. For certain parameter configurations, equilibria with substantial levels of participation exist, even in large electorates (PR83). With cost and preference uncertainty, however, turnout levels are very low and the 'paradox of voting' persists (PR85).

The experimental literature on participation games is scarce. Bornstein (1992) reports the first experiment on participation behavior in small groups. Subsequent studies investigate variations in group and electorate size (Schram and Sonnemans, 1996a; Hsu and Sung, 2002); the effect of proportional representation *vs.* winner-takes-all elections (Schram and Sonnemans, 1996a); the role of group identification and communication (Schram and Sonnemans, 1996b; Goren and Bornstein, 2000); and the introduction of payoff uncertainty under various voting costs (Cason and Mui, forthcoming). Other recent studies extend the model to allow for endogenous information about other voters' turnout (Großer and Schram, 2004); and endogenous policy making and group formation (Großer and Giertz, 2005). In all these experiments, relatively high participation is observed, albeit lower than in most general elections around the world. Typically, the standard (Bayesian-) Nash equilibrium concept finds little empirical support. However, Goeree and Holt (forthcoming) and Cason and Mui (forthcoming) show that logit equilibria can account for some of the data. To the best of our knowledge, this study is the first to experimentally investigate the effects of uncertainty about the level of disagreement in a participation game.

The organization of the remainder of this paper is as follows. Section 2 defines the participation game with preference uncertainty (PU) and discusses its equilibria. We will distinguish between Bayesian-Nash and logit equilibria for the game. In section 3, we describe the experimental design and derive specific equilibria for the parameters of our experiment. Section 4 presents our experimental results and compares them to the equilibrium predictions. Section 5 concludes.

2. THE PU-PARTICIPATION GAME

GROUP STRUCTURE

Consider a democracy in which elections are decided by simple majority rule and ties are broken by a coin-toss. The electorate consists of two groups, each supporting one of two exogenous candidates. We will distinguish between two scenarios. In the first, there are no alliances between voters and candidates. Each voter can belong to one supporter group in one election and to the other group in the next. The second scenario is where there is a number of fixed voters in each group. This number is equal and commonly known. One interpretation is that these are 'allied' voters, whereas the others are 'floating' voters. Because of this asymmetry within a group, our model will allow allied and floating voters to follow distinct decision rules.

THE GAME

The players in the participation game (PR83) are the E (risk neutral) voters in an electorate, each seeking to maximize the own payoff. Each voter belongs to one of two supporter groups $i = A, B$. Let integer $N_i \geq 1$, $i = A, B$, be the number of voters in i, with $N_A + N_B = E$. The notation j_i, $i = A, B$, will be used to represent voter $j \in i$. Each voter j_i faces a strategy set consisting of two pure strategies $v_{j_i} \in \{0,1\}$, where $v_{j_i} = 1$ denotes participation in favor of candidate i and $v_{j_i} = 0$ denotes abstention. A mixed strategy profile for j_i is given by the probabilities of participation q_{j_i} and abstention $1 - q_{j_i}$, where $0 \leq q_{j_i} \leq 1$. All voters in the electorate make their decisions simultaneously. Aggregate participation in $i = A, B$ is

$$V_i \equiv \sum\nolimits_{j_i} v_{j_i} \qquad (1)$$

and, for later use, aggregate participation by other voters in i than j_i is denoted by

$$V_i^{-j_i} \equiv V_i - v_{j_i}. \qquad (2)$$

Payoffs are determined by the outcome of the election and the cost of voting. Normalizing the individual benefit from having one's preferred candidate win (lose) the election to 1 (0), the election outcome determines an (expected) benefit for voter j_i, $i = A, B$, given by

$$w_{j_i}(V_i, V_{-i}) = \begin{cases} 0 & \text{if} \quad V_i < V_{-i} \\ 1/2 & \text{if} \quad V_i = V_{-i} \\ 1 & \text{if} \quad V_i > V_{-i}, \end{cases} \qquad (3)$$

where V_{-i} denotes aggregate participation in the group j_i does not belong to. Note that $w_{j_i}(V_i, V_{-i})$ is non-decreasing (non-increasing) in its first (second) argument and symmetric to the groups, $w_{j_{-i}}(.,.) = 1 - w_{j_i}(.,.)$. Furthermore, assume identical participation costs to all voters within the range

$$c \in (0,1), \tag{4}$$

$\forall j_i$, $i = A, B$. The payoff for any voter j_i, $i = A, B$, is then given by

$$\pi_{j_i} = w_{j_i}(V_i, V_{-i}) - v_{j_i} c. \tag{5}$$

We now introduce preference uncertainty (PU), thus creating a Bayesian-Nash game. PU is formally characterized by common knowledge about:

(*i*) a minimal group size denoted by integer $\underline{N}_i \geq 1$, which implies a maximal group size $\overline{N}_{-i} = E - \underline{N}_i$, with $1 \leq \underline{N}_i < \overline{N}_i \leq E - 1$, $i = A, B$;

(*ii*) a discrete probability distribution over all possible electoral compositions (N_i, N_{-i}) from the set $\{(\underline{N}_A, \overline{N}_B), (\underline{N}_A + 1, \overline{N}_B - 1), ..., (\overline{N}_A, \underline{N}_B)\}$, with $prob(.,.) > 0$ for each element in the set.

Throughout, we will only consider the symmetric case, where $\underline{N}_i = \underline{N}_{-i} \geq 1$.

(BAYESIAN-) NASH EQUILIBRIA

PR83 analyze the participation game without PU and, among others, derive two types of Nash equilibria. For $c > 1/2$, the only equilibrium is in pure strategies where nobody votes. In contrast, when $c < 1/2$ there is no equilibrium in pure strategies, unless $N_A = N_B$, in which case the only such equilibrium is one where everyone votes.[6] Moreover, for $c < 1/2$ PR83 also derive 'totally quasi-symmetric mixed strategy equilibria' (denoted by '$(q, \tilde{q})$-equilibria') where all voters within one group vote with probability $q \in (0,1)$ and all voters in the other group with $\tilde{q} \in (0,1)$. They derive two such equilibria. First, for the symmetric case $N_A = N_B$, two totally quasi-symmetric equilibria exist if c is large enough. Both involve equal voting probabilities in both groups $(q = \tilde{q})$. No such equilibrium exists for low costs. Second, without restriction to equal group sizes, if c is small enough, two totally quasi-symmetric mixed strategy equilibria exist with the members of one group voting with probability q and those in the other group with probability $\tilde{q} = 1 - q$.

The derivation of equilibria for the PU-participation game is a straightforward modification of that in PR83 and PR85. Here, we provide the optimization conditions

[6] The case where $c = 1/2$ is straightforward, but laborious.

involved. A more detailed analysis with examples is presented in appendix A. In section 3, we will present the equilibria for our experimental parameters. Throughout, we will follow PR83 and PR85 in assuming risk neutrality.

In the equilibrium analysis, the central condition is that a player will vote with certainty if the expected payoff is higher than the expected payoff of abstaining. Formally, voter j_i, $i = A, B$, will vote with probability 1 iff

$$Exp_{size}\left[Exp_{strat}\left[\pi_{j_i} \middle| v_{j_i} = 1\right]\right] > Exp_{size}\left[Exp_{strat}\left[\pi_{j_i} \middle| v_{j_i} = 0\right]\right], \qquad (6)$$

and will abstain with probability 1 if the reverse is true. Expectation operators refer to strategic uncertainty ($strat$) and PU ($size$).[7] Elaboration gives

$$\sum_{x=\underline{N}_i}^{\overline{N}_i} prob(x)\left[prob(V_i^{-j_i} = V_{-i} \mid x) + prob(V_i^{-j_i} + 1 = V_{-i} \mid x)\right] > 2c, \qquad (7)$$

where $prob(x)$ is the probability that electoral composition $(x, E-x)$ occurs, and $prob(V_i^{-j_i} = V_{-i} \mid x) + prob(V_i^{-j_i} + 1 = V_{-i} \mid x)$ gives voter j_i's probability of being pivotal, given *i*'s own group size x. The first of these terms gives the probability that j_i can turn a tie into a victory, and the second the probability that she can turn a defeat into a tie. For fixed *x*, (7) simply reduces to the equilibrium condition for the 'standard' participation game (PR83). Note that the expected payoff from voting is always negative for $c > 1/2$, implying that a risk neutral voter will abstain in this case. Hence, for high costs, the only Bayesian-Nash equilibrium is for every voter to abstain.

Condition (7) can be used to determine pure strategy Bayesian-Nash equilibria for the PU-participation game with $c < 1/2$. Appendix A (proposition A1) gives a comprehensive overview of these equilibria for various values of *c*. For example, if 2*c* is smaller than the probability that both groups are of equal size (or that there is one voter more in the own group), then one equilibrium in pure strategies is for everyone to vote. This is an intuitive extension of the PR83 full participation equilibrium discussed above. There are other pure strategy equilibria with some voters participating. The pure strategy equilibria that we derive for our experimental parameters will be described in section 3.

[7] PR85 refer to the uncertainty about group sizes (preferences) as 'strategic', because it enters the participation decision of voters. We agree and only use a different terminology for notational clarity.

Next, consider equilibria in totally quasi-symmetric mixed strategies. A necessary and sufficient condition for Bayesian-Nash equilibria in such strategies to exist is that each voter j_i, $i = A, B$, is indifferent between participation and abstention (*i.e.,* condition (7) holds as an equality). Since in the experiment we use a symmetric group size distribution we focus on symmetric (q, q)-equilibria. Then, elaboration and specification of (7) implicitly defines best response q:

$$\sum_{x=\underline{N}_i}^{\overline{N}_i} prob(x)\Bigg[\sum_{k=0}^{\min[x-1,E-x]}\binom{x-1}{k}\binom{E-x}{k}q^{2k}(1-q)^{E-1-2k} + \sum_{k=0}^{\min[x-1,E-x-1]}\binom{x-1}{k}\binom{E-x}{k+1}q^{2k+1}(1-q)^{E-2-2k}\Bigg] = 2c. \qquad (8)$$

The first term in the large square brackets gives the (binomial) probability that there is a tie of k votes between the $E-x$ members in the other group and the $x-1$ other members of j_i's own group (j_i can turn a tie into a victory). The second term gives the (binomial) probability that the other group outvotes j_i's co-members by one vote (j_i can turn a defeat into a tie). Numerical calculations show that these equilibria exist for a variety of parameter values (see appendix A for examples).

Until now, we have allowed for PU without distinguishing between allied and floating voters. All voters were assumed to choose a candidate on election eve, under the condition that at least $\underline{N}_i$ voters would choose *i*. This describes a world with only floating voters. We are now assuming that the minimal group of $\underline{N}_i$ voters has determined their choice beforehand. Hence, a situation of PU appears, with the minimal group representing the number of allied voters in i. There is a subtle, but important difference in the information set of the two voter types: each floating voter has private information about the candidate she supports, which she can use to (subjectively) update the probability distribution of the electorate's composition. Allied voters, on the other hand, must rely on the common prior distribution. Because of this difference, we consider mixed strategy equilibria with distinct voting probabilities for the two voter types.

In our analysis (and experiments) we use a binomial distribution in which a priori each floating voter belongs to $i = A, B$ with equal probability $p = 0.5$ (*cf.* Börgers, 2004). We restrict our analysis to the symmetric case with an equal number of allied voters in each group: $\underline{N}_A = \underline{N}_B \geq 1$.[8] We again refer to appendix A (proposition A2) for an overview of all pure strategy equilibria of the game. As in the case without allied voters, the only equilibrium when $c > 1/2$ is for everyone to abstain and a full participation equilibrium exists if 2*c* is smaller than the probability of a tie or a win by 1 vote. Once again, other equilibria in pure strategies exist for specific cost levels.

For mixed strategy equilibria, we focus on quasi-symmetric cases were all allied voters participate with the same probability $q_a \in (0,1)$ and all floating voters with the same probability $q_f \in (0,1)$. A necessary and sufficient condition for Bayesian-Nash equilibria in such strategies to exist is that each allied voter and each floating voter is indifferent between participation and abstention. Elaboration and specification of (7) as an equality gives implicit functions for the best responses q_a and q_f that can be solved numerically for a wide range of parameters. These implicit functions and some examples of equilibria are presented in appendix A.

QUANTAL RESPONSE EQUILIBRIA

Goeree and Holt (forthcoming) and Cason and Mui (forthcoming) show that quantal response equilibria, in particular logit equilibria, predict behavior in experimental participation games better than (Bayesian-) Nash equilibria do. Here, we show how such equilibria can be derived for the PU-participation game. In the next section, we will present specific equilibria for the parameters of our experiment.

Starting point for the quantal response analysis is the comparison of expected payoffs for voting and abstaining described in condition (6). A stochastic term $\mu\varepsilon_{j_i}$ is added to the expected payoff of each decision (vote or abstain) to allow for the possibility that voters perceive these payoffs subject to noise. It includes an error parameter $\mu \geq 0$ common to all and ε_{j_i} as a realization of j_i's individually specific random variable, which is identically and independently distributed per voter and decision (*cf.* Goeree and Holt, forthcoming). Voter j_i will participate iff the expected payoff from voting is higher than that of abstaining:

$$Exp_{size}\left[Exp_{strat}\left[\pi_{j_i} \middle| v_{j_i} = 1\right]\right] + \mu\varepsilon_{j_i}^1 > Exp_{size}\left[Exp_{strat}\left[\pi_{j_i} \middle| v_{j_i} = 0\right]\right] + \mu\varepsilon_{j_i}^0, \qquad (9)$$

[8] The generalization to asymmetric cases through unequal minimal group sizes or $p \neq .5$ is straightforward, however, more laborious best response conditions and notations are needed.

where ε's superscript '1' ('0') refers to the realization of the random variable in the stochastic term that is added to the payoff from voting (abstaining). In the absence of noise (μ = 0), (9) reduces to condition (6) for a Bayesian-Nash equilibrium. Hence, the equilibria described above are a limit case of the quantal response equilibria described here (McKelvey and Palfrey, 1995; Goeree and Holt, forthcoming).

For $\mu > 0$ it follows from (9) that voter j_i will vote iff

$$\varepsilon^0_{j_i} - \varepsilon^1_{j_i} < \frac{Exp_{size}\left[Exp_{strat}\left[\pi_{j_i} \middle| v_{j_i} = 1\right]\right] - Exp_{size}\left[Exp_{strat}\left[\pi_{j_i} \middle| v_{j_i} = 0\right]\right]}{\mu}. \qquad (10)$$

Denoting the distribution function of the difference $\varepsilon^0_{j_i} - \varepsilon^1_{j_i}$ by F, this gives the probability q that voter j_i will vote:

$$q = F\left[\frac{Exp_{size}\left[Exp_{strat}\left[\pi_{j_i} \middle| v_{j_i} = 1\right]\right] - Exp_{size}\left[Exp_{strat}\left[\pi_{j_i} \middle| v_{j_i} = 0\right]\right]}{\mu}\right], \qquad (11a)$$

or, after elaboration (*cf.* condition 7),

$$q = F\left[\sum_{x=\underline{N}_i}^{\overline{N}_i} prob(x)\left[\frac{prob(V_i^{-j_i} = V_{-i}|x)}{2} + \frac{prob(V_i^{-j_i} + 1 = V_{-i}|x)}{2}\right] - c\right] \Big/ \mu. \qquad (11b)$$

This equation describes the voting probability q as a 'noisy best response' to the expected payoff difference between voting and abstaining. Assuming symmetry not only within but also between groups (because all voters face exactly the same decisions) and using the binomials in eq. (8), the right hand side of (11b) is a function of the probability, q, that a randomly drawn other voter will vote. A quantal response equilibrium (McKelvey and Palfrey, 1995) for some specification of error distribution F occurs if the participation probability on the right hand side is equal to the q that shows up on the left hand side. This can be found numerically for specific values of the error parameter μ.

The quantal response equilibrium for the case with allied and floating voters can be derived in a similar way. Each type is symmetric across groups due to the symmetric group size distribution. Then, to calculate the noisy best responses for allied and floating voters, q_a respectively q_f, two equations similar to (11b) have to be solved simultaneously.

3. EXPERIMENTAL DESIGN

PROCEDURES AND TREATMENTS

The computerized[9] experiment was run at the laboratory of the Center for Research in Experimental Economics and political Decision making (CREED), University of Amsterdam. 288 undergraduate students were recruited in 12 sessions of 24 subjects. Each session lasted about 2 hours (*cf*. appendix B for the read-aloud instructions). Earnings in the experiment were expressed in tokens. At the end of a session token earnings were transferred to cash at a rate of 4 tokens to one Dutch Guilder. Subjects earned an average of 56.01 Dutch Guilders (≈ € 25.42).

In each session, the 24 subjects were randomly divided into two electorates of 12 voters. Each electorate consisted of two groups. There was no interaction of any kind between subjects in different electorates, and this was known to all of them. Given that we do not know the structure of the correlations across observations, we treat the electorate as the only independent unit of observation. Hence, each session provides us with two independent observations.

The experimental design employed a full 2×2 between subject factorial design with 3 sessions (6 electorates) per cell. A first factor (‘preference (un)certainty’) manipulated the information about the level of disagreement (*i.e*., realized group sizes). This information was either given at the beginning of each round (‘informed’) or not at all (‘uninformed’, which yields the case of preference uncertainty). The second factor (‘voter alliance’) manipulated the presence of allied voters. In one treatment (‘floating’), there were no allied voters, while in the other (‘mixed’), there were 3 allied voters in each group that stayed in their groups throughout the session. Floating voters were reallocated to groups at the beginning of each round, whereas allied voters were assigned to a group once and for all at the beginning of the first round. Each subject was either an allied or floating voter throughout the session and knew her type from the beginning of the first round. Note that floating voters were always reallocated within the same electorate.

PREFERENCE UNCERTAINTY (PU)

Preference (un)certainty and voter alliance were both varied in a between subject design. To create the possibility of PU, we varied group sizes in a within subject design. In any given

[9] The experimental software was programmed using RatImage (Abbink and Sadrieh, 1995).

round, each group i consisted of a minimum of $\underline{N}_i = 3$ voters and a maximum of $\overline{N}_i = 9$. Any integer group size $N_i \in \{3,4,...,9\}$ was possible. The randomization used to determine group sizes proceeds in the following two steps:

- Step 1: 3 subjects are put into each group. Each subject in the electorate has an equal chance of being chosen and of being allocated to either group.
- Step 2: The remaining 6 subjects are independently and randomly allocated, with equal probability for each group.

This procedure was known to all subjects. The way it is applied is different for our 'floating' and 'mixed' treatments. In 'mixed' sessions, step 1 determined the 6 subjects to take the role of allied voters and their group allocation. This step was performed only once, at the beginning of the first round; step 2 took place at the beginning of each round and reallocated the 6 floating voters. In 'floating' sessions, both steps were performed at the beginning of each round and, importantly, subjects did not know at which step they were allocated to the groups. Notice that step 2 produces a binomial distribution of group sizes with $p = 0.5$, where electoral composition (6-6) occurs with probability .3125, (5-7) and (7-5) each with .2344, (4-8) and (8-4) each with .0938, and (3-9) and (9-3) each with .0156.

Each session consisted of 100 decision rounds.[10] The distribution of the composition of the electorate across rounds was varied in a random, but predetermined manner (see appendix C for the complete sequence). 33 rounds use the composition (6-6), 23 use (5-7), 22 use (7-5), 9 use (4-8), 9 use (8-4), 2 use (3-9), and 2 use (9-3). Whether subjects knew the actual group sizes when making their decisions depended on the preference (un)certainty treatment. In the 'informed' sessions, the actual 'own' and 'other' group sizes were announced at the beginning of each round.[11] Note that subjects in 'uninformed' faced the same decision problem in each round, because they never knew the actual level of disagreement when making their decisions. Moreover, 'uninformed' allied and floating voters had different information about group sizes. An allied voter knew that there are 'at least 3' ('at most 9') voters in each group. A floating voter, on the other hand, knew that there 'are at least 4' voters in her own group and at most 8 in the other. As a consequence, in the 'mixed' treatment allied voters had an expected 'own' group size of 6, whereas for floating voters

[10] Due to a computer crash, one session had to be stopped after 94 rounds.
[11] Groups were labeled 'own' and 'other' to avoid floating voters associating with either group.

Bayesian updating yields an expected 'own' group size of 6.5. When there are only floating voters, this expectation is 6.25.

PAYOFF PARAMETERS

In each round, each member of the winning group received revenue of 4 tokens and each member of the losing group received 1 token. As the cost of participation was 1 token (independent of a subject's type), negative payoffs were avoided. Table 1 summarizes treatments and parameters.

TABLE 1: SUMMARY OF TREATMENTS AND PARAMETERS*

Treatment	Acronym	#Floating voters	#Allied voters	Preference uncertainty?
Uninformed Floating	UF	12	0	yes
Uninformed Mixed	UM	6	6	yes
Informed Floating	IF	12	0	no
Informed Mixed	IM	6	6	no

*All treatments had 100 rounds and electorates of 12 voters, with a minimum (maximum) of 3 (9) in each group. A victory (loss) payed 4 (1) to each voter in the group and the individual costs of voting were equal to 1. We have observations from 6 independent electorates per treatment.

EQUILIBRIUM PREDICTIONS

For these parameters, we can derive (Bayesian-) Nash equilibria and quantal response equilibria as described in section 2. Following PR83 we conclude that 'everybody votes' is the pure strategy equilibrium in 'informed' when both groups have equal size 6.[12] For the other (asymmetric) realizations of group sizes where voters know the level of disagreement there is no equilibrium in pure strategies. For both uninformed treatments (UF and UM) it follows from propositions A1 and A2 in appendix A that there is no pure strategy Bayesian-Nash equilibrium with full participation.[13] For UF, from proposition A1 we know that the only Bayesian-Nash equilibria in pure strategies are those, where one voter of either group participates and all others abstain. From proposition A2 we conclude that no Bayesian-Nash equilibria in pure strategies exist for UM. All in all, pure strategy equilibria do not provide us with testable predictions about our treatment effects.

We use eqs. (8) and (11b) to numerically derive (Bayesian-) Nash equilibria in totally quasi-symmetric mixed strategies and quantal response equilibria for our treatments. As discussed in section 2, the former equilibria are a limit case of the latter, for $\mu = 0$. At the

[12] Normalizing revenue to 1 (0) in case of a victory (defeat), we have $c = 1/3$.
[13] This follows from $2c > .3125$ (with normalized $c = 1/3$), which is the probability of equal group sizes.

other extreme, when $\mu \to \infty$ the equilibrium probability of voting approaches 0.5, which represents pure random behavior. For the quantal response, we derive logit equilibria by choosing a double exponential distribution for error distribution F (McKelvey and Palfrey, 1995). For equilibrium prediction, we use estimates for error parameters μ reported in Goeree and Holt (forthcoming).[14] They determine logit equilibria for data presented in Schram and Sonnemans (1996a) for an experimental participation game with symmetric group sizes equal to 6 and benefits and costs similar to the parameters used in our experiment. The model fits early round data best for $\mu = .8$ and later round data for $\mu = .4$. Therefore, we derive logit equilibria for our treatments using $\mu = .4$ and $\mu = .8$ in addition to the (Bayesian-) Nash equilibria (μ=0).[15] For ease of presentation, we will refer to these equilibria as E_μ, for $\mu \in \{0, 0.4, 0.8\}$. *E.g.*, E_0 refers to the (Bayesian-) Nash equilibrium. Appendix D gives a table providing a full overview of these equilibria per treatment. Here we summarize them by focusing on three characteristics: participation probabilities, the probability of victory, and efficiency.

Figures 1 and 2 summarize the equilibrium probabilities of voting. For the informed treatments in figure 1, these are weighted averages for the distinct electoral compositions.

FIGURE 1: EQUILIBRIUM PARTICIPATION PER TREATMENT

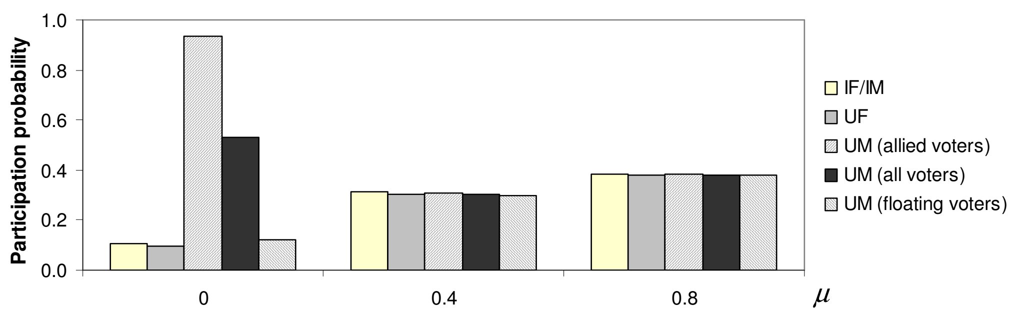


Figure 1 shows participation probabilities per treatment and noise-level. Note that in informed (IF and IM) there is no difference in equilibrium participation between allied and floating voters, because both know the group sizes prior to making their decisions. For this

[14] We intend to confront our experimental data with comparative statics derived from equilibrium analysis. For this reason, we cannot estimate μ using our own data and take the value found to derive equilibrium predictions. Instead, we use the exogenous values found elsewhere. In section 4.6, we will give maximum likelihood estimates of μ for our data, however.

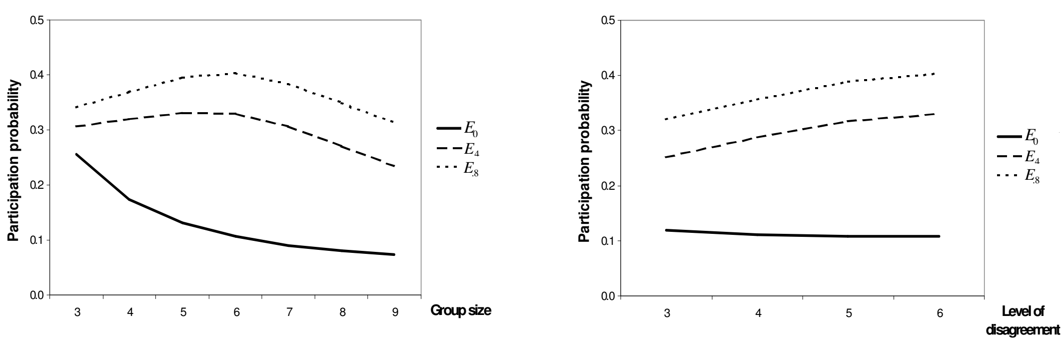
[15] Because we normalized our revenues to 1 (0) in case of a victory (defeat) for the calculations, we normalized μ to 0.4/3 and 0.8/3, respectively. Our presentation in the main text refers to non-normalized parameters.

reason, we present the equilibrium results as one case. In E_0, average expected turnout is substantially higher when there is a mix of allied and floating voters with PU (53% in UM) than when all voters are floating and face PU (10% in UF) or when all voters are informed (11% in IF/IM). This difference is entirely due to the allied voters, who vote with very high probability (94%) in the Bayesian-Nash equilibrium. Floating voters in UM on the other hand, vote at a low rate of 12%, which is similar to voters in the other treatments. After introducing noise, the logit equilibria show that the turnout probabilities of all voter types and conditions converge and are hardly distinguishable (30-31% in $E_{.4}$, and 38-39% in $E_{.8}$). These probabilities increase with μ and are higher than the low probabilities derived in E_0 for all voters except allied voters in UM.

Figure 2 shows equilibrium turnout as a function of group size (left panel) and ex ante closeness (right panel) for informed voters. Note that ex ante closeness is determined by the level of disagreement.

FIGURE 2: EQUILIBRIUM PARTICIPATION OF INFORMED VOTERS

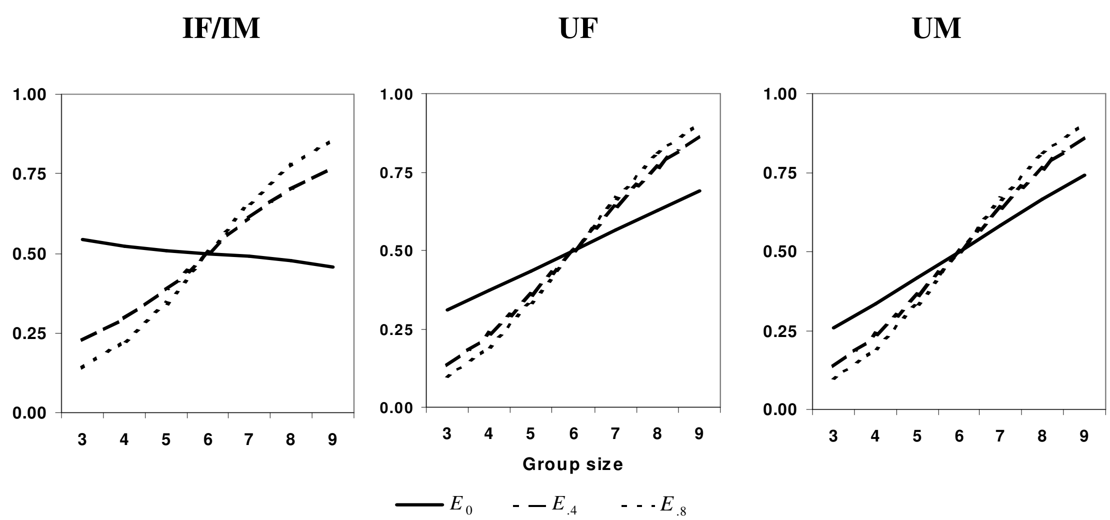
GROUP SIZE LEVEL OF DISAGREEMENT



The left panel shows that in E_0 the voting probability decreases monotonically with group size when voters are informed about the level of disagreement. This is a consequence of increased free riding incentives. When adding noise, the monotonicity disappears. Both logit equilibria as a function of group size are an inverted U-shape. The highest voting probability in $E_{.4}$ (33%) is found for group size 5 and in $E_{.8}$ (40%) for equal group sizes of 6. For each μ, the turnout probability is higher for voters in a minority than in the opposing majority, though the difference decreases with μ (this can be seen in the left panel of figure 2 by observing that the participation curve becomes more symmetric around group size 6 as μ increases). Finally, for any group size, the probability of voting increases with μ. The right panel in figure 2 shows that turnout (slightly) decreases with the level of disagreement in E_0 and increases as minorities become larger in $E_{.4}$ and $E_{.8}$.[16]

From the probabilities of voting, we can derive the equilibrium probability that a group will win the election. Because informed minority voters participate at a higher probability than their majority opponents, the former may achieve a victory more often than the latter. Figure 3 gives the equilibrium probabilities of winning the election per treatment, noise-level, and group size. In E_0 with informed groups (left panel), minorities indeed have a higher probability of winning the election than majorities. When noise is introduced, the smaller difference in equilibrium participation by voters in opposed groups together with an overall increase in the level of participation, turns the tide in favor for the majority: the group size effect dominates the free riding effect and the majority has a (substantially) higher equilibrium probability of victory. Finally, the middle and right panels of figure 3 show victory probabilities when voters do not know group sizes at the time they vote. In this case, voters participate with the same probability, since they cannot respond to unknown group sizes (though allied and floating voters may vote at different rates). Hence, majorities win more often. Moreover, the figures confirm that treatment effects diminish as μ increases. And, when we introduce allied voters (UM), in E_0 minorities have a lower probability of winning than when all voters are floating (UF). This is a direct consequence of the high probability with which allied voters vote (figure 1).

FIGURE 3: EQUILIBRIUM VICTORY PROBABILITIES

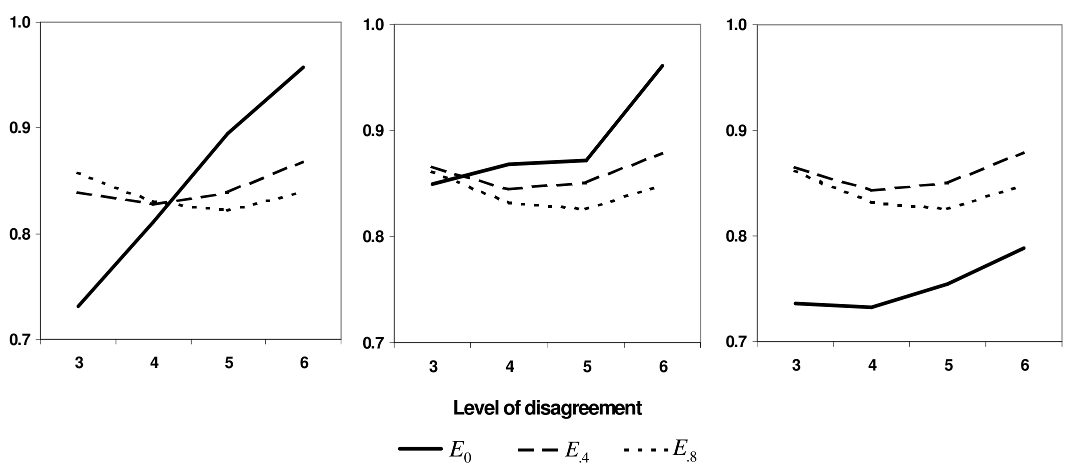


[16] For equal group sizes appendix D shows a second Nash equilibrium, not presented in the figure.

These voting and victory probabilities can be used to calculate voters' expected payoffs, which determine the electoral efficiencies in equilibrium (PR83). Figure 4 shows these efficiencies per treatment, noise-level, and level of disagreement. Efficiency is calculated as the electorate's aggregate payoff in equilibrium, divided by its socially optimal (efficient) total payoff. For unequal group sizes, surplus is maximized when one voter in the majority participates and all other voters abstain. For example, for electorates with group sizes (3-9) or (9-3), the efficient aggregate payoff is $3\times1+9\times4-1=38$ [similarly, 35 for (4-8)/(8-4), and 32 for (5-7)/(7-5)]. For equal group sizes (6-6), it is efficient if nobody participates, in which case total payoff is $6\times1+6\times4=30$.

FIGURE 4: EQUILIBRIUM EFFICIENCY

IF/IM **UF** **UM**



Note from figure 4 that once again the treatment effect is strongest in E_0. When voters are informed about group sizes, efficiency is lowest (73%) when the level of disagreement is lowest and monotonically increases with the size of the minority to 96% when there are 6 voters in each group. This pattern occurs in E_0, because overall expected participation decreases (right panel, figure 2) and the majority's probability of winning increases (left panel, figure 3) in the level of disagreement. Compare this to the treatments with PU. In UF efficiency in E_0 is very high (96%) for equal group sizes, whereas intermediate values of efficiency between 85% and 87% are observed for all other levels of disagreement. In UM, efficiency in E_0 is low (between 73% and 79%) for all levels of disagreement. This is because allied voters participate extensively. When noise is introduced, the differences across treatments are minor. In all cases the efficiency curves are U-shaped and show intermediate values (83-88% for $E_{.4}$ and 82-86% for $E_{.8}$).

Obviously, the (comparative static) predictions we derive depend on the equilibrium concept used and level of noise assumed. We summarize these theoretical results (TR) below. Confronting them with our experimental results (ER) will allow us to determine which equilibrium predicts the comparative statics best.

TR1: Each equilibrium predicts that turnout will be virtually the same in IF, IM and UF. The logit equilibria predict that turnout will also be similar in UM, E_0 predicts higher turnout for UM.

TR2: Introducing noise substantially increases participation in UF, IF, and IM. In UM the relationship between participation and the level of noise is non-monotonic.

TR3: In E_0, participation by informed voters decreases in group size. The relationship between group size and turnout is non-monotonic in $E_{.4}$ and $E_{.8.}$

TR4: In E_0, $E_{.4}$ and $E_{.8.}$, voters in the minority participate more than those in the opposing majority.

TR5: In E_0, participation by informed voters decreases in the level of disagreement. The reverse holds in $E_{.4}$ and $E_{.8.}$

TR6: In UM, allied voters participate much more than floating voters in E_0. Both types participate at similar levels in $E_{.4}$ and $E_{.8}$.

TR7: In E_0, informed minorities win more often than informed majorities. In $E_{.4}$ and $E_{.8}$ the reverse holds. In both treatments with PU, the majority wins more often than the minority irrespective of μ.

TR8: In E_0, electoral efficiency is highest when the level of disagreement is highest and voters are informed or there are only uninformed floating voters. Average electoral efficiency in E_0 is lowest in UM and highest in UF.

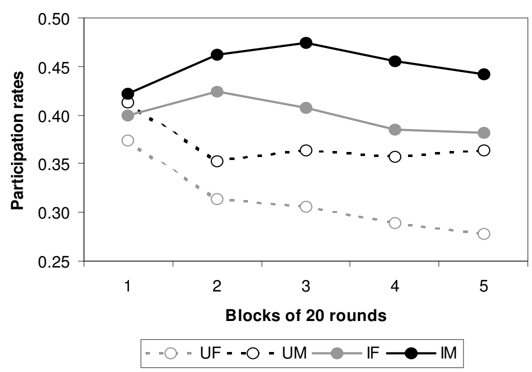
4. EXPERIMENTAL RESULTS

The presentation and analysis of our experimental results is organized along the same lines as the equilibrium predictions, above. First, we discuss observed participation rates (sections 4.1-4.3), then victory probabilities for minorities and majorities (4.4), and finally electoral efficiency (4.5). In 4.6, we analyze how the logit equilibrium fits our data. Our statistical tests in 4.1-4.5 are based on nonparametric statistics as described in Siegel and Castellan, Jr. (1988). For the reasons mentioned above all our tests will be conducted at the electorate level (qualitative conclusions are based on one-tailed tests).

4.1 AGGREGATE PARTICIPATION RATES

Figure 5 shows participation rates averaged over blocks of 20 rounds each. We observe higher participation rates when group size information is provided than when preferences are uncertain. This holds for all blocks of rounds: the rate is always higher in IF (IM) than in UF (UM). Though all treatments start at similar levels, a difference in participation rates of approximately 10%-points exists between treatments where voters are informed and those with PU from the second block onward for both the IF-UF and IM-UM comparisons.

FIGURE 5: AGGREGATE PARTICIPATION RATES



EXPERIMENTAL RESULT 1: *Information about the level of disagreement increases participation.*

SUPPORT. Wilcoxon-Mann-Whitney tests reject the null hypothesis of no difference in average participation in favor of higher rates in informed at the 5% significance level for the IF-UF comparison and at the 10% level for IM-UM.

Figure 5 also shows higher participation rates in the mixed treatments than in the corresponding treatments with only floating voters. The differences start out relatively small, but increase to 7%-points for each information treatment in blocks 3-5.

EXPERIMENTAL RESULT 2: *In the presence of allied voters, informed voters participate more in later rounds.*

SUPPORT. Across all rounds, Wilcoxon-Mann-Whitney tests cannot reject the null hypothesis of no difference for the UM-UF or IM-IF comparisons at the 10% significance level. Considering blocks 3-5 only, rates are significantly higher in IM than in IF (5% level) but the difference UM-UF is not significant (10% level).

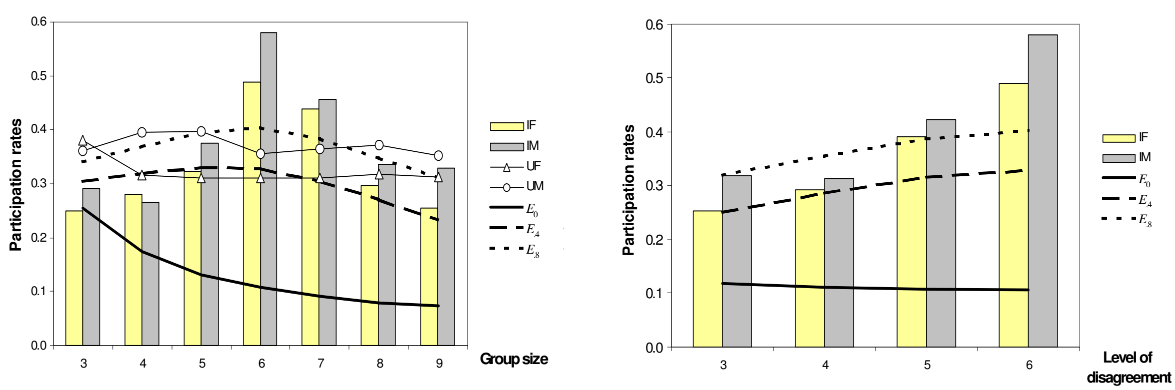
We can compare these first two experimental results (ER) to theoretical results TR1 and TR2, derived from our equilibrium analysis. TR1 (that participation is the same across treatments in $E_{.4}$ and $E_{.8}$, and only higher in UM in E_0) is rejected by both ER1 and ER2. We will discuss these two discrepancies between our data and the equilibrium predictions in section 4.6. As for TR2, we have the more general observation that participation levels in figure 5 are much closer to the ones predicted by the logit equilibria $E_{.4}$ and $E_{.8}$ than to those predicted by the (Bayesian-) Nash equilibrium E_0 (figure 1).

4.2 PARTICIPATION RATES PER GROUP SIZE AND LEVEL OF DISAGREEMENT

Figure 6 shows aggregate participation rates per group size (left panel) and level of disagreement (right panel).

FIGURE 6: PARTICIPATION RATES

GROUP SIZE | LEVEL OF DISAGREEMENT



Note: The bars in both figures indicate participation rates when voters know their group size (left panel) and level of disagreement (right panel). In the left panel, the lines with markers show participation levels for voters who are unaware of the actual group size and level of disagreement. The remaining lines are taken from figure 2 and show participation levels in the equilibria E_0, $E_{.4}$ and $E_{.8}$.

For uninformed voters there is no reason to expect participation to vary across size because subjects face PU. This is confirmed in the left panel. When informed about the level of disagreement, subjects participate most for group size 6 (IF: 49%; IM: 58%).

EXPERIMENTAL RESULT 3: *When informed about the level of disagreement, participation is highest when group sizes are equal.*

SUPPORT. A Friedman two-way analysis of variance by ranks cannot reject the null hypothesis of no difference in participation rates across group sizes in UF or UM (10%

significance), but rejects it for IF (IM) at the 1% (0.1%) level. Pairwise comparisons of participation rates across group sizes using Wilcoxon signed ranks tests show that in IF (IM) the null hypothesis of no differences is rejected in favor of higher rates in groups of 6 in comparison to any other size at least at the 5% (10%) significance level.

The left panel of figure 6 also shows (slightly) higher participation by voters in the majority than in the opposing minority. This gives:

EXPERIMENTAL RESULT 4: *When informed about the level of disagreement, participation is not higher for minority members.*

SUPPORT. A Wilcoxon signed ranks test cannot reject the null hypothesis of no difference in participation rates between minority members and the opposing majority members at the 10% significance level for any level of disagreement in IF and IM.

ER3 and ER4 can be compared to TR3 and TR4, which essentially means comparing the equilibria to our data in the left panel of figure 6 (note that these equilibria are copied from the left panel of figure 2). Once again, we observe that observed participation seems much closer to the logit equilibria than to E_0 because of the non-monotonic relationship with group size (TR3). Contrary to ER4, however, all of the equilibria we considered predicted higher participation rates for the minority members. Our experimental results do not support the idea that the free riding incentive is larger in majorities (this is a third discrepancy between the equilibria and our data, to be addressed in 4.6).

The right panel in figure 6 relates participation rates of informed voters to the level of disagreement and compares them to the equilibrium predictions. A general conclusion is that the content of information about other voters' preferences has a strong impact on turnout: observed participation increases (substantially) in the level of disagreement.

EXPERIMENTAL RESULT 5: *Participation by informed voters increases in the level of disagreement.*

SUPPORT. A Page test for ordered alternatives rejects the null hypothesis of no difference in participation rates across levels of disagreement in favor of increasing participation rates at the 0.1% significance level for both IF and IM.

Relating this result to TR5 provides additional support for the logit equilibria in comparison to E_0.

4.3 PARTICIPATION RATES OF ALLIED AND FLOATING VOTERS

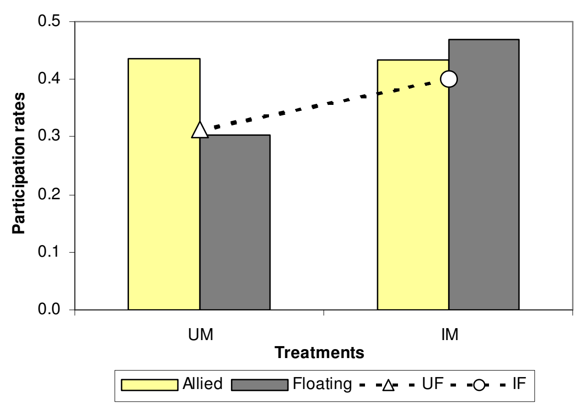
Figure 7 compares average participation rates of allied and floating voters for the mixed treatments. Rates for the treatments without allied voters are shown for comparison. We observe slightly higher participation (4%-points) by floating than by allied voters in IM. Allied voters participate substantially more than floating voters when facing PU (13%-points). The difference is not statistically significant, however.

EXPERIMENTAL RESULT 6A: *When there are allied voters, they participate at similar levels as floating voters.*

SUPPORT. Neither in IM nor in UM can Wilcoxon signed ranks tests reject the null hypothesis of no difference in rates between allied and floating voters at the 10% significance level.

Comparing ER6A to TR6, we see that the large difference in turnout of allied and floating voters predicted by the Bayesian-Nash equilibrium for UM is not supported statistically. The negligible differences predicted by $E_{.4}$ or $E_{.8}$ are supported.

FIGURE 7: PARTICIPATION RATES FOR ALLIED AND FLOATING VOTERS



Another way to see the participation rates in figure 7 is by noting that participation is generally 40% or more, but that floating voters facing PU vote at a rate of about 30% (both in UF and in UM). Recall that uninformed floating voters have extra information about their own group size, compared to informed and allied voters. This may cause them to vote less. We conclude that the lower participation rates in UF and UM compared to their informed counterparts (*cf.* ER1 and figure 5) are caused by floating voters, behaving differently in these distinct cases. This conclusion is tested in the following result.

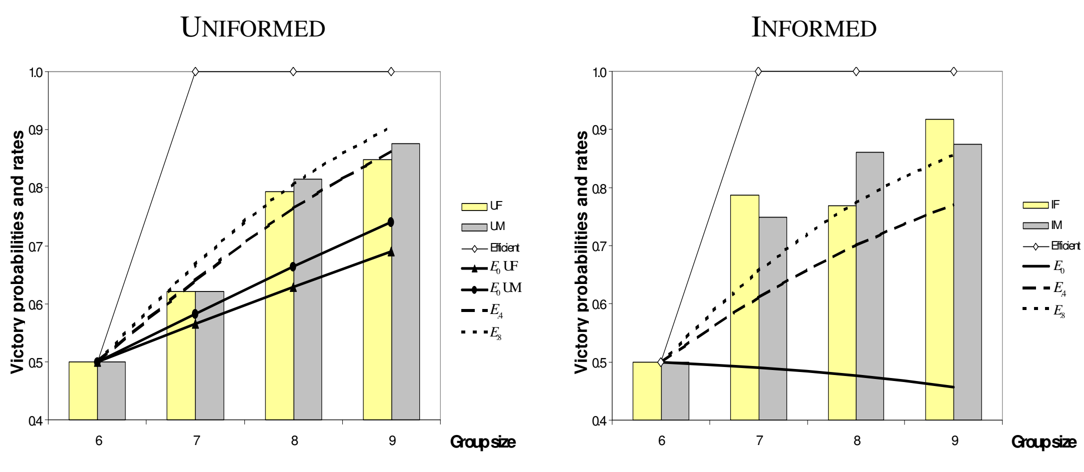
EXPERIMENTAL RESULT 6B: *Floating voters participate more when informed about the level of disagreement. Preference uncertainty has no effect on allied voters.*

SUPPORT. A Wilcoxon-Mann-Whitney test cannot reject the null hypothesis of no difference in participation rates of allied voters in the IM-UM comparison (10% significance level), but rejects it for floating voters in favor of higher rates for informed in both the IF-UF and IM-UM comparisons (both 5% level; *cf*. ER1).

4.4 VICTORY PROBABILITIES

In each of the three equilibria considered, the participation rate for informed voters was higher in the minority than in the opposing majority. In E_0, the difference was large enough to give the minority a higher probability of winning the election than the majority. ER4, however, showed that realized participation rates are not higher in the minority. This means that the majority achieves a victory more often. This is visible in Figure 8, where we show the realized fractions of majority victories. For comparison, the probabilities implied by E_0, $E_{.4}$ and $E_{.8}$ as well as the efficient solution are also shown. We add the (6-6) case for comparison, defining the majority victory rate as 50%.

FIGURE 8: VICTORY RATES



Note: Because the victory rates in the logit equilibria for UF and UM are almost identical (*cf*. figure 3), we only present them for UF, here.

The figure shows that majority victory rates are always larger than 50% and (with one exception) increase in group size. For the uninformed treatments, this is a direct consequence of equal average participation rates across group sizes (since subjects cannot respond to what they don't know). For the informed treatments, this follows from ER4. Comparing the left and right panels, note that only for a majority of 7 voters do the victory rates depend on whether or not other voters' preferences are known. An informed majority of 7 has a 17%-point (13%-point) higher probability of beating the minority of 5 in the IF-UF (IM-UM) comparison.[17] These observations give:

EXPERIMENTAL RESULT 7: *Majorities have a higher probability of winning the elections than the opposing minorities.*

SUPPORT. This follows from the discussion above.

Comparing this result to TR7, we once again observe that the logit equilibria predict the comparative statics better than E_0.

4.5 ELECTORAL EFFICIENCY

As outlined in section 3, electoral efficiency is calculated as an electorate's realized earnings, divided by its socially optimal (efficient) earnings. Table 3 gives realized efficiencies and their standard deviations. The data are pooled across the two voter alliance treatments, since virtually no differences in patterns are observed for this variable.

TABLE 3: ELECTORAL EFFICIENCY

Treatment	Uninformed				Informed			
Level of disagreement	3	4	5	6	3	4	5	6
Efficiency (standard dev.)	.854 (.081)	.842 (.051)	.831 (.046)	.867 (.042)	.887 (.066).	.861 (.042)	.835 (.045)	.786 (.059)
Weighted average	.846				.826			

EXPERIMENTAL RESULT 8: *Preference uncertainty has little effect on aggregate electoral efficiency. With informed voters, efficiency decreases in the level of disagreement.*

SUPPORT. A Wilcoxon-Mann-Whitney test cannot reject the null hypothesis of no difference in aggregate efficiency between uninformed and informed at the 10% significance level. With

[17] For a majority of 7, Wilcoxon-Mann-Whitney tests reject the null hypothesis of no difference in victory rates in favor of higher rates in informed, for both the IF-UF (5% significance level) and IM-UM (10% level) comparisons. No significant differences are found for majority sizes of 8 and 9 (10% level).

regard to the level of disagreement, a Page test for ordered alternatives rejects the null hypothesis of no difference in efficiencies across these levels in favor of the order 3>4>5>6 for informed at the 1% significance level, but not for PU (10% level).

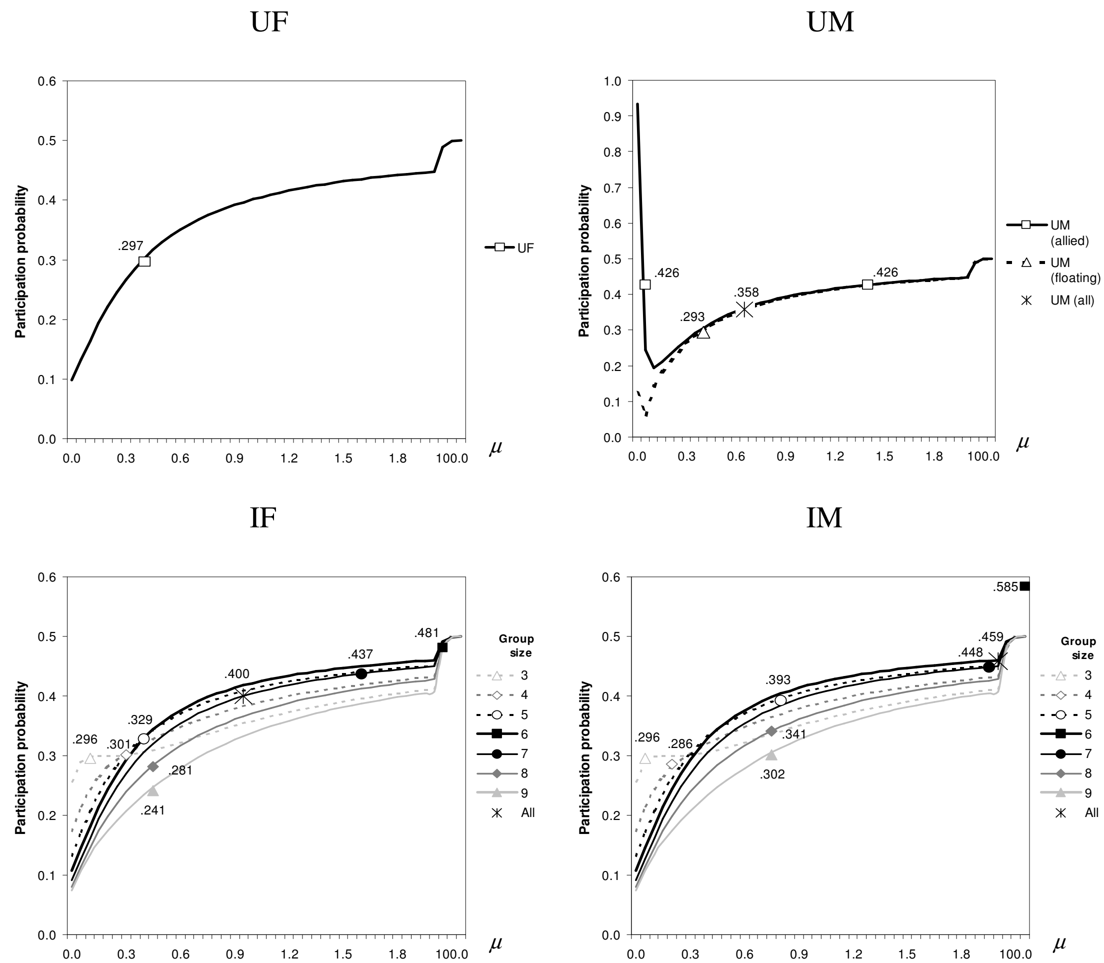
The experimental result that electoral efficiency decreases with the level of disagreement for informed voters contradicts prediction TR8 for E_0. It is closer to the patterns shown in $E_{.4}$ and $E_{.8}$ in figure 4, though we only observe the U-shape predicted by these logit equilibria in the case with preference uncertainty.

To conclude, the impact of preference uncertainty on electoral efficiency varies with the actual level of disagreement in the electorate. Compared to the case with PU, information about this level is disadvantageous (advantageous) for efficiency when group sizes are (un)equal. Aggregated across all levels of disagreement, PU has little effect on efficiency. These experimental results have implications for the effects that opinion polls have on electoral efficiency. We will discuss these effects in the concluding discussion.

4.6 Interpreting the equilibrium predictions

The observed comparative statics in ER1-8 together with theoretical predictions TR1-TR8 provide strong evidence in favor of the logit equilibria $E_{.4}$ and $E_{.8}$, compared to the (Bayesian-) Nash equilibrium E_0. However, we made three observations that cannot be justified by the logit predictions either: (*i*) higher participation in IF than UF and in IM than in UM (TR1 and ER1); (*ii*) higher participation in IM than in IF (TR1 and ER2); and (*iii*) participation rates are not higher in an informed minority than in the opposing majority (TR4 and ER4). One possible explanation for these discrepancies lies in our assumption that the error rate μ is equal across treatments and environments. In the experiment, however, some situations may appear more difficult to subjects than others, causing more noise in their responses. Alternatively, noise may be accompanied by systematic behavior that is not accounted for in the equilibrium concepts we use. We will evaluate both explanations. To start with the possibility that error rates vary across treatments and environments, figure 9 shows for each of our treatments how the logit equilibrium changes with the noise parameter μ (including $E_{.4}$ and $E_{.8}$; *cf.* appendix D). Recall that $\mu = 0$ yields the (Bayesian-) Nash equilibrium and that as $\mu \rightarrow \infty$ the equilibrium probability of voting approaches 0.5, which represents pure random behavior.

FIGURE 9: LOGIT EQUILIBRIA



Note: The lines give the logit equilibria for varying μ from 0 to 2 and the discrete cases 10, 100, and 1000. The latter show up as a ‘jump’ upwards, with participation probabilities close to 0.5. The symbols show the maximum likelihood estimates $\hat{\mu}$ for rounds 21 to 100, the corresponding numbers give the observed participation rates. ‘All’ represents the $\hat{\mu}$ for the (weighted) average participation of the treatment.

The lines in figure 9 give equilibrium participation probabilities as a function of μ. We can use these levels to find the $\hat{\mu}$ (*i.e.*, find the logit equilibrium) that best predicts our observations. This is equivalent to determining maximum likelihood estimates of μ for our treatments.[18] To allow for an initial phase of confusion and orientation we estimate μ for observations after round 20.[19] The symbols with the numbers in figure 9 give the location of

[18] Denote by q_μ the logit equilbrium (or equilibria) for μ. Let q_o be the observed participation rate for n observations. The Log-likelihood that q_μ yields q_o is $\ln(L) = \ln\binom{n}{q_o n} + q_o n \ln q_\mu + (1-q_o) n \ln(1-q_\mu)$, which is maximized for $q_\mu = q_o$, if such a μ exists (otherwise a corner solution maximizes the Log-likelihood). The Log-likelihoods of our estimates for the various treatments are presented in appendix D.

[19] Many experimental games show a decrease in noise over the rounds, commonly attributed to learning (*e.g.*, McKelvey and Palfrey, 1995; Goeree and Holt, forthcoming; see Anderson *et al.*, 2004, for the relation between

our experimental observations on the equilibrium line. For example, in UF, we observed a participation rate of 29.7%. This is a logit equilibrium level of turnout for $\hat{\mu}$=0.39. As expected from the observed participation rates, the maximum likelihood estimates $\hat{\mu}$ appear to be quite diverse, often outside of the range [0.4, 0.8], implying distinct levels of noise in the best response functions. One possible (*ex post*) explanation for the three discrepancies between our data and the logit predictions is that treatments where the estimated level of noise is higher were more difficult decision making environments for our subjects. An alternative explanation is that the discrepancies are deviations through systematic behavior not captured by the 'pure' logit equilibrium. For the three cases, we suggest the following possible explanations:

Discrepancy (*i*): The drop in participation for the uninformed treatments (TR1 and ER1) was shown to be attributable to floating voters (ER6B). Floating voters may be reacting to their larger expected 'own' group size. Because this Bayesian updating is included in the logit equilibrium calculations, it cannot by itself explain the discrepancy. However, floating voters may overweigh this information, which would lead to the pattern of behavior we observe.

Discrepancy (*ii*): Allied voters *interact repeatedly* in the mixed treatments. If they try to 'coordinate' across rounds, the stage game equilibria described in section 2 are not applicable. Repeated interaction yields higher turnout in experimental participation games, even in a finitely repeated setting (Schram and Sonnemans, 1996a,b). Our UM and IM treatments are best seen as a mix between a partners and strangers design (Andreoni, 1988), *i.e*, we have partially repeated interaction. This may explain the observed higher participation in IM than IF (TR1 and ER2).

Discrepancy (*iii*): Another potential source of variation in noise is group size. Recall that a higher participation rate in majorities than in the opposing minorities was our third discrepancy (TR4 and ER4). Figure 9 and appendix D show higher estimates of the error parameters for the majority. Alternatively to more noise, an explanation may be that some

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directional learning and the logit equilibrium). If learning reduces errors, one might expect subjects in the uninformed treatments to learn faster, because they make the same decision in every round. Moreover, informed subjects may learn faster about electorates with higher levels of disagreement due to the binomial distribution of group sizes. Learning may also be quicker for the case where voters are in a majority, which occurs more often than being in a minority. Contrary to what we observe in our experiment (figure 9 and appendix D), this would imply lower noise for higher levels of disagreement and for majorities. Instead, it appears in our data that the major changes in participation occur in the first 20 rounds in all treatments, suggesting that learning is not bound to a particular electoral composition or one's 'own' situation.

subjects have altruistic preferences. In linear public goods experiments, contributions often increase with group size (Isaac and Walker, 1988; Isaac *et al*., 1994). This cannot be explained by a logit equilibrium, unless payoffs are adjusted to allow for altruism (Anderson *et al*., 1998). Whether this result carries over to participation game experiments is a matter to be investigated. Assuming that some subjects direct pure altruism towards all voters in the electorate, one would expect minority (majority) members to decrease (increase) their participation compared to the logit predictions to support the more efficient outcome. Our data suggest this as a reasonable conjecture.[20]

In this paper, it was not our aim to enrich the logit model to capture every discrepancy with observed data. We stress the fact that the logit equilibrium, even with homogeneous noise levels across treatments, substantially improves the (comparative statics) predictions compared to the (Bayesian-) Nash model. This supports previous findings for experimental participation games (Goeree and Holt, forthcoming; Cason and Mui, forthcoming) and shows that the logit equilibrium remains a reliable predictor also for richer voting environments.

5. CONCLUSIONS

Theoretical work on simple majority voting suggests that information about group sizes prior to elections, *e.g*. through polls, can have negative effects on welfare in certain environments (Goeree and Großer, 2004). The reason is that expected overall participation may increase and voters who are reinforced in their beliefs of being part of the majority (realize they are in a minority) may be stimulated to participate less (more) than before. We extend this literature by theoretically and experimentally analyzing participation games and comparing situations where the electorate only knows the a priori distribution of preferences to those where it is informed about the exact realization of group sizes. Our theoretical predictions for this comparison depend strongly on the equilibrium concept used and the decision environment. For the logit predictions, though participation (efficiency) probabilities are higher (lower) when voters are informed about other voters' preferences than when they are not, these

[20] In a model with (some) voters who care about the welfare of their own group, Feddersen and Sandroni (2004) derive comparative static predictions very similar to those of the logit equilibrium derived here. For example, they predict that participation is increasing in the level of disagreement, and that the participation probability is higher in the minority than in the majority, but that the majority wins more often. Due to distinct assumptions between their model and our design (for example, the probability of being pivotal is zero in their model), we cannot directly test their predictions, however. For an empirical test of their model see Coate and Conlin (2005).

differences are minor across voter types (*i.e.* allied and floating voters) and environments. For the (Bayesian-) Nash predictions, the effect of full information on participation and efficiency is much stronger and depends on voter types and environment. For example, participation of uninformed allied voters is much higher than of any voter in any other situations. This high equilibrium turnout causes serious efficiency losses.

The experiment allowed us to measure the effect of information about other voters' preferences on participation and efficiency and at the same time to select the equilibrium concept that best predicts behavior in the various environments. We find strong evidence in favor of the (comparative statics) predictions of the logit equilibria in comparison to the (Bayesian-) Nash equilibria. The quantal response equilibrium seems to capture many qualitative features of voter behavior.

Our experimental results show higher overall participation when information about other voters' preferences is provided. Moreover, in line with the logit predictions but contrary to the Nash predictions, (informed) majorities achieve a victory more often than their opposed minorities. The difference between participation in electorates with and without preference uncertainty is most striking for equal supporter group sizes. This yields severe welfare losses in case the level of disagreement is known, because voters participate much more than is socially optimal. However, overall, information about other voters' preferences has no systematic effect on welfare.

When distinguishing between allied and floating voters in our experiment, we find that the floating voters are the cause of the observed lower participation in electorates with preference uncertainty. This can be attributed to their private information about their own preference, rather than to an unstable preference or, as often argued, to them having lower stakes in the election outcome than allied voters. Our laboratory control allowed us to distinguish allied from floating voters on one dimension: the instability in their preferences, causing uncertainty about the level of disagreement in the electorate. We kept the stakes across types constant. Our experimental results show that this uncertainty *per se* is enough to yield lower turnout for floating voters, with consequences for the electoral outcome.

Overall, this study shows that the impact of information about other voters' preferences, *e.g.* through polls, on participation and efficiency depends on the underlying, actual group sizes. In this respect, our experimental results suggest that the publication of polls will increase electoral efficiency only when the tendency is that the level of disagreement is low, but will harm efficiency when it is high. Of course, our study is limited to some extent. For example, we ignored the possibility that poll outcomes may be biased by strategic voter responses, and hence may be less informative than in our study. Future research should also relax some of the assumptions in our set-up (*e.g.*, using larger electorates with a larger variety of group sizes, asymmetric size distributions, endogenous poll outcomes, and allowing for voters with 'non-selfish' preferences). Nevertheless, we report important evidence showing that (*a*) participation decreases as a consequence of uncertainty about others' preferences; (*b*) when polls reveal supporter group sizes in the electorate, participation increases strongly with the level of disagreement; and (*c*) that majorities manage to win more often than minorities.

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APPENDIX A:
BAYESIAN-NASH EQUILIBRIA OF THE PU-PARTICIPATION GAME

This appendix provides Bayesian-Nash equilibria for the PU-participation game with and without allied voters. We derive pure strategy equilibria (propositions A1 and A2) and present numerical estimations of totally quasi-symmetric mixed strategy equilibria for varying choices of parameters. We define total aggregate participation by $V \equiv V_i + V_{-i}$. Our presentation starts with equilibria for the PU-participation game without allied voters and then proceeds to the case with allied voters. First, proposition A1 describes all pure strategy Bayesian-Nash equilibria.

PROPOSITION A1 (pure strategy Bayesian-Nash equilibria in the PU-participation game without allied voters):[1]

(*i*) If $c > 1/2$, the only Bayesian-Nash equilibrium in pure strategies is $v_{j_i} = 0$, $\forall j_i$, $i = A, B$ (nobody participates).

(*ii*) If $c \leq prob(N_i = N_{-i})/2$ $\{ c \leq prob(N_i = N_{-i} + 1)/2 \}$ for E even {odd}, there is a Bayesian-Nash equilibrium in pure strategies with $v_{j_i} = 1$, $\forall j_i$, $i = A, B$ (everybody participates).

(*iii*) If $c = prob(x = V/2)/2$ $\{ c = prob(x = \lceil V/2 \rceil)/2 \wedge c \leq prob(x = \lfloor V/2 \rfloor)/2 \}$ for E even {odd}, there are Bayesian-Nash equilibria in pure strategies with $v_{j_i} = 1$, $\forall j_i$, and $v_{j_{-i}} = 1$ for some or none of the voters in $-i$, $i \neq -i$ (everybody in *i* participates and possibly some in $-i$).

(*iv*) If $c < 1/2$, then for E even {odd} any turnout level V, $0 < V < E$ $\{ 0 < V \leq E/2 \}$, is an outcome of a Bayesian-Nash equilibrium in pure strategies if $c \in [c(V)_{\min}, c(V)_{\max}]$ with $c(V)_{\min}$ ($c(V)_{\max}$) is $1/2$ times the probability that an abstainer (a participant) can change the election outcome by participating (abstaining) {for turnout levels $E/2 < V < E$ such equilibria exist for some specification of symmetric group size distributions}.

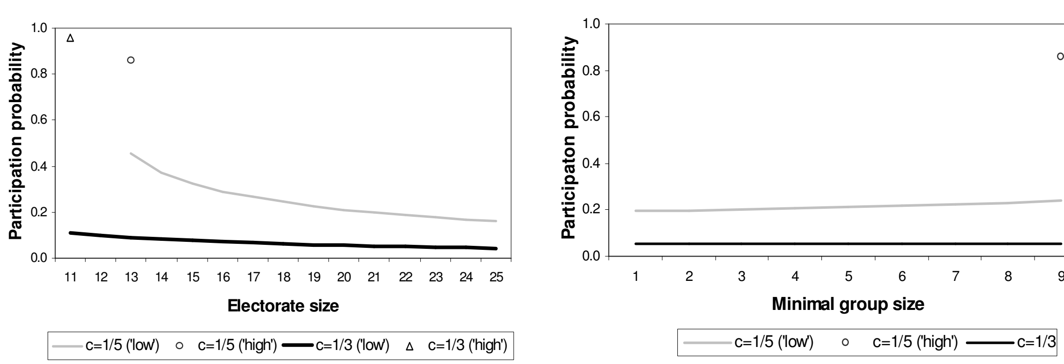
(*v*) No other Bayesian-Nash equilibria in pure strategies exist.

Proof to be found at (http://ockenfels.uni-koeln.de/person.php?i=3)

Totally quasi-symmetric mixed strategy equilibria ((*q,q*)-equilibria) can be derived numerically using eq. (8). Figure A1 depicts examples of such equilibria for the PU-participation game without allied voters. The left panel varies the electorate size $E \in \{11,12,...,25\}$, using fixed and equal minimal group sizes of 5 voters in each group ($\underline{N}_A = \underline{N}_B = 5$) and an equal probability of being in either group for each voter (binomial distribution with *p* = 0.5).

FIGURE A1: (q,q)-EQUILIBRIA IN THE PU-PARTICIPATION GAME WITHOUT ALLIED VOTERS AND WITH BINOMIAL GROUP SIZE DISTRIBUTION

VARYING ELECTORATE SIZES AND $\underline{N}_A = \underline{N}_B = 5$ | VARYING MINIMAL GROUP SIZES AND $E = 20$



Participation probabilities are shown for voting costs $c = 1/5$ and $c = 1/3$. Participation is quite low for $c = 1/3$ and slightly decreasing in E. For $c = 1/5$, we find (q,q)-equilibria only for $E \geq 13$. With lower

[1] The trivial but laborious cases with $c = 1/2$ are not discussed in this appendix.

voting cost, participation is always higher as compared to higher costs and also decreasing in E. Aside from these equilibria with low levels of participation, we find two 'high'-participation (q,q)-equilibria: one for $E=11$ when $c=1/3$ and one for $E=13$ when $c=1/5$. The right panel of figure A1 shows equilibrium participation probabilities when the electorate size is kept constant ($E=20$), but the minimal group sizes are varied and equal across groups ($\underline{N}_A=\underline{N}_B=1,2,...,9$). As in the left panel, we use $p=0.5$ and show participation for $c=1/5$ and $c=1/3$. We observe higher participation for the lower voting costs. Moreover, participation appears quite constant across different minimal group sizes. Again, we find a 'high-participation' (q,q)-equilibrium, in this case for $\underline{N}_A=\underline{N}_B=9$, when $c=1/5$.

Next, we consider pure and totally quasi-symmetric mixed strategy equilibria for the PU-participation game with allied voters. Let F denote the number of floating voters: $F\equiv E-2\underline{N}_i$. Proposition A2 gives pure strategy equilibria using a binomial group size distribution ($p=.5$). It is a straightforward extension of proposition A1. We only need to account for the differences in beliefs that allied and floating voters have about group sizes.

PROPOSITION A2 (pure strategy Bayesian-Nash equilibria in the PU-participation game with allied voters): Assume an equal number of allied voters $\underline{N}_A=\underline{N}_B\geq 1$, and binomially distributed floating voters with $p=.5$.

(*i*) If $c>1/2$, the only Bayesian-Nash equilibrium in pure strategies is $v_{j_{i,a}}=v_{j_{i,f}}=0$, $\forall j_{i,a}$, $\forall j_{i,f}$, $i=A,B$ (nobody participates).

(*ii*) If $c\leq\binom{F}{\lfloor F/2\rfloor}(1/2)^{F+1}$, the only Bayesian-Nash equilibrium in pure strategies is $v_{j_i,a}=v_{j_{i,f}}=1$, $\forall j_{i,a}$, $\forall j_{i,f}$, $i=A,B$ (everybody participates).

(*iii*) Other, more specific Bayesian-Nash equilibria in pure strategies exist.

Proof to be found at (http://ockenfels.uni-koeln.de/person.php?i=3)

Similar to proposition A1(*i*), proposition A2(*i*) states that all voters abstain if the voting costs are too high. As propositions A1(*ii*), A2(*ii*) is an intuitive extension of the full participation equilibrium for $N_i=N_{-i}$ analyzed in PR83. If all voters have high enough (compared to costs) expectations that both groups are of equal size respectively that there is one voter more in the own group, a full participation equilibrium exists.

Next, we consider totally quasi-symmetric mixed strategy equilibria for the game with allied and floating voters. Elaboration and specification of condition (6) for this case gives implicit functions for the best responses q_a (A1a) and q_f (A1b):

$$\sum_{y=0}^{F}\binom{F}{y}(.5)^{y}(.5)^{F-y}$$

$$\times\Bigg[\sum_{k=0}^{\min[\underline{N}_i+y-1,\underline{N}_i+F-y]}\ \sum_{k_i=\max[0,k-\underline{N}_i+1]}^{\min[y,k]}\ \sum_{k_{-i}=\max[0,k-\underline{N}_i]}^{\min[F-y,k]}\binom{\underline{N}_i-1}{k-k_i}\binom{y}{k_i}\binom{\underline{N}_i}{k-k_{-i}}\binom{F-y}{k_{-i}}$$

$$\times\, q_a^{2k-k_i-k_{-i}}(1-q_a)^{2\underline{N}_i-1-2k+k_i+k_{-i}}q_f^{k_i+k_{-i}}(1-q_f)^{F-k_i-k_{-i}}$$

$$+\sum_{k=0}^{\min[\underline{N}_i+y-1,\underline{N}_i+F-y-1]}\ \sum_{k_i=\max[0,k-\underline{N}_i+1]}^{\min[y,k]}\ \sum_{k_{-i}=\max[0,k+1-\underline{N}_i]}^{\min[F-y,k+1]}\binom{\underline{N}_i-1}{k-k_i}\binom{y}{k_i}\binom{\underline{N}_i}{k+1-k_{-i}}\binom{F-y}{k_{-i}}$$

$$\times\, q_a^{2k+1-k_i-k_{-i}}(1-q_a)^{2\underline{N}_i-2-2k+k_i+k_{-i}}q_f^{k_i+k_{-i}}(1-q_f)^{F-k_i-k_{-i}}\Bigg]=2c \qquad \text{(A1a)}$$

$$\sum_{y=1}^{F}\binom{F-1}{y-1}(.5)^{y-1}(.5)^{F-y}$$

$$\times\ \Bigg[\sum_{k=0}^{\min[\underline{N}_i+y-1,\underline{N}_i+F-y]}\ \sum_{k_i=\max[0,k-\underline{N}_i]}^{\min[y-1,k]}\ \sum_{k_{-i}=\max[0,k-\underline{N}_i]}^{\min[F-y,k]}\binom{\underline{N}_i}{k-k_i}\binom{y-1}{k_i}\binom{\underline{N}_i}{k-k_{-i}}\binom{F-y}{k_{-i}}$$

$$\times\ q_a^{2k-k_i-k_{-i}}(1-q_a)^{2\underline{N}_i-2k+k_i+k_{-i}}\,q_f^{k_i+k_{-i}}(1-q_f)^{F-1-k_i-k_{-i}}$$

$$+\ \sum_{k=0}^{\max[\underline{N}_i+y-1,\underline{N}_i+F-y-1]}\ \sum_{k_i=\max[0,k-\underline{N}_i]}^{\min[y-1,k]}\ \sum_{k_{-i}=\max[0,k+1-\underline{N}_i]}^{\min[F-y,k+1]}\binom{\underline{N}_i}{k-k_i}\binom{y-1}{k_i}\binom{\underline{N}_i}{k+1-k_{-i}}\binom{F-y}{k_{-i}}$$

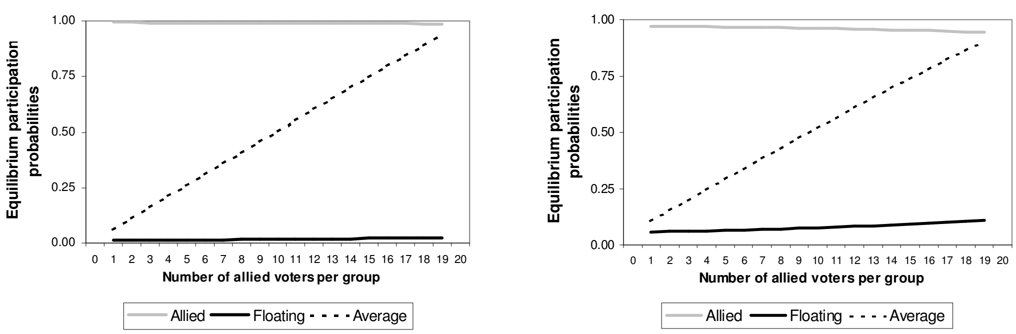
$$\times\ q_a^{2k+1-k_i-k_{-i}}(1-q_a)^{2\underline{N}_i-2k-1+k_i+k_{-i}}\,q_f^{k_i+k_{-i}}(1-q_f)^{F-1-k_i-k_{-i}}\Bigg]\ =\ 2c\,. \qquad (\mathrm{A1b})$$

To understand theses conditions, consider $(\mathrm{A1a})$ ($(\mathrm{A1b})$ is a similar application to floating voters). The equation elaborates the condition that the probability of being pivotal is equal to $2c$ for a mixed strategy to be a best response. The left-hand side of $(\mathrm{A1a})$ shows this probability for an allied voter. The term outside of the square brackets gives the probabilities of y $(F-y)$ floating voters being 'allocated' to the same (other) group as this allied voter. For each y, the first (second) term inside the square brackets gives the probability that this voter can break (create) a tie. In the first term, all ties with k votes are considered. In the allied voter's own group, the k votes consist of k_i votes by floating voters and $k-k_i$ by the other allied voters. In the other group k_{-i} $(k-k_{-i})$ of the floating (allied) voters turn out. The first term gives the probability for each event (k,k_i,k_{-i}), given the best responses. In a similar way, the second term inside the square brackets represents the probabilities of all events where k other voters in the allied voter's own group vote and $k+1$ in the other group.

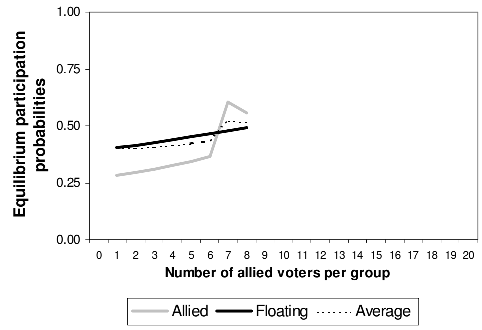
Once again, these equilibria cannot be derived analytically. Numerical estimations show that they do exist for a wide range of parameter values, however. Figure A2 shows numerical examples of such '(q_a,q_f,q_a,q_f)-equilibria' for a fixed electorate size $E=40$ and varying numbers of allied and floating voters. The number of allied voters per group is from the set $\underline{N}_i\in\{1,,2,...,19\}$ and equal across groups ($\underline{N}_A=\underline{N}_B$). We present participation probabilities for allied and floating voters for voting costs $c=.40$ (upper left panel), $c=.25$ (upper right panel), and $c=.10$ (lower panel). The figure indicates very high (low) participation for allied (floating) voters for both higher costs cases. Only for $c=.10$, equilibrium participation is in the middle range and similar for the two types. We find no equilibrium for these costs for $\underline{N}_i>8$.

FIGURE A2: (q_a,q_f,q_a,q_f)-EQUILIBIRA IN THE PU-PARTICIPATION GAME WITH ALLIED VOTERS, BINOMIAL GROUP SIZE DISTRBITUION, $E=40$, AND $\underline{N}_A=\underline{N}_B\in(1,2,...,19)$

$c=.40$ $\qquad$ $c=.25$



$c = .10$



APPENDIX B:
Instructions for treatment UF [*UM*, *IF*, *IM*]

Welcome to our experiment on decision-making. Depending on your *own choices* and the *choices of other participants*, you may earn money today. Your earnings in the experiment are expressed in *tokens*. 4 tokens are worth one Guilder. At the end of the experiment your total earnings in tokens will be exchanged into Guilders and paid to you in cash. The payment will remain *anonymous*. No other participant will be informed about your payment.

Please remain quiet and do not communicate with other participants during the entire experiment. Raise you hand if you have any questions. One of us will come to you to answer them.

Rounds, 'your group' and the 'other group'

The experiment consists of *100 rounds*. At the *beginning of the experiment* the computer program will randomly split all participants into two different *populations* of 12 participants. In addition, at the *beginning of each round* the computer program will randomly divide the participants in each population into *two groups*. The group you are part of will be referred to as *your group* and the group in your population which you are not part of will be called the *other group*. You will not know which of the participants belongs in the other group and which to your group. You will have nothing to do with participants in the other population in this experiment.

Number of participants in 'your group' and the 'other group'

At the beginning of *each round* the computer program will randomly determine the *number of participants* in your group and the number of participants in the other group. **At no point in time will you or anybody else receive information about the number of participants in your group and the number of participants in the other group.** [*In IF and IM instead*: **You and all other participants in both groups will then receive information about the number of participants in your group and the number of participants in the other group.**]

However, you and all other participants know that [*in IF and IM*: There is the following structure of group sizes]:
(1) Independent of the round, the sum of participants in *both* groups (your group and the other group) is always **12**.
(2) Both groups contain a *minimum* of **3** participants and a *maximum* of **9** participants.

Because the sum of participants in both groups is always twelve, there are the following **7** possible *combinations of group sizes*:

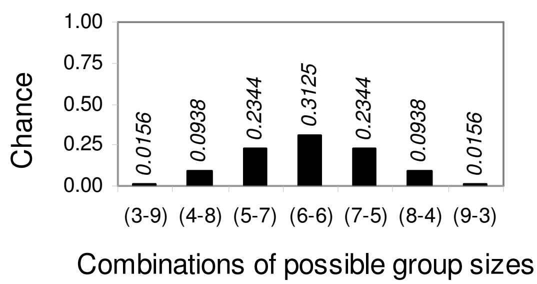
(3-9) (4-8) (5-7) (6-6) (7-5) (8-4) (9-3),

whereby the first number represents the group size of the first group and the second number the group size of the second group.
The arrangement of a population (12 participants) into two groups by the computer program proceeds in the following two steps:
(1) Both groups are randomly filled with **3** participants, the minimal number of participants per group (in total **6** participants). Each participant has the *same* chance of being selected.
(2) Each of the remaining **6** participants is randomly put into one of the two groups, with a chance of 50% for each group.

[*For UM and IM instead*:
(1) At the beginning of the *first round* both groups are randomly filled with **3** participants, the minimal number of participants per group (in total **6** participants). Each participant has the *same* chance of being selected. The chosen participants will be called '*FIX*'-participants, because they will not change groups during the whole experiment.
(2) At the beginning of *each round* each of the remaining **6** participants is randomly put into one of the two groups, with a chance of 50% for each group. These participants will be called '*VAR*'-(=variable) participants, because they will randomly change groups during the whole experiment.

At the beginning of the *first round* you will receive information about your own type *FIX* or *VAR*. Your own type will not change during the whole experiment.]

The following figure shows for all seven possible combinations of group sizes the chance that a particular combination occurs.



Note again that group sizes will be randomly determined at the beginning of *each* round. Hence, the group sizes may change from one round to another.

Choices and earnings

In *each round* you and all other participants will face an identical choice problem. You will be asked to make *one choice*. You can choose between the following two alternatives:

- '*Choice A*': no costs involved (**0 tokens**).
- '*Choice B*': costs are **1 token**.

When making your choice, nobody else in your group or in the other group will know this choice. After *all* participants have made their choices, the computer program will count the number of *B*-choices in your group and in the other group and will compare the numbers in both groups. There are **3** possible outcomes that are relevant for your *revenue* in the following way. You will receive the revenue irrespective of the choice you made.

(1) The number of *B*-choices in your group **exceeds** the number of *B*-choices in the other group. In this case *each* participant in your group (including yourself) will get revenue of **4 tokens**. *Each* participant in the other group will get **1 token**.
(2) The number of *B*-choices in your group **is smaller than** the number of *B*-choices in the other group. In this case *each* participant in your group (including yourself) will get revenue of **1 token**. *Each* participant in the other group will get **4 tokens**.
(3) The number of *B*-choices in your group **is equal to** the number of *B*-choices in the other group. In this case the computer program will randomly determine the group in which *each* participant gets revenue of **4 tokens** (each group has the same chance of 50% of being chosen). *Each* participant in the group that is not chosen will get **1 token**.

Your _round earnings_ are calculated in the following way: *round earnings = round revenue – round costs*. Your _total earnings_ are the sum of all of your round earnings.

The following table gives your possible round earnings:

Your possible round earnings:

Your choice	*Your group has more B-choices*	*Your group has less B-choices*	*Equal number of B-choices in both groups*
Choice *A*	**4 tokens**	**1 token**	**4** *or* **1 token** (50% chance each)
Choice *B*	**3 tokens**	**0 token**	**3** *or* **0 token** (50% chance each)

Computer screen

The computer screen has four main windows.
(1) The _Status_ window shows [*for UM and IM*: your _type_ (*FIX* or *VAR*),] the actual _round number_ and the _total earnings_ up to the previous round.
(2) The _Previous round_ window depicts the following information about the previous round:
   (a) The number of *B-choices in your group* [*in IF and IM*: and, in brackets, the size of your group].
   (b) The number of *B-choices in the other group* [*in IF and IM*: and, in brackets, the size of the other group].
   (c) Your *choice*.
   (d) Your *revenue*.
   (e) Your *costs*.
   (f) Your *round earnings*.
   Note that no information about the group sizes will be given [*this sentence not for IF and IM*].
(3) In the _Choice_ window you will find two *buttons*. Press the button "Choice *A*" or the button "Choice *B*" with the mouse, or press the key "*A*" or "*B*". When you have chosen you will have to wait until all participants have made their choices.
(4) The _Result_ window shows the result of the *current* round, hence after each participant has made a choice. Each *yellow* rectangle shown represents one *B*-choice of your group and each *blue* rectangle represents one *B*-choice of the other group. After a few seconds the result will also appear in numbers.

At the top of the screen you will find a _Menu bar_. You can use this to access the _Calculator_ and _History_ functions. The calculator can be handled with the numerical pad on the right side of your keyboard or with the mouse buttons. The function 'history' shows all information of the last *sixteen* rounds as this had appeared in the window 'Previous round'. At the bottom of your screen the _Information bar_ is located. There you are told the actual status of the experiment.

Further procedures

Before the 100 rounds of the experiment start, we will ask you to participate in _three training-rounds_. You will have to answer questions in order to proceed further in these training-rounds. In the training-rounds you are not matched to other participants but to the computer program. **You cannot draw conclusions about choices of other participants based on the results in the training-rounds.** The training-rounds will not count for your payment.

We will now start with the three trainings-rounds. If you have any questions, please raise your hand. One of us will come to you to answer them.

APPENDIX C:

TABLE: SEQUENCE OF ELECTORAL COMPOSITIONS

Round	Elect. comp.	Round	Elect. comp.	Round	Elect. comp.	Round	Elect. comp.	Round	Elect. comp.
1	*6-6*	21	*5-7*	41	*6-6*	61	*7-5*	81	*4-8*
2	*7-5*	22	*6-6*	42	*6-6*	62	*7-5*	82	*6-6*
3	*6-6*	23	*4-8*	43	*7-5*	63	*5-7*	83	*6-6*
4	*5-7*	24	*7-5*	44	*6-6*	64	*6-6*	84	*6-6*
5	*6-6*	25	*6-6*	45	*4-8*	65	*4-8*	85	*5-7*
6	*4-8*	26	*9-3*	46	*5-7*	66	*5-7*	86	*5-7*
7	*6-6*	27	*6-6*	47	*7-5*	67	*5-7*	87	*6-6*
8	*7-5*	28	*7-5*	48	*7-5*	68	*6-6*	88	*8-4*
9	*7-5*	29	*5-7*	49	*6-6*	69	*6-6*	89	*6-6*
10	*5-7*	30	*5-7*	50	*8-4*	70	*5-7*	90	*6-6*
11	*5-7*	31	*5-7*	51	*6-6*	71	*5-7*	91	*7-5*
12	*7-5*	32	*8-4*	52	*5-7*	72	*6-6*	92	*6-6*
13	*4-8*	33	*8-4*	53	*6-6*	73	*8-4*	93	*5-7*
14	*7-5*	34	*6-6*	54	*7-5*	74	*5-7*	94	*8-4*
15	*7-5*	35	*7-5*	55	*7-5*	75	*7-5*	95	*7-5*
16	*8-4*	36	*5-7*	56	*4-8*	76	*7-5*	96	*3-9*
17	*6-6*	37	*6-6*	57	*6-6*	77	*4-8*	97	*5-7*
18	*5-7*	38	*5-7*	58	*3-9*	78	*5-7*	98	*6-6*
19	*4-8*	39	*6-6*	59	*6-6*	79	*6-6*	99	*6-6*
20	*9-3*	40	*7-5*	60	*7-5*	80	*8-4*	100	*8-4*

APPENDIX D:
TABLE: OVERVIEW OF EQUILIBRIUM PREDICTIONS, OBSERVED PARTICIPATION RATES FOR ROUNDS 21 TO 100, AND MAXIMUM LIKELIHOOD ESTIMATES OF THE NOISE PARAMETERS

Treatment	Group size	Observed participation rates	Predictions			Estimated μ	# observations	Log-likelihood		
			(Bayesian) Nash	QRE				(Bayesian) Nash	QRE	Random play
			E_0	$E_{.4}$	$E_{.8}$					
UF	-	.297	.098	.302	.381	.39	5616	-4286.62	-3418.01	-3892.71
UM	-	*Allied*: .426	.934	.306	.382	.05 or 1.38	2880	-4576.80	-1964.64	-1996.26
		Floating: .293	.123	.299	.380	.38	2880	-2033.92	-1741.13	-1996.26
	All	.359	.529**	.303**	.381**	.65	5760	-4094.34	-3761.61	-3992,53
IF	3	.296	.256	.306	.341	.08	54	-33.04	-32.82	-37.43
	4	.301	.173	.320	.369	.29	336	-221.73	-205.42	-232.90
	5	.329	.132	.331	.395	.39	1050	-798.61	-664.82	-727.80
	6	.481	.107*	.329	.404	4.28	2016	-2285.99	-1395.95	-1397.38
	7	.437	.091	.306	.384	1.58	1470	-1619.11	-1007.13	-1018.93
	8	.281	.080	.271	.349	.44	672	-517.75	-399.26	-465.79
	9	.241	.074	.234	.313	.43	162	-111.11	-89.41	-112.29
	All	.400	.109**	.313**	.385**	.95	5760	-5501.26	-3875.74	-3992.53
IM	3	.296	.256	.306	.341	.08	54	-33.04	-32.82	-37.43
	4	.286	.173	.320	.369	.22	336	-213.92	-201.02	-232.90
	5	.393	.132	.331	.395	.79	1050	-926.74	-703.73	-727.80
	6	.585	.107*	.329	.404	∞	2016	-2731.49	-1397.38	-1397.38
	7	.448	.091	.306	.384	1.96	1470	-1658.27	-1011.05	-1018.93
	8	.341	.080	.271	.349	.74	672	-615.48	-431.12	-465.79
	9	.302	.074	.234	.313	.73	162	-136.43	-99.30	-112.29
	All	.459	.109**	.313**	.385**	2.55	5760	-6215.60	-3972.84	-3992.53

* Or, .893; **weighted overall participation probability.