Presentation Chapter 3 of Moulin by Otto de Smeth

Exercise 3.11a

Divide $100,- between two agents with the following utilities for money,

and no initial endowments: $u_1(x)=v(x_1)$, $u_2(x)=2v(x_2)$

So, the constraint here is: $x_1+x_2=100$

(Note that the solution-concepts only care about the utilities of the agents, and not about the amount of dollars that each agent gets.)

Solution:	X1	X2	U1	U2
Utilitarian	20	80	4.47	17.89
Egalitarian	80	20	8.99	8.99
Nash	50	50	7.07	14.14
KS	50	50	7.07	14.14

Exercise 11d: What happens to the solutions when the cash prize

increases?

So suppose now that $200.- has to be divided so that the constraint now is

$x_1+x_2=200$

The new outcomes are then:

Solution:	X1	X2
Utilitarian	40	160
Egalitarian	160	40
Nash	100	100
KS	100	100

In the previous example, there was “the” Nash solution, whereas the book discerns

1) Nash collective utility function
2) Nash bargaining solution

Because there were no initial endowments, there was no difference, but consider the following example:

A student and a millionaire are dividing $100,-. (in $1,- coins)

Assume now: $u_1(x)=v(x_1)$, $u_2(x)=v(1{,}000{,}000+x_2)$

Then under the **Nash collective utility function**:

Maximize $u_1*u_{2.}$ This has the obvious solution: $x_1=100$, $x_2=0$

(notice the implicit constraint: $x_{1,}x_2 =_0$)

But under the **Nash bargaining solution:**

Maximization of Nash c.u.f. under normalization of individual zero’s, so:

Max. $(u_1-u_1^0 * u_2-u_2^0)$. $u_1^0=0$ and $u_2^0=1000$.

But $x_1=x_2=50$ is not the solution, because then the utility-gain of the millionaire would be v1,000,050-1,000=0,025 and 0,025*v50=0,176 but and allocation of (67,33) would yield 0,0335*v33=0,1924

Notice that the KS-solutions equalizes relative utility-gains, so the KS-solution would be 1 for the student and 99 for the millionaire

Three properties of some welfare functions have appeared throughout the text:

1) **independence of individual zero's**

if we square negative utilities, then those who had a zero utility, suddenly have another ranking than the negative ones.

2) **independence of common utility pace**

this property means that there is no rescaling possible of ***all*** utilities ***at the same time*** that would affect the choice of allocation. So all utilities could be squared, and that does not matter.

3) **independence of individual scales of utility**

this property means that the intensity of a ***single*** preference does not play a role in the decision of an allocation. Eliminates strategic incentives to overstate one's preferences.

Which of the 3 welfare functions has which property?

	Utilitarianism	Egalitarianism	Nash c.u.f.
Individual zero's	Yes	No	No
Common utility pace	No	Yes	No
Individual scales of utility	No	No	Yes

Pitfalls:

There are several kinds of monotonicity mentioned in the book:

1) (p.47) resource monotonicity: when the surplus t increases, each agent i gets a larger share y_i
2) (p.47) population monotonicity: one agent dead is an extra bite of the pie for all.
3) (p.66) monotonicity: increase of u_i and for all $u_{j?i}$? u_j=0, then W(u) increases

page 73: Of course….does not hold. This is so, because when:

u={1,2}

u*={1,3}

then $W_e(u)=W_e(u^*) = \min_i u_i=1$,

but not u=u* because the egalitarian solution would prefer u*

P.94 “any social welfare ordering strictly (only) improves under a Pigou-Dalton transfer.” Why?

The P-D transfer means that a redistribution can only take place when the allocation after the redistribution is preferred over the allocation before the redistribution. So when new allocations become possible, those new allocations themselves have to be preferred, otherwise they are irrelevant. The KS-solution is not a welfare function, so it has no preference over distributions. More bargaining strength is relevant for the KS-solution, but not for a welfare distribution.