

STRUCTURE IN LEGISLATIVE BARGAINING

Adrian de Groot Ruiz*

Roald Ramer⁺

Arthur Schram*

ABSTRACT

We study how the degree of structure of a legislative bargaining process affects the outcome. We compare a high-structure alternating offers game to a low-structured continuous-time bargaining game. Both non-cooperative games correspond to the same cooperative game: a three-player median voter setting with an external disagreement point. The divergence of interests (polarization) determines whether the core is empty (if so, we consider the uncovered set). In the high-structure game, the (refined) subgame perfect equilibrium converges to the core when this exists. In the low-structure game, a large range of outcomes can be supported in equilibrium. Our main experimental finding is that structure matters. First, the median player is significantly better off with low structure. Second, the cooperative solution concepts (core and uncovered set) perform significantly better when structure is low. This result provides support for the intuition that cooperative game theory is more applicable to settings with less structure.

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*Center for Research in Experimental Economics and political Decision making (CREED), Faculty of Economics and Business, University of Amsterdam

⁺ Center for Nonlinear Dynamics in Economics and Finance (CeNDEF), Faculty of Economics and Business, University of Amsterdam

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ADDRESS

Amsterdam School of Economics
Roetersstraat 11
1018 WB Amsterdam
The Netherlands

Email: De Groot Ruiz (corresponding author): a.degrootruiz@uva.nl; Ramer: R.Ramer@uva.nl; Schram: Schram@uva.nl.

1. INTRODUCTION

In legislative decision making processes, each member typically negotiates to obtain majority support for the individually preferred outcome, or something as close to it as possible. The extent to which a member can direct the final outcome towards her own preference depends on the procedures governing the process, however. In particular, since the research boom on spatial voting in the late 1970s it has been recognized that the structure of the bargaining process can be highly relevant. If the structure favors specific negotiators (*e.g.*, through the order of voting, agenda-setting power, or proposal and voting rights) the outcome may crucially depend on it (*e.g.*, McKelvey 1976; 1979; Schofield 1978; Nolan 2000). Much less is known about the effect of structure when procedures are ‘neutral’ and favor no particular negotiator *prima facie*, such as procedures where agenda-formation is endogenous and all players have equal proposal and voting opportunities.

In this paper we consider legislative bargaining procedures that are neutral in this respect. The main question we address is whether structure still matters in such an environment. In particular, can the freedom players have as to the order and timing of proposals still affect the legislative outcome? If structure matters in these situations, does it do so for purely strategic reasons or do psychological effects of structure play a role? To provide a first answer to these questions we analyze legislative bargaining both theoretically and in a controlled laboratory experiment. Specifically, we compare two legislative bargaining processes. One has a (neutral) low structure, the other a (neutral) high structure. In the process with low structure players can freely make and accept proposals at any time. In the high-structure process proposals and voting are regulated by a Baron-Ferejohn (1987) alternating offers scheme. Parliamentary voting procedures are an example of highly structured bargaining, whereas backroom dealings may be seen as a form with low structure.

Understanding the relevance of structure is important when studying institutional choice and parliamentary procedures. Though most legislatures are *prima facie* neutral and impose fixed rules on their bargaining process, these typically allow for a mixture of highly and lowly structured bargaining. During the legislative process, members may move back and forth between backroom negotiations and formal voting

on proposals, for example. The choice between how much weight to put on either of the two may well be determined by strategic considerations (Elster 1998; Stasavage 2004). For instance, parties with a strong bargaining position may prefer backrooms and wish to reserve formal voting for well negotiated deals. On the other hand, parties with more extreme positions might prefer to avoid backrooms and follow the more formal procedures in order to allow their proposals to have a chance of success. By studying the effects of structure on bargaining outcomes, this study intends to help us better understand this kind of preferences for low or high structure.

Our study also points to the validity of distinct solution concepts for the analysis of legislative bargaining. The bargaining problem can be modeled as a cooperative game. Imposing a specific bargaining structure (in the form of rules) results in a bargaining process that can be modeled as a non-cooperative game (Baron and Ferejohn 1987, 1989; Okada 1996; Banks and Duggan 2000, 2006; Miyakawa 2008). However, the non-cooperative solution obtained may depend critically on the specific bargaining process chosen. In contrast, the solutions obtained when analyzing the bargaining problem as a cooperative game do not depend on structure. Hence, a single cooperative game will yield diverse non-cooperative predictions under distinct bargaining processes. These predictions may or may not correspond to the solution concept of the cooperative game. The validity of cooperative models then depends on the (ir)relevance of the bargaining process, which is the central issue of this study.

Moreover, though non-cooperative models explicitly take the procedure into account, they typically need to simplify reality in order to keep the analysis tractable. This means that they often impose more structure on the bargaining process than is present in actual fact. The force of the Baron-Ferejohn model, for instance, lies in the fact that it can make clear, tractable predictions by offering a highly stylized model of real legislative bargaining. Whether such simplifications made in non-cooperative models are justified, depends on the sensitivity of the bargaining outcome to the degree of structure. Again, our results will pertain to this issue.

We study the effect of structure in the context of a three-player legislative bargaining setting, in which we can naturally vary whether or not the core is empty. This game is interesting in its own right and has the following motivation. In the classic one-dimensional median voter setting (Black, 1948; 1958), the degree of

polarization (the divergence of players' preferences) is irrelevant to the outcome and its stability. The median player's ideal point is the unique (strong) core outcome, no matter how strong polarization is. However, in real life one may expect that the outcome of a legislative bargaining process or the coalition supporting this outcome is less stable if preferences are far apart, even if the policy space seems unidimensional. One explanation, which has not been appreciated in the literature, is that the disagreement point may well be a point outside of the line on which all policy proposals may be defined. First, a decision often involves a new type of policy or project so that the status quo may not fall in the space considered for the new policy. Second, if the disagreement point consists in the termination of a project or a coalition, then it may involve significant transaction costs (eg. involving new elections). If so, the disagreement point will be of a qualitatively different nature than the issue under negotiation.

An example serves to illustrate the environments we are looking at. Imagine a legislature that consists of three factions (doves, moderates and hawks) and is deciding on the renewal of a budget for an ongoing war. No single faction holds a majority and any coalition of two does. Doves prefer a reduction of the current war budget, moderates want no change and hawks would like an increase. Preferences are single-peaked with respect to budget revisions. The option to end the war ('retreat') serves as a disagreement point, which cannot simply be represented as a budget revision. (Retreating is qualitatively distinct from a reduction of the budget to zero and spending 5 billion on retreating is rather distinct than spending this amount on war efforts.) Preferences are such, that all parties prefer some revisions to retreating. How much interests diverge, will drive the stability of the outcome. Polarization is defined as the distance between the ideal revisions of the factions (relative to the attractiveness of retreating) and captures divergence of interests. If polarization is weak, then the factions' preferences lie close together and retreating is a relatively unattractive agreement. Hence, all coalitions will prefer the median preference to retreating. In this case, we can use Black's Median Voter Theorem (1948; 1958) to predict that the moderates' ideal point will prevail. If polarization is strong, the ideal revisions are very far apart. In this case retreating is relatively attractive and there are no revisions that any coalition prefers to retreating. With moderate polarization, we

get a cyclical pattern. Both doves and hawks prefer retreating to the unaltered budget; however, moderates and doves prefer some negative revisions to retreating; and moderates and hawks prefer the unaltered budget to negative revisions. In this case, it is not clear what the legislature may decide. Intuitively, one may expect the moderates to have the highest bargaining power, as doves and hawks can only coordinate on retreating.

Our model and experimental design take the same basic form as this example. The point of departure is a three-player cooperative game where we can use cooperative solution concepts to predict outcomes. We use a median voter setting, modified to have an exterior disagreement point. This allows us to obtain varying outcomes with respect to the core by varying the level of polarization. Then, we introduce two bargaining structures that differ in the rules they impose on the bargaining process. The low-structure (non-cooperative) game allows players to freely make and accept proposals in continuous time. The high-structure (non-cooperative) scenario is an alternating offers game where agenda-setting and voting is regulated according to a Baron-Ferejohn scheme.

The bargaining outcome will typically depend on specific characteristics of the environment (*i.c.*, the extent of polarization). As a consequence, the effect of structure may also depend on these characteristics. In particular, the relevance of structure may arguably be dependent on whether or not the bargaining problem has a core, which in turn may be affected by the extent of polarization.¹ When the core is empty, all outcomes can be supported by some agenda-setting institution (McKelvey 1976, 1979; Schofield 1978). This is important, because the core will be empty unless extreme symmetry conditions are satisfied (Gillies 1953; Plott 1967; LeBreton 1987; Saari 1997) and, hence, many real life bargaining problems are bound to have an empty core (Riker 1980). Nevertheless, the core may be non-empty in some bargaining games. Theoretically, the (non-cooperative) equilibrium outcome for many procedures then tends to lie in the core (Perry and Reny 1994; Baron 1996; Banks and Duggan 2006). There is indeed experimental evidence on the stability of core-outcomes (*e.g.*, Fiorina and Plott 1978, McKelvey and Ordeshook 1984, Palfrey

¹ In the example, when polarization is: (i) low, the unique core element is the moderates' preference for no change in budget; (ii) moderate, the core is empty; (iii) high, the unique core element is the decision to retreat.

2006). In our setting, the core consists of a unique element if it is non-empty, and we may therefore expect various procedures to yield the same outcome. On the other hand, in some situations, the outcome has been shown to be sensitive to fairness considerations (Isaac and Plott 1978; Eavey and Miller 1984). Structure may then matter even if there is a core, for instance, if some procedures are considered fairer than others (Bolton *et al.* 2005).

As in the example, in our model the variations in polarization indeed determine whether or not the core is empty. We find the following theoretical outcomes for the cooperative game (irrespective of structure).² When polarization is weak, the core consists of the median preference and with strong polarization the core is the disagreement point. With moderate polarization, the core is empty. The uncovered set³ is equal to the core when the latter exists and consists of three or four points when the core is empty. As discussed above, the non-cooperative predictions depend on the structure. For the high-structure game, we derive a unique (refined) subgame perfect equilibrium (SPE) that converges (with the number of bargaining rounds) to the core element when this exists. It typically does not converge at all when the core is empty. In the low-structure game the disagreement point and all points between the players' ideal points can be supported as a SPE-outcome, irrespective of the extent of polarization. Our interpretation is that low structure offers so much strategic flexibility that strategic considerations alone cannot identify an outcome.

From our experiments, we have two main findings: polarization matters and structure matters. Polarization has a strong impact on the outcome. In accordance with theory, the median player is significantly worse off with moderate than with weak polarization. However, we find that polarization hurts the median player and does so even at weak levels when her most preferred outcome is the unique core element. Our experimental findings attribute this to intra-coalitional fairness considerations. Such

² Two theoretical breakthroughs have allowed us to overcome challenges in studying structure. First, for a long time many cooperative solution concepts have been advanced for situations in which the core is empty but none found broad theoretical and empirical support. Miller's (1980) uncovered set as a generalization of the core drew theoretical support (Sheples and Weingast 1984, Banks 1985, Cox 1987, Scott *et al.* 1987). However, systematic empirical tests were problematic, as it was impossible to compute the uncovered set in most cases. By developing an algorithm to find the uncovered set, Bianco *et al.* (2006, 2007) managed to find solid empirical support for this solution concept using data from many old and new experiments. Second, continuous time bargaining has made the non-cooperative analysis of low-structure settings possible (Simon and Stinchcombe 1989; Perry and Reny 1993).

³ We will formally introduce the concept of the uncovered set in section 3.

considerations become less important as negotiators gain more experience, however. After players have repeatedly played the game (in ever-changing groups), competition between coalitions is strengthened and the position of the median player also becomes stronger within the set of weak polarization levels.

Our second group of experimental results show that structure matters. It does so, on two accounts, even when the core is nonempty. First, the median player is significantly better off with low than with high structure. One plausible cause seems to be that flexibility in making proposals at any time increases her ability to exploit her superior bargaining position, as observed by Drouvelis et al (2010) in a different setting. This points to the more general idea that parties in a superior bargaining position will prefer institutions that impose less structure on the bargaining procedure.

The second way in which structure matters is that cooperative solution concepts fare better when there is less structure in the bargaining process. Specifically, the core (for weak polarization) and the uncovered set (for moderate polarization) perform significantly better with low than with high structure. In the final periods of the game, they both attract approximately 60% of all outcomes in the low-structure bargaining games, whereas they attract just 30% of all outcomes when structure is high. Though it is intuitively appealing that cooperative theory predicts better when it is not ‘hindered’ by structure, note that this is neither predicted by cooperative game theory nor by non-cooperative game theory. The ‘Nash-program’ attempts to make a link between the two types of game theory (Serrano 2005), but it has –to the best of our knowledge– not yet compared distinct non-cooperative games applied to the same cooperative game. Our study does exactly this.

The remainder of this paper is organized as follows. Section 2 models the bargaining problem as a cooperative game and applies various solution concepts. Section 3 describes and solves the non-cooperative games for the low- and high-structure bargaining processes. Our experimental design is presented in section 4 and the experimental results in section 5. Section 6 concludes.

2. THE BARGAINING PROBLEM AS A COOPERATIVE GAME

Formally, the bargaining problem is represented by $\Gamma = \Gamma(N, Z, u_i, \mathcal{W})$ and consists of a finite set N of players, thought of as factions in a legislature; a collection \mathcal{W} of subsets of N , thought of as winning coalitions; a set Z of alternatives; and utility functions u_i , one for each player $i \in N$ representing i 's preferences over Z . Note that although winning coalitions have been specified, nothing as yet has been said about the decision making process itself. Structures governing this process will be described and formalized in the next section.

In the bargaining problems studied here three players ($N = \{1,2,3\}$) bargain over the set of alternatives represented by $Z \equiv R \cup \delta$, with R denoting the set of real numbers and δ a disagreement point. Each player $i \in N$ has an ideal point $z_i \in R$. Without loss of generality we normalize by setting $z_1 = -a < 0$, $z_2 = 0$, and $z_3 = b > 0$, with $b \geq a$. Hence, the ideal point of player 2, the *median* player, is $z = 0$. For players 1 and 3, the *wing* players, z_1 is normalized to lie closer to 0 than z_3 . This distance, a , between the closer wing player and the median player will be interpreted as a measure of the *polarization* of players' preferences. We shall distinguish the three cases of respectively, weak ($a \leq 1$), moderate ($1 < a < 2$), and strong polarization ($a \geq 2$).

Preferences of all players are single-peaked on R and represented by piecewise linear von Neumann-Morgenstern utility functions $u_i(z) = 1 - |z - z_i|$. We further assume that the utility attributed to the disagreement point is normalized at 0, that is, $u_i(\delta) = 0$, for all $i \in N$. Hence, each player has an open interval of outcomes with strictly positive values; to wit, $(-a - 1, -a + 1)$ for player 1, $(-1, 1)$ for player 2, and $(b - 1, b + 1)$ for player 3. Note that the endpoints of these intervals yield utility of 0, while the outcomes outside of these intervals are strictly worse for the respective players than the disagreement point δ .

As for winning coalitions \mathcal{W} , we assume that any majority of two players can implement any $z \in Z$ as the outcome. This can be achieved in various ways, determined by the structure of the bargaining process (section 3).

Γ can be regarded as a cooperative game and more precisely as a coalitional game without transferable payoff.⁴ We start by defining the dominating and covering relations for any given $\Gamma(N, Z, u_i, W)$.

DEFINITION 1. Let $z, z' \in Z$. We say that

(i) z' dominates z , and write $z' \succ z$, if there is a winning coalition $M \in W$ such that all members in M strictly prefer z' to z ;

(ii) z' covers z , and write $z' Cz$ if $z' \succ z$ and $z'' \succ z' \Rightarrow z'' \succ z$, for all $z'' \in Z$.

Note that the assumption that the disagreement point δ does not coincide with a point on the line R is important. If δ did lie in R , then we would have a standard median voter setting and the median preference 0 would dominate all other possible outcomes.

The dominance relations in our bargaining problem are listed in table 1. These depend on the polarization parameter a and to some degree on the distance b between the ideal points of the median player and that of player 3, the wing player furthest away.

TABLE 1: DOMINANCE

	Position	polarization
$z' \succ z$	$ z' < z $ for $z, z' \in R$	
$z \succ \delta$	$z \in (-1, 1)$ $z \in (-1, -a + 1) \cup (b - 1, 1)$ $z \in (-1, -a + 1)$	$a + b < 2$ $b < 2$ $a < 2$
$\delta \succ z$	$z \in (-a + 1, b - 1)$ $z \in (-a + 1, 1)$ $z \in [-1, 1]$ $z \in (-\infty, -a - 1) \cup (b + 1, \infty)$	$a, b \leq 2, a + b > 2$ $a \leq 2, b > 2$ $a > 2$ all a, b

Notes: The table summarize dominance relations between alternatives in the cooperative game.

⁴ Note that individual players cannot achieve any outcome by themselves and hence the pay-offs available to singleton coalitions are not independent of the actions of the complementary coalition. Hence, under some definitions it would fall outside of the class of coalitional games with nontransferable utility.

For example, the second row states that when two alternatives are real numbers, the one closest to the median preference of 0 dominates the alternative further away. The third and fourth rows compare real numbers to the disagreement point. For example, row 3 shows for $a < 2$ that any real number between -1 and $-a+1$ dominates δ (because it yields players 1 and 2 strictly positive utility). On the other hand, if $a > 2$, any real number between -1 and 1 gives both wing players negative utility, so they both prefer the disagreement point, which then dominates the real number (row 4).

The counter-positive equivalent of definition 1 reads as follows.

DEFINITION. *For an alternative $z \in Z$ we say that*

- (i) z is undominated if for every z' the set of players who strictly prefer z' over z is not a winning coalition;*
- (ii) z is uncovered if for every z' which dominates z there is z'' which dominates z' and does not dominate z .*

To obtain a solution for the game we look at the *core* and, when the core is empty, at the *uncovered set*, which is a generalization of the core. The core and the uncovered set are defined by:

DEFINITION. *(i) The core $\mathcal{C}(\Gamma)$ of Γ is the set of all points in Z that are not dominated;*

(ii) The uncovered set $\mathcal{U}(\Gamma)$ of Γ is the set of all points in Z that are not covered.

Intuitively, the uncovered set is a ‘two step core.’ If an outcome z is uncovered, there might be an outcome z' that dominates it, but this outcome z' will itself be dominated by an outcome z'' that does not dominate z . This means for instance, that forward-looking negotiators might be hesitant to move away from a point in the uncovered set. The uncovered set has several appealing theoretical properties. It is never empty, is equal to the core if the latter is nonempty and strict (Miller, 1980), contains all *Von Neumann Morgenstern sets* (McKelvey, 1986) and it subsumes the *Banks set* (Banks, 1985)⁵. McKelvey argues that the uncovered set could be seen as a

⁵ These different papers prove these relations under slightly differing conditions.

“useful generalization of the core when the core does not exist” (1986). Recently, this concept has also attracted significant empirical support (Bianco et. al., 2006, 2008).

Whereas the uncovered set is typically large and difficult to calculate, in our bargaining problems it is small and simple. In addition, in our game the uncovered set is refined in a nice way by the *von Neumann Morgenstern set* ($\mathcal{L}(\Gamma)$)⁶ and the *bargaining set* ($\mathcal{B}(\Gamma)$), both of which are unique.⁷

All four solutions are finite subsets of the set of alternatives for all three player bargaining problems discussed in this paper. The size of each depends on the polarization parameter a . The four solutions coincide whenever the core is non-empty. This is the case in both extremes, i.e., for weakly and for strongly polarized preferences. In the remaining case of moderately polarized preferences ($1 < a < 2$), the other three solution sets are non-empty and satisfy the following inclusions.

$$\emptyset = \mathcal{C}(\Gamma) \subset \mathcal{B}(\Gamma) \subset \mathcal{A}(\Gamma) \subset \mathcal{U}(\Gamma)$$

It turns out that all inclusions are strict. Under the general additional assumption that $a < b$, the bargaining set $\mathcal{B}(\Gamma)$ consists of a single point, the von Neumann Morgenstern set $\mathcal{A}(\Gamma)$ consists of two points, and the uncovered set $\mathcal{U}(\Gamma)$ consists of three points. In the special case when $1 < a = b < 2$ all three sets contain an additional solution point, due to symmetry considerations. The solution sets are listed in table 2.

TABLE 2: COOPERATIVE SOLUTIONS

Polarization	$\mathcal{C}(\Gamma)$	$\mathcal{B}(\Gamma)$	$\mathcal{A}(\Gamma)$	$\mathcal{U}(\Gamma)$
weak: $a \leq 1$	0	0	0	0
mode- rate $1 < a < 2$	\emptyset	$a < b$	$\{-a + 1\}$	$\{-a + 1, \delta\}$
		$a = b$	$\{-a + 1, b - 1\}$	$\{-a + 1, 0, \delta\}$
strong: $b \geq 2$	δ	δ	δ	δ

Notes: For $\Gamma = \Gamma(N, Z, u, W)$, the table gives the elements in the core ($\mathcal{C}(\Gamma)$), bargaining set $\mathcal{B}(\Gamma)$, vNM set ($\mathcal{A}(\Gamma)$) and uncovered set ($\mathcal{U}(\Gamma)$) for the levels of polarization distinguished in the first column.

⁶ Formally, a subset L of Z is a von Neumann Morgenstern set if elements of L do not dominate each other and every element of $Z \setminus L$ is dominated by at least one element of L .

⁷ The bargaining set is the set of efficient points z in Z such that for any z' which dominates z and player $k \in N$ who prefers z over z' there exists z'' which weakly dominates z' and is for player k at least as good as z . (We use Maschler's, 1992, definition of the bargaining set.)

The results for weak and strong polarization are straightforward. In the former case, the median preference (0) dominates all other alternatives and the same holds for the disagreement point in the latter case. Here, we briefly explain the arguments underlying the results for moderate polarization, specifically for a and b such that $1 < a < 2 < b$.⁸ The results for parameters where equalities hold in this relationship are straightforward and can be obtained from the authors upon request.

- (C) All $z \in R$ unequal 0 are dominated by 0, 0 itself is dominated by δ , which in turn is dominated by any z between -1 and $(-a + 1)$. Hence, the core is empty.
- (B) The proposal $-a + 1$ is dominated by any alternative z' closer to 0 than itself. Player 1 who prefers $-a + 1$ above z' has a counter-objection $z'' = \delta$ which dominates z' and which gives him the same utility of 0 as the original proposal z . For any other proposal $z \in Z$ an objection exists for which there is no counter-objection justifying the original proposal. Hence, $(-a+1)$ is the unique element of the bargaining set $\mathcal{B}(\Gamma)$.
- (L) Player 1 is indifferent between $(-a + 1)$ and δ while players 2 and 3 have opposite preferences for these alternatives, hence, they do not dominate each other. Points on R between $(-a + 1)$ and $(b - 1)$ are dominated by δ , those beyond these limits are dominated by $(-a + 1)$ itself. Hence, $\{-a + 1, \delta\}$ is a vNM set of Γ . One can easily verify that in Γ there is no other vNM set.
- (U) (i) $z = -a + 1$ is only dominated by $z' \in (-a + 1, 1 - a)$, which in turn are dominated by $z'' = \delta$ which does not dominate z ,
(ii) $z = 0$ is dominated only by $z' = \delta$, which in turn is dominated by, for example, $z'' = -a / 2$ which does not dominate z ,
(iii) $z = \delta$ is dominated by any $z' \in (-1, -a + 1)$, all of which are dominated by $z'' = 0$ which does not dominate z ,

⁸ The properties of the core and the uncovered set known from the literature on cooperative games are invariably obtained under the assumption of convexity of the values of all coalitions, which does not hold here. The bargaining sets and the von Neumann Morgenstern sets have been studied mostly in the context of TU games. Hence, all results in table 2 must be verified case by case. We do not claim validity for any of these relations beyond the scope of bargaining problems as described here, with a one-dimensional set of alternatives augmented with a disagreement point and single-peaked preferences for the trio of players.

(iv) all z with $|z| > -a + 1$ are covered by $-a + 1$, while all $z \in (-a + 1, 0) \cup (0, a - 1]$ are covered by 0.

Hence, the uncovered set $\mathcal{U}(I)$ consists of the triple $\{-a + 1, 0, \delta\}$.

3. BARGAINING PROCESSES AS NON-COOPERATIVE GAMES

We now impose structure on the bargaining problem described in section 2 and analyze the resulting *bargaining process*. One may think of this as the legislature selecting exactly one element of the set of feasible alternatives Z by means of a procedure established (or agreed upon) in advance. Formally, such a procedure can be regarded as an extensive game. We shall present and discuss two such games, exemplary for two main frameworks for collective decision processes, voting and open bargaining. We do so by introducing bargaining procedures with, respectively, high and low structure to $\Gamma = \Gamma(N, Z, u_i, W)$.

HIGH STRUCTURE

We begin with a *high-structure* framework for the selection of an outcome in Z , represented by a sequential voting game Γ^H . The game – similar to that in Baron and Ferejohn (1989)– consists of multiple rounds, with a predetermined maximum of T rounds. Each round comprises of three stages.

At stage 1 one player is randomly selected with equal probability across players. At stage 2 the selected player i submits her proposal. At stage 3 players vote independently on this proposal. It becomes the final choice if it is accepted by at least two players. Because the player who submitted it supports her own proposal (by assumption), support by one other player suffices to pass the proposal and end the game. Whenever the proposal is voted down by the two other players the game proceeds to round $t+1$, where a player submits a new proposal, and so on. If the game reaches round T and the final proposal is also rejected, the game ends and the disagreement point δ is implemented.

For any given T and bargaining problem Γ , the game Γ_T^H is an extensive form game of finite length with random moves by nature at the first stage and simultaneous

moves by all three players at the third stage of each round.⁹ Actions played at any stage are observed before the next stage or round begins. In general, players' best responses will not be unique so one can expect multiple Subgame Perfect Nash Equilibria (SPE) in some cases, possibly with distinct outcomes. In order to select a single best response at each stage, and ultimately to select consistently a single SPE for every given T we shall adopt a number of tie-breaking rules known from the literature on voting games (Baron and Ferejohn 1989, Baron 1996, Banks and Duggan 2006).

- (i) A player accepts a proposal submitted in round t if it provides to her a payoff equal to her expected equilibrium payoff in the subgame beginning at stage 1 of round $t + 1$.
- (ii) Whenever a player has two best proposals, one that will be accepted and one that will be rejected, she submits the proposal that will be accepted.
- (iii) Whenever $-c$ and c are both best proposals for player 2 she submits each of them with an equal probability.
- (iv) Whenever δ and $c \in R$ are both best proposals for a player she submits δ .

The first assumption guarantees that a SPE exists, the remaining assumptions imply that it is unique.¹⁰ From now on we will refer to this equilibrium simply as 'the equilibrium'. Note that equilibrium strategies in a round do not depend on what happened in previous rounds or on the total number of rounds (T), but only on the number of rounds left before the game ends.

The equilibrium outcome of Γ_T^H can be characterized by the probability distribution of the equilibrium outcomes $\mu^T : Z \rightarrow [0, 1]$.¹¹ If all equilibrium proposals are accepted in the first round, μ^T simply allots equal probability to each of the

⁹ Though the player that made a proposal has an actions set consisting of one element ('accept') at the 3rd stage.

¹⁰ Assumption 4 is particular to our game (as $\delta \notin R$) and is a convenient tie breaking rule for the case $a = 2$. For other parameter values it is not essential which rule one assumes for these ties.

¹¹ μ^T is a probability mass function. As we show in Appendix A, μ^T has countable support.

players' equilibrium proposals in the first round. There is, however, the possibility of delay. Though for some values of parameters a and b and T , all three equilibrium proposals are immediately accepted, for other values an equilibrium proposal, in particular that of player 3, will be rejected. In this case, μ^T has a more complicated support.

The equilibrium outcome of Γ_T^H may depend in complicated ways on the number of bargaining rounds, T .¹² Hence, our approach is to look at whether the equilibrium outcome converges as T increases. We say that for given values of a and b the equilibrium outcome *converges* if there exists a probability distribution $\mu_{a,b}^*$ on Z such that $\lim_{T \rightarrow \infty} \mu^T = \mu_{a,b}^*$ in the sense of weak convergence of probability measures (Billingsley 1999). If no such limit exists, then we say that the outcome does not converge. The equilibrium outcome converges to some single $z \in Z$, if $\mu_{a,b}^*$ is concentrated at $z \in Z$, i.e. $\mu_{a,b}^*(z) = 1$. This allows us to summarize the SPE of Γ_T^H in the following proposition:

PROPOSITION 1

- (i) *If $0 \leq a < 1$ or $a = b = 1$, the equilibrium outcome converges to 0. For T sufficiently large the first round proposals are accepted without delay.*
- (ii) *If $1 \leq a \leq b < 2$ and $b > 1$, the equilibrium outcome does not converge except for some patches of the values of a and b , and never to a single outcome in Z .*
- (iii) *If $a \geq 2$, the equilibrium outcome is δ for sufficiently large T .*

Proof: The proof is given in Appendix A, which is a terse and long exercise in backward induction, since the non-convexity of the outcome set precludes the use of standard techniques and results. All appendices are provided in an online appendix.¹³

Comparing Proposition 2 to table 2, we conclude that the equilibrium outcome converges to the single element of the core for $a < 1$ and $a \geq 2$, and that it does not

¹² Numerically, the equilibrium outcome can be calculated for each value of a , b and T . Simulations show that the equilibrium outcome always appears to converge to a cycle in T , the length of which depends in erratic ways on a and b . To illustrate, Appendix A provides simulation results that show how the period of the cycles depends on a and b .

¹³ See http://www1.fee.uva.nl/creed/pdf/Structure_Appendices.pdf

converge to a single outcome if the core is empty. In the exceptional case of $a = 1$ and $b > 1$ the core consist of 0 but the equilibrium outcomes do not converge as T goes to infinity.

LOW STRUCTURE

Next, we turn to the alternative bargaining process with low structure. The idea underlying the game with low structure, which we denote by Γ_T^L , is that players can make and accept proposals in continuous time. It has by now been well established that such a game with continuous time cannot be solved without further assumptions, however (Simon and Stinchcombe 1989, Perry and Reny 1993, 1994). Drawing on Perry and Reny's two-player game (1994), we introduce a reaction and waiting time. Our game is in important ways different from Perry and Reny (1994), nonetheless, as Γ can have an empty core and does not have transferable payoffs. The basic tenet of Γ_T^L is a triple of proposals (p_1^t, p_2^t, p_3^t) on the table at all times $t \in [0, T]$ until one of the proposals is accepted lest the game ends with the disagreement point δ at time T .

The rules of the game are as follows. Player i can either be silent (ζ), $p_i^t = \zeta$, have a proposal on the table, $p_i^t = z, z \in Z$, or accept the proposal of another player j , $p_i^t = a_j$. For each player i , p_i^t as a function of time is assumed to be piecewise constant and to be right-continuous. We say that a player moves at time t when $p_i^t \neq p_i^{t-} \equiv \lim_{s \uparrow t} p_i^s$. It is natural to only allow only such discrete changes in proposals, since actual negotiations (face-to-face or computerized) consist of discrete actions ('I propose x', 'I accept', 'I withdraw y') in a continuous time.

Players start with no proposal on the table: $p_i^{0-} \equiv \zeta$. Building on Perry and Reny (1993), we introduce a uniform reaction and waiting time of ρ . In particular, if some player moves at time t , no player can move in time $t \in (s, s + \rho)$.¹⁴ This models the fact that players cannot react (or act again) immediately after a player has moved and

¹⁴ We also allow a player to resubmit her old proposal and induce the reaction time ρ . Intuitively, this is the strategic move "I still propose z." Technically, if $p_i^{t-} = z$, then we define the move that resubmits the same proposal as $p_i^t = z^*$. (We set $z^{**} \equiv z$, such that if $p_i^{t-} = z^*$, then resubmitting z is $p_i^t = z$.) If accepted, z^* just induces z as outcome.

that the time it takes to process information, take a decision and execute it is roughly the same for all players at all times. Essential is that we allow ρ to be arbitrarily small.¹⁵

Player i accepts j 's proposal by setting $p_i^t = a_j$. In order to ensure that a player knows which proposal she accepts, if player i plays $p_i^t = a_j$ she accepts p_i^{t-} . To ensure a unique, well-defined outcome a player i can only accept a proposal at time t if she is silent herself ($p_i^{t-} = \zeta$).¹⁶ In addition, one can (obviously) not accept a proposal from a player who is silent. As soon as a proposal has been accepted, the accepted proposal is the outcome of the game. If no proposal has been accepted before or at $t = T$, then the outcome is the disagreement point δ . After a proposal has been accepted or when $t > T$, no player can move anymore. Formally, we always let the game end at $t = T + \rho$.¹⁷

To define strategies and derive equilibria, we need to introduce some further definitions.

DEFINITIONS

1. A *history*, h , consists of a specification of $\{p_1^t, p_2^t, p_3^t\}$ for $t \in [0, \tau(h))$, where $\tau(h) \in [0, T + \rho]$ is the history's end.
2. $\tilde{t}(h) \equiv \sup\{t < \tau : p_i^t(h) \neq p_i^{t-}(h)\}$ is the *last moment* before $\tau(h)$ that any *player moved* (if no player has moved, we conveniently set $\tilde{t}(h) \equiv -\rho$).
3. A proposal function is *right-continuous and piecewise linear* if for all $\tau < T$ and each i , there is an $\varepsilon > 0$ such that $p_i^s(h) = p_i^t(h) \forall s \in [t, t + \varepsilon)$. As discussed above, we will only consider such proposals.

¹⁵ In our model the waiting time is exactly equal to the reaction time, unlike in Perry and Reny (1993). What is important is that we exclude the possibility of making a proposal and then withdrawing it before it can be accepted. For this purpose, any reaction time smaller than the waiting time would suffice as well; the analysis would just be (unnecessarily) more complex.

¹⁶ If a player were not required to be silent when accepting a proposal, her proposal could be accepted while she were accepting another. Essentially, we are requiring that a player removes her own proposal before accepting another. Given that ρ can be arbitrarily small, this assumption is not behaviorally restrictive.

¹⁷ This is because at time t , it is not yet known what happens at t itself. Any time after T would do.

4. History h is an active history if $\tilde{t}(h) + \rho \leq \tau(h) \leq T$ and no proposal has been accepted.
5. The set of admissible proposals at each history for player i is called $Z_i(h)$, with $Z_i(h_\tau) \subset \hat{Z}$, where $\hat{Z} = Z \cup \{\zeta, a_1, a_2, a_3\}$.¹⁸ $Z_i(h)$ is subject to the following restrictions:
 - a. $Z_i(h_\tau) = \{p_i^{\tau-}\}$ if h_τ is not an active history
 - b. $Z \cup \{\zeta, p_i^{\tau-}(h_\tau)^*\} \subset Z_i(h_\tau)$ if h_τ is an active history
 - c. $a_j \in Z_i(h_\tau)$ if h_τ is an active history, $i \neq j$, $p_j^{\tau-}(h_\tau) \neq \zeta$ and $p_i^{\tau-}(h_\tau) = \zeta$
6. The *outcome* of a history h is $z(h)$:
$$z(h) \equiv \begin{cases} \text{The unique element of } \{p_j^{\tau-}(h): p_i^{\tau-}(h) = a_j \text{ for some } t < \tau(h)\} \text{ if it exists} \\ \emptyset \text{ if } \{p_j^{\tau-}(h): p_i^{\tau-}(h) = a_j \text{ for some } t < \tau(h)\} = \emptyset \text{ and } \tau(h) \leq T \\ \delta \text{ if } \{p_j^{\tau-}(h): p_i^{\tau-}(h) = a_j \text{ for some } t < \tau(h)\} = \emptyset \text{ and } \tau(h) > T \end{cases}$$
7. Let \bar{H} be the set of histories in which all proposals are right-continuous and piecewise linear and admissible and have $\tau(h) = T + \rho$. Call any $\bar{h} \in \bar{H}$ a *resolved history*.
8. Let H be the set of histories in which all proposals are right-continuous, piecewise linear and admissible, and that have no outcome; any $h_\tau \in H$ is called an *unresolved history*.
9. h is a *subhistory* of h' , or $h \subseteq h'$, if $\tau(h) \leq \tau(h')$ and $p_i^t(h) = p_i^t(h')$ for each i and for all $t \in [0, \tau(h))$. Furthermore, h is a '*proper subhistory*' of h' , $h \subset h'$, if $h \subseteq h'$ and $\tau(h) < \tau(h')$.
10. $h_s(h)$ is the unique history such that $\tau(h_s(h)) = s$, $h \subseteq h_s(h)$ and no player moves in $[\tau(h), s)$.
11. $H_{\tau'} \equiv \{h \in H : \tau(h) = \tau'\}$ is the set of *histories ending at τ'* .

¹⁸ Because we allow players to repeat their previous proposal, we actually have $\hat{Z} = Z \cup Z^* \cup \{\zeta, \zeta^*, a_1, a_2, a_3\}$, where Z^* includes all asterisked outcomes (cf. fn 14).

12. $\tilde{H} \equiv \{h \in H : \tau(h) \geq \tilde{t}(h) + \rho\}$ is the set of *active histories* in H (hence those containing only admissible proposals). From now, we will only refer to active histories in \tilde{H} .

A strategy of player i is a mapping from the set of unresolved histories to the set of admissible proposals, $\sigma_i : H \rightarrow \hat{Z}$. It meets the following two conditions:

(S1) For all $h_\tau \in H$, $\sigma_i(h_\tau) \in Z_i(h_\tau)$

(S2) For all $h_\tau \in H$, a time $\varepsilon > 0$ exists such that $\sigma_i(h_{\tau'}) = \sigma_i(h_\tau) \forall h_{\tau'}$ with $h_\tau \subseteq h_{\tau'} \subseteq h_{\tau+\varepsilon}(h_\tau)$.

(S1) ensures moves are in the set of admissible proposals. (S2) ensures that the strategies result in well-defined histories with piece-wise constant, right-continuous paths. By Σ we denote the set of all profiles that meet (S1) and (S2).

Proposition 3 shows that $\sigma \in \Sigma$ induces from any well-defined unresolved history $h_\tau \in H$ a unique well-defined resolved history $\bar{h} \in \bar{H}$:

PROPOSITION 2

The game Γ_T^L is a well-defined mapping $\Gamma_T^L : H \times \Sigma \rightarrow \bar{H}_{T+\rho}$

Proof: See Appendix B.

The intuition underlying the proof is that for a given strategy profile and unresolved history h_τ , either no player would do anything after h_τ until the game ended or one can find a well-defined first action at or after τ . Such an action results in a new history, which is either a resolved history or an unresolved history. Hence, one can repeat this procedure of searching for the first action until it yields a resolved history.

We can now define subgames and equilibria of Γ_T^L . A subgame $\Gamma_{T|h_\tau}^L \{h \in H : h \supseteq h_\tau\} \times \Sigma \rightarrow \bar{H}_T$, represents the game that starts at h_τ . Let $U_i(\sigma_i; \sigma_{-i} | h_\tau) \equiv u_i(z(\Gamma_{T|h_\tau}^L(h_\tau, \sigma)))$ be the payoff player i receives from the outcome that σ induces on h_τ and let $U_i(\sigma_i; \sigma_{-i}) \equiv U_i(\sigma_i; \sigma_{-i} | h_0)$. A Nash Equilibrium of

$\Gamma_{T \setminus h_\tau}^L$ is a strategy profile σ such that $U_i(\sigma_i; \sigma_{-i} | h_\tau) \geq U_i(\sigma'_i; \sigma_{-i} | h_\tau)$ for each i and all $\sigma'_i \in \Sigma_i$. σ is a Subgame Perfect Equilibrium (SPE) of Γ_T^L if it is a Nash Equilibrium in all of its subgames: $U_i(\sigma_i; \sigma_{-i} | h_\tau) \geq U_i(\sigma'_i; \sigma_{-i} | h_\tau)$ for each i and all $\sigma'_i \in \Sigma_i$ in all $h_\tau \in H$.

Finally, we show by an explicit construction that every point of the interval $[-a, b] \subset R$ and the disagreement point δ itself can be an outcome of a subgame-perfect equilibrium of $\Gamma_T^L(B)$.

PROPOSITION 3

The set of SPE outcomes contains $[-a, b] \cup \{\delta\}$ for every continuous game Γ_T^L with $T > \rho$.

Proof: See Appendix B.

Many of the SPEs in proposition 3 may seem unintuitive. For instance, the at first sight unlikely outcome b (which is the ideal point of the wing player furthest from the median player) can be supported by an equilibrium in which players 1 and 2 always propose b . Player 3 will accept b as soon as she can, while players 1 and 2 cannot individually profitably deviate, as the other player will anyhow propose b , which will be readily accepted by player 3.

This large set of subgame-perfect equilibria cannot be refined in any standard way. A simple set of standard tie-breaking rules, such as those used for Γ_T^H , would be much too weak to have any effect. Stationarity only has a very small bite (b and δ can, for instance, be sustained by stationary strategies for any a and b). A procedure of iterated elimination of weakly dominated strategies, as proposed by Hervé Moulin (1979), and used by Baron and Ferejohn (1989), is of little avail in our case, due to the fact that typically in many subgames of Γ_T^L multiple actions per player will survive, so that hardly any strategy will eventually be eliminated in the complete game. Finally, trembling-hand perfection (adapted to continuous time and space) will not eliminate these unintuitive equilibrium-outcomes either. For instance, the reason that player 1 does not propose 0 instead of b in the equilibrium discussed above, could be that she is afraid that player 2 might tremble and play $b + \varepsilon$ instead of b .

Hence, there are few strategic restrictions on the equilibrium strategies when there is low structure. Still, there are some points in the outcome set that strike us as more ‘likely’ than others (for instance points in the uncovered set). Their plausibility might be the result of their *focal* nature due to the constellation of preferences and winning coalitions (which is captured by the cooperative game Γ). Our experiment provides us with a tool to investigate whether such outcomes are indeed more likely to be observed than other equilibrium outcomes.

OVERVIEW OF THEORETICAL RESULTS

In table 3 we summarize the main results obtained for the outcomes of the subgame-perfect equilibria of the two strategic games analyzed in this section, Γ_T^H and Γ_T^L , together with the solutions of the cooperative game Γ .

TABLE 3: THEORETICAL RESULTS MAIN CASES.

		Polarization		
		Weak $a < 1$	Moderate $1 < a < 2$	Strong $a > 2$
Cooperative Game $\Gamma(N, X, u_i, W)$	Core	0	\emptyset	δ
	Uncovered Set	0	$\{1-a, 0, a-1^*, \delta\}$	δ
High Structure (convergence) Γ_T^H		0	No convergence **	δ
Low Structure (for $T \geq \rho$) Γ_T^L		$[-a, b] \cup \{\delta\}$ ***		

Notes: Cells give the ‘equilibrium outcome’ for the three games as derived above. Solution concepts used are described in the previous subsections.

* $a-1$ is only included if $a=b$;

** There are some exceptions, in which case the outcome may converge but never to a single outcome in Z .

***Outcomes can also lie in the interval $[b-1, a]$ if $-b < b-1 < -a$

4. EXPERIMENTAL PROCEDURES AND DESIGN

The experiment was run at the Center for Research in Experimental Economics and political Decision making (CREED) of the University of Amsterdam. They were computerized using Z-tree (Fischbacher 2007). An English translation of the Dutch experimental instructions is provided in Appendix C. Subjects had to correctly answer

a quiz before proceeding to the experiment. In total, 102 subjects were recruited from CREED's subject pool.¹⁹ They participated in six sessions, and earned a €5 show-up fee plus on average €11.65 in 90-120 minutes. In the experiment payoffs are in 'francs'. The cumulative earnings in francs are exchanged for euros at the end of the session at a rate of 1 euro per 10 francs.

Each session consists of 24 periods. In each period subjects are rematched in groups of three. For statistical reasons, we use matching groups of 6 or 9 subjects.²⁰ After groups have been formed, subjects are randomly appointed the role of player 'A', 'B', or 'C'. To avoid focalness players do not play the normalized game described above (*e.g.* B's position is not set equal to 0 and it is not necessarily the case that A's ideal value is closer to B's value than C's is). For analysis, the game subjects play can easily be normalized to correspond to the model of section 2.

Each player is appointed an 'ideal value', which is an integer between 0 and 100 (inclusive).²¹ Player A's ideal value is always the smallest and player C's the largest. Players know all ideal values. Each group has to choose an integer between 0 and 100 (inclusive). If the group chooses a player's ideal value, this player receives 20 francs. For every unit further from the ideal point, one franc is subtracted. Hence, earnings are negative for a player if the group chooses a number that is more than 20 larger or smaller than her ideal value. To avoid negative earnings overall, each subject starts with a positive balance of 100 francs.²²

The extent of polarization is varied in a within-subjects design by using 12 sets of ideal values. Each set was used once in the *first half* (first 12 periods) and once in the

¹⁹ The subject pool consists of around 2000 individuals who have voluntarily registered. Almost all of these are undergraduate students at the University of Amsterdam, approximately 40% major in Economics or Business. When the experiment was announced, all received an invitation to sign up and participation was on a first-come, first-serve basis.

²⁰ Subjects are only told that they are in a session with 15 or 18 participants and will be rematched in every round.

²¹ The restriction to natural numbers is done for practical purposes. It is sufficiently fine-mazed to avoid affecting the equilibria derived in the previous section in any relevant way. One difference is that if the outcome converges in T to 0, it converges in finite time and, for our parameter values, in fact it already converges for $T < 10$. In table 4, we report the equilibria of the high structure game with a discretized line and 10 rounds (which we calculated numerically).

²² Still, five subjects ended their session with negative earnings. They were sent off with no pay other than the €5 show-up fee. Data which involved these individuals were deleted from the sample due to possible incentive problems. Including these individuals makes little difference, except that statistical results become somewhat less conclusive due to the extreme behavior of one subject who would have earned -14.70 euros and showed erratic behavior after his earnings became negative.

second half (last 12 periods) of a session. The sets were chosen such that for the normalized parameters there were six with $a < 1$ and six with $1 < a < 2$ (cf. table 4, below). We chose not to use parameters with $a \geq 2$ in the experiment because it seems obvious that participants will always agree on the disagreement point of no earnings if there is no outcome where at least two players have positive earnings.

The level of structure was varied in a between subjects design. Three sessions involved a highly structured bargaining process (called *High*) and three had a process with low structure (*Low*).²³ In both cases proposals are made, consisting of any integer between 0 and 100 (inclusive) or δ (called “end”). If the disagreement point is the outcome, each player receives a payoff of zero.

To help a subject to determine her choices in the negotiations for a group decision, her monitor always shows a history of previous rounds, current earnings, a scrollable help-box with instructions, a history of offers in the current round and a device to calculate pay-offs for any proposal.

In the high-structure sessions, subjects play the game Γ_T^H of section 3 for a maximum of $T = 10$ negotiation rounds per period. We use the strategy method (for proposals) where in every round every player is asked to make a proposal, after which one proposal is randomly selected and put to the other two group members to vote on. If at least one of the two accepts this proposal, it becomes the group choice and the game ends. If the proposal is rejected by both players, a new round begins, unless 10 rounds have been finished. In the latter case, the outcome is the disagreement point.

In the low-structure sessions subjects are given two-and-a-half minutes to reach an agreement. At any time during these 150 seconds, any group member can make a proposal, change a previous own proposal or accept one made by another member. As soon as a proposal has been accepted, this becomes the group choice for the period and negotiations are finished. If no proposal is accepted within the time-span, the disagreement point is the outcome.

²³ In both treatments, a round of bilateral messages precedes group negotiations: each player may send a private message (consisting of a number between 0 and 100 or δ) to either or both other player(s). This is meant to reflect pre-negotiation lobbying. This cheap-talk does not affect the theoretical analysis presented in section 3.

Table 4 gives the (normalized) parameters used, the periods in which they were used and the theoretical predictions for each set. These predictions will be compared to the experimental outcomes in the next section.

TABLE 4: PARAMETERS AND PREDICTIONS

Parameters		Periods	Cooperative ¹	High-structure ²			Low-structure
a	b			1 starts	2 starts	3 starts	
0.2	1.4	5, 23	0	0	0	0	$[-0.2, 1.4] \cup \delta$
0.5	1.1	3, 21	0	0	0	0	$[-0.5, 1.1] \cup \delta$
0.5	1.7	11, 13	0	0	0	0	$[-0.5, 1.7] \cup \delta$
0.8	0.8	1, 19	0	0	0	0	$[-0.8, 0.8] \cup \delta$
0.8	1.4	9, 15	0	0	0	0	$[-0.8, 1.4] \cup \delta$
0.8	2	8, 24	0	0	0	0	$[-0.8, 2.0] \cup \delta$
1.1	1.1	7, 17	$\{-0.1, 0, 0.1, \delta\}$	-0.3	0	0.3	$[-1.1, 1.1] \cup \delta$
1.1	1.7	10, 22	$\{-0.1, 0, \delta\}$	-0.4	-0.15	0.4	$[-1.1, 1.7] \cup \delta$
1.1	2.3	2, 14	$\{-0.1, 0, \delta\}$	-0.45	-0.15	0.45	$[-1.1, 2.3] \cup \delta$
1.4	1.4	12, 20	$\{-0.4, 0, 0.4, \delta\}$	δ	0	δ	$[-1.4, 1.4] \cup \delta$
1.4	2	4, 16	$\{-0.4, 0, \delta\}$	-0.55	0.2	δ	$[-1.4, 2.0] \cup \delta$
1.7	1.7	6, 18	$\{-0.7, 0, 0.7, \delta\}$	δ	$\{-0.5, 0.5\}$	δ	$[-1.7, 1.7] \cup \delta$

Notes: Cells give the theoretical prediction (*cf.* table 1) applied to the experimental parameter set. The High-structure predictions are for the game with a discretized outcome set and 10 rounds, as played in the experiment.

¹For $a < 1$ the prediction is given by the core (=uncovered set); for $a > 1$ it is given by the uncovered set.

²The column gives the (refined) SPE conditional on the player chosen to make the first offer. Player 2 is defined as the median position, 1 is the other player closest to the median.

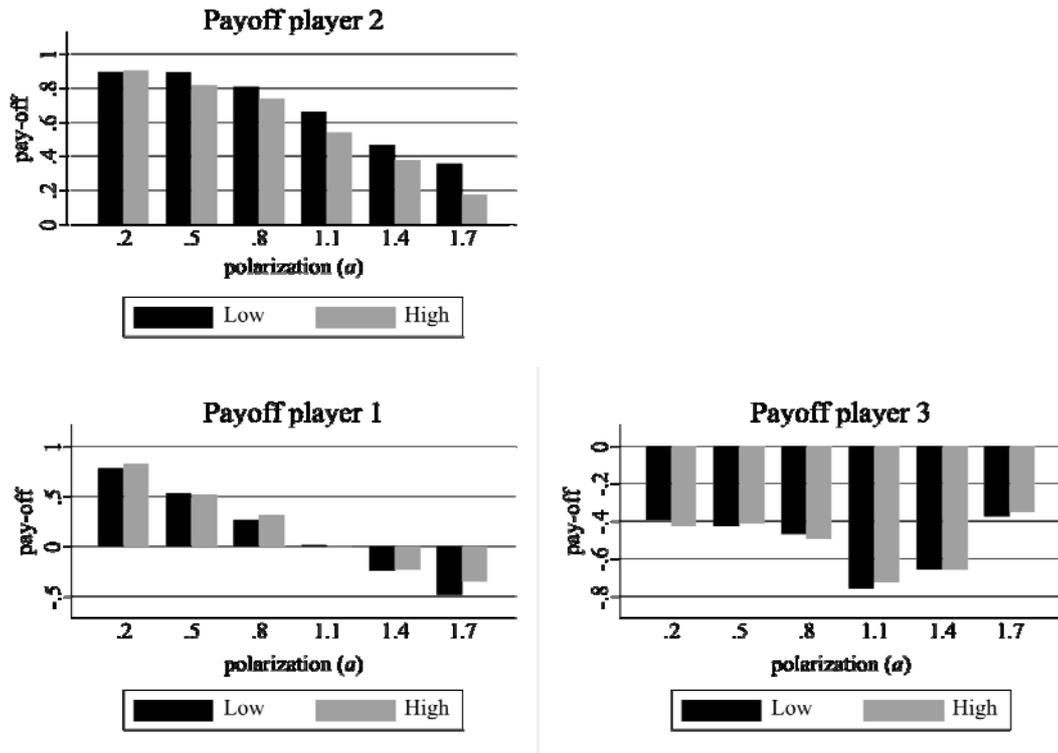
5. EXPERIMENTAL RESULTS

We focus on two issues. First, we look at how structure (and its interaction with polarization) affects the ability of the median player to reach agreements close to her ideal point. Second, we look at the performance of solution concepts and, in particular, how structure and polarization affect the performance of the Core and Uncovered Set. All tests used below are two-sided and non-parametric, using each matching group (of six or nine participants) as one independent data point. We use the Wilcoxon signed rank tests for within comparisons and the Mann-Whitney test for between comparisons. Whenever we report statistically significant results for pooled *Low/High* data only, the results are also significant at the 0.05 level for the disaggregated data where *Low* and *High* structure are tested separately. *P*-values that are (unrounded) smaller than 0.05 are marked by an asterisk.

EARNINGS

We start with players' earnings from negotiations. Figure 1 shows the payoffs for different levels of polarization (captured by a) and the two treatments.

FIGURE 1: PAYOFFS



Notes: Bars show the average payoffs of players per period (*i.e.*, one round of negotiations). Player 2 is the median player and player 1 is the other player closest to her.

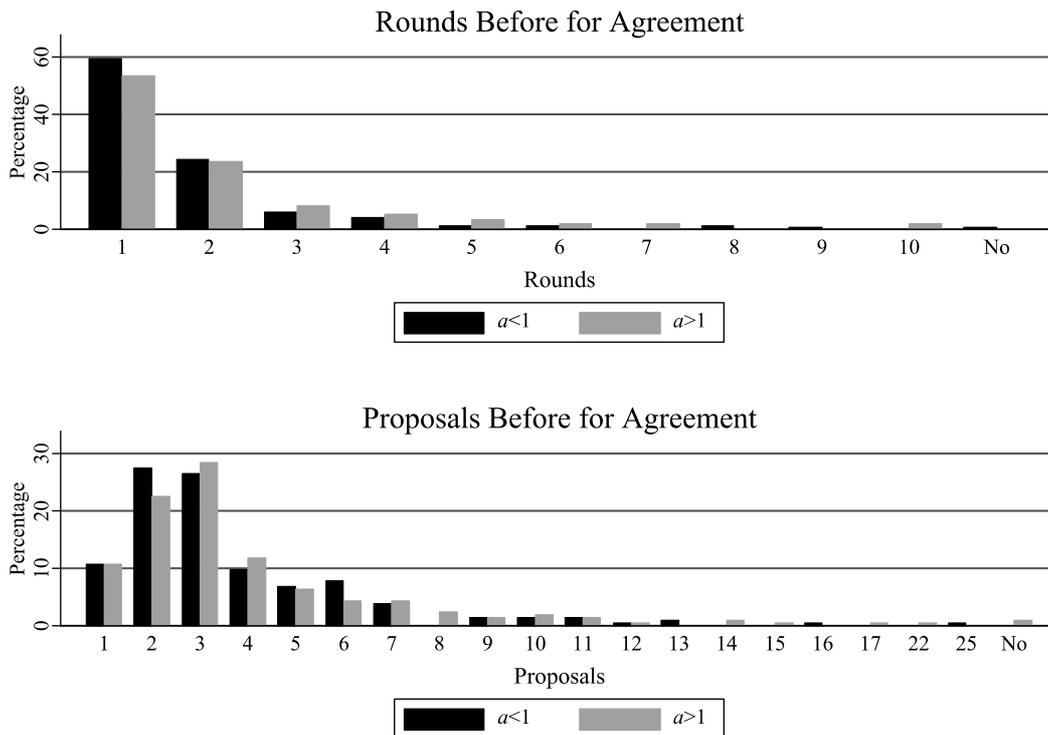
Most relevant are the payoffs of the median player. First consider the effect of polarization. Theory predicts that for weak polarization ($a < 1$) the median player will be able to secure her maximum pay-off (of 1), whereas moderate levels ($1 < a < 2$) of polarization would hurt her. The experimental results show no obvious change at $a = 1$. Increasing polarization clearly affects the median player (player 2) negatively, even when $a < 1$. For example, Player 2 earns approximately 0.9 (close to the maximum of 1) when $a=0.2$ (for both *High* and *Low*) but only just over 0.79 for $a=0.8$ when structure is low. Her earnings are significantly lower for $a=0.8$ than for $a=0.2$ ($p=0.01^*$). As predicted by theory, the median's payoff is significantly lower for moderate ($a > 1$) than for weak ($a < 1$) polarization ($p < 0.01^*$).

Second, structure also has a clear effect. The median player is (significantly) better off in the low-structure treatment than in the high-structure treatment ($p=0.03^*$). The difference between treatments seems to increase with the extent of polarization. When polarization is very weak ($a=0.2$) structure does not affect player 2's earnings from negotiations. When it is relatively strong ($a=1.7$) the median earns more than twice in *Low* than in *High*. Next, we further explore what drives these results.

HOW POLARIZATION AND STRUCTURE AFFECT THE MEDIAN PLAYER

We start by looking at whether participants manage to reach an agreement before the deadline. Figure 2 shows the number of proposal needed to reach agreement.

FIGURE 2: ROUNDS/PROPOSALS BEFORE AGREEMENT



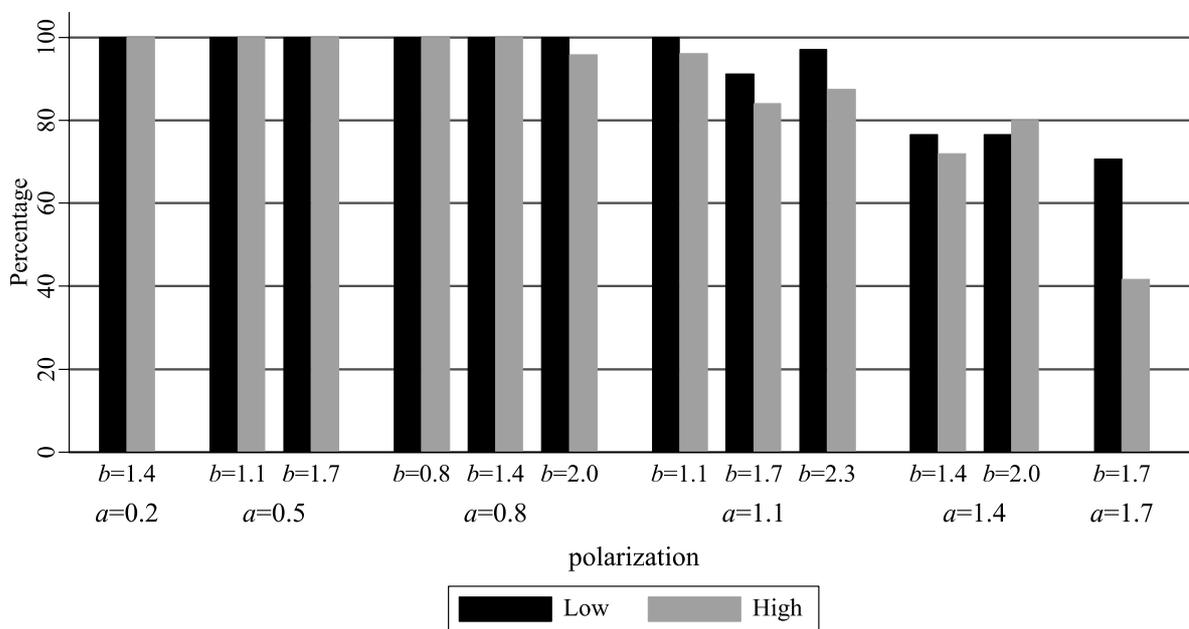
Notes: Bars show the fraction of agreements using the number of proposals depicted on the horizontal axis for *High* (top panel) and *Low* (bottom panel). In *High*, a proposal in any period could only be made by the player selected to do so and there was a maximum of 10 periods. In *Low*, any player could make a proposal at any time and there was a maximum of 150 seconds.

In both *Low* and *High*, agreement was reached within the limit (150 seconds and 10 rounds, respectively) in 99% of all cases. Hence, it almost never occurred that the disagreement point was forced upon the negotiators for missing their limit. Moreover,

agreement was generally reached very quickly. In *High* agreement was reached in at most 3 rounds in 88% of the cases and in *Low* agreement was reached in at most 30 seconds in 82% of the cases. Consequently, binding (time) limits do not appear to be of any influence (in treatment effects). Players make significantly more proposals in *Low* (4) than in *High* (2) ($p < 0.01^*$), however.

The outcome of the game has three dimensions: whether it is a *real number* (as opposed to disagreement) and, if so, its value (*'location'*) and its distance to the median position, i.e., its absolute value (*'distance'*). We will look at each of these in turn. Figure 3 shows the percentages of outcomes that were a real number. As long as polarization is weak ($a < 1$), virtually all outcomes are real numbers and polarization is immaterial. This percentage is, however, clearly and statistically significantly lower for moderate than for weak polarization ($p < 0.01^*$). Hence, a decrease in real number agreements may partly explain why moderate polarization is worse for the median player than weak polarization. However, it cannot explain why she cannot obtain her optimal payoff even when polarization is weak.

FIGURE 3: REAL NUMBER OUTCOMES (PERCENTAGE)



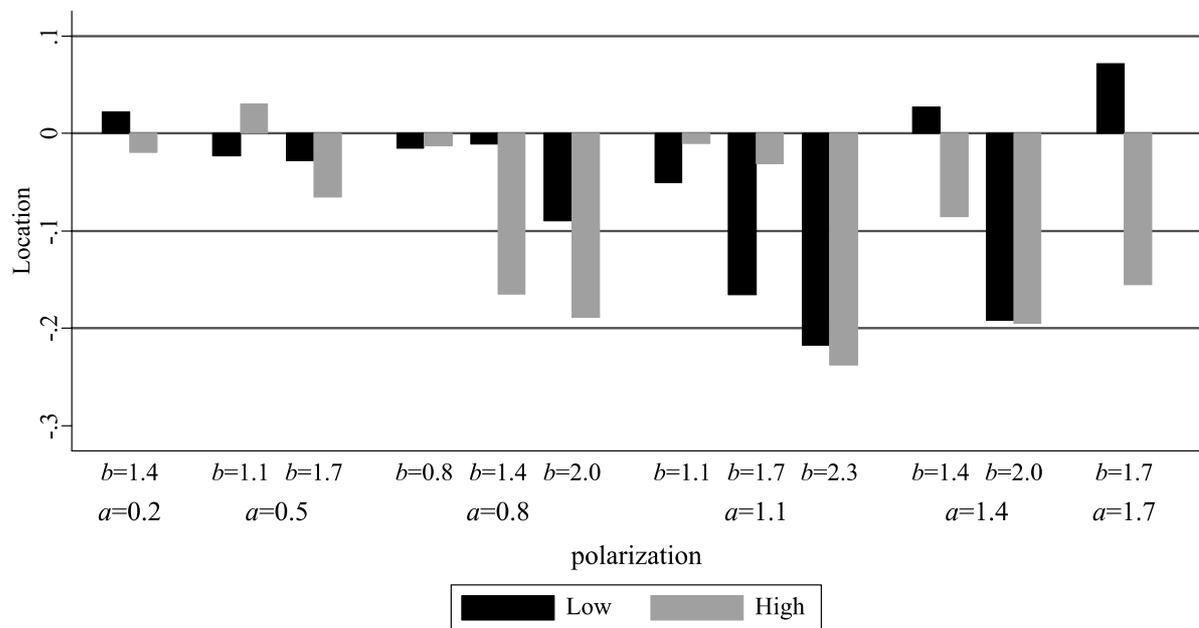
Notes: Bars show the fraction of outcomes that were a real number.

Furthermore, there is no clear treatment effect. Real number outcomes are somewhat less likely in *High*, but the effect is small and insignificant ($p = 0.33$ for $a < 1$ and

$p=0.22$ for $a>1$). Hence, the percentage of real number outcomes cannot explain why the median player is better off in *Low*.

In search of such an explanation, we take a closer look at the real number outcomes players agreed upon. For completeness sake, we first depict the location of real number agreements in figure 4 (although the location *per se* is not relevant for player 2's payoff). This clearly shows that the average agreement is typically between the ideal points of the median player and player 1 (the other player closest to the median). In fact, there are only two cases with $a \neq b$ where the average agreement lies between the ideal points of the median player and player 3. In both cases, player 1 still earns more than player 3. We will discuss the coalitions observed in more detail, below.

FIGURE 4: LOCATION (REAL NUMBER OUTCOMES)

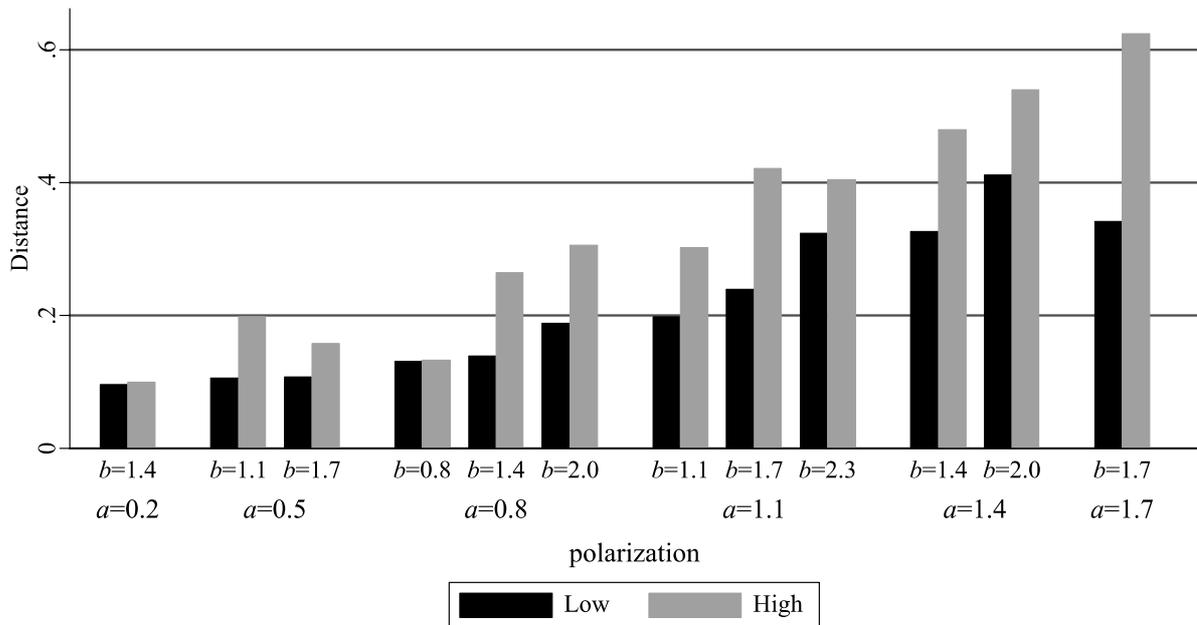


Notes: Bars show the average normalized location of agreements, when groups agreed on a real number. Negative (positive) numbers indicate an agreement in between the ideal points of players 1 (3) and 2. The median position is an agreement at 0. Whenever $a=b$, the non-median players are randomly located as players 1 and 2, so any agreement is equally likely to be normalized to a positive or negative number.

Given that an outcome is a real number, the median player's payoff is fully determined by its distance to the median preference (0). This is shown in figure 5. This figure clearly shows that the distance increases with polarization. Distance is significantly higher for moderate than for weak polarization ($p<0.01^*$). Distance

matters even within weak levels of polarization: it is significantly higher for $a=0.8$ than for $a=0.2$ ($p=0.01^*$ (pooled), $p=0.12$ (*Low*), $p=0.05^*$ (*High*)).

FIGURE 5: DISTANCE (REAL NUMBER OUTCOMES)



Notes: Bars show the average absolute distance between agreements and the median point, when groups agree on a real number.

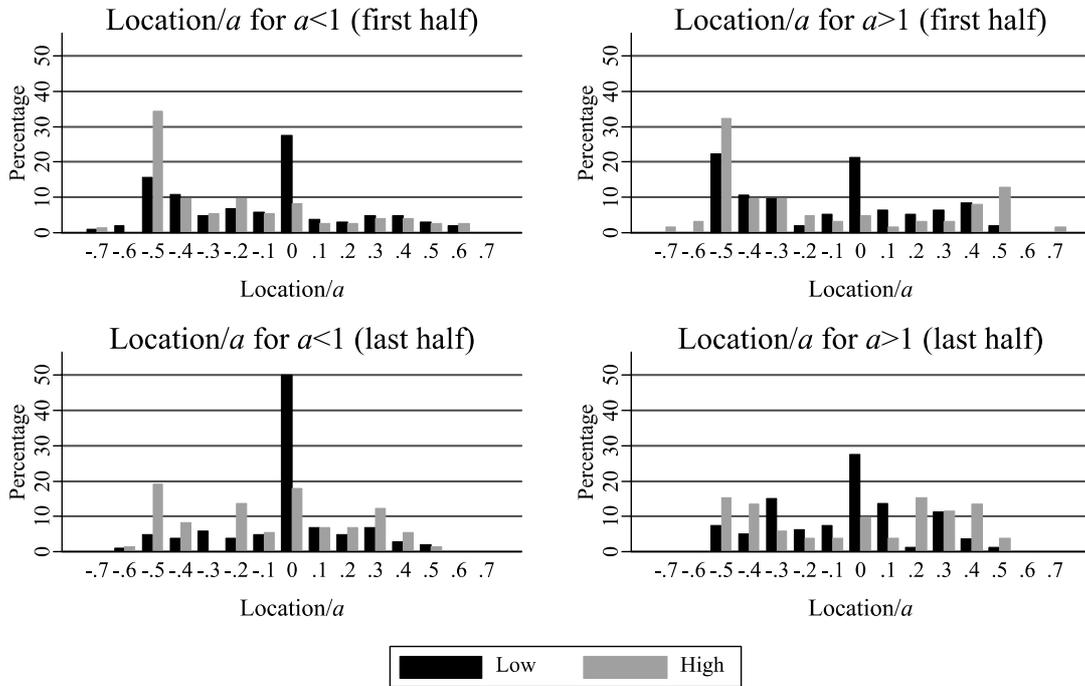
Figure 5 also shows a clear treatment-effect. Distance is significantly lower for *Low* than for *High*. Hence, player 2 seems to exploit her superior bargaining position better in *Low* than in *High*. A possible explanation is that players are freer to make proposals in *Low* than in *High*, so that they can negotiate better. Recall that players make significantly more proposals in *Low* than in *High*. We conclude that the main driving force underlying the higher profits for the median player in the *Low* treatment is that the more flexible negotiation structure allows her to secure real number agreements closer to her preference.

INTRACOALITIONAL FAIRNESS VS INTERCOALITIONAL COMPETITION

One intriguing question that remains is why even weak polarization hurts the median player, while her preference is the unique core element. To address this question, we consider coalitions and the way in which outcomes distribute pay-offs within them.

Figure 6 shows the distribution of real number agreements divided by a . Hence, -1 represents an agreement at $-a$ (*i.e.*, player 1's ideal point), 0 represents the median preference 0 and 1 represents a .

FIGURE 6: DISTRIBUTION OF REAL NUMBER OUTCOMES (LEARNING)

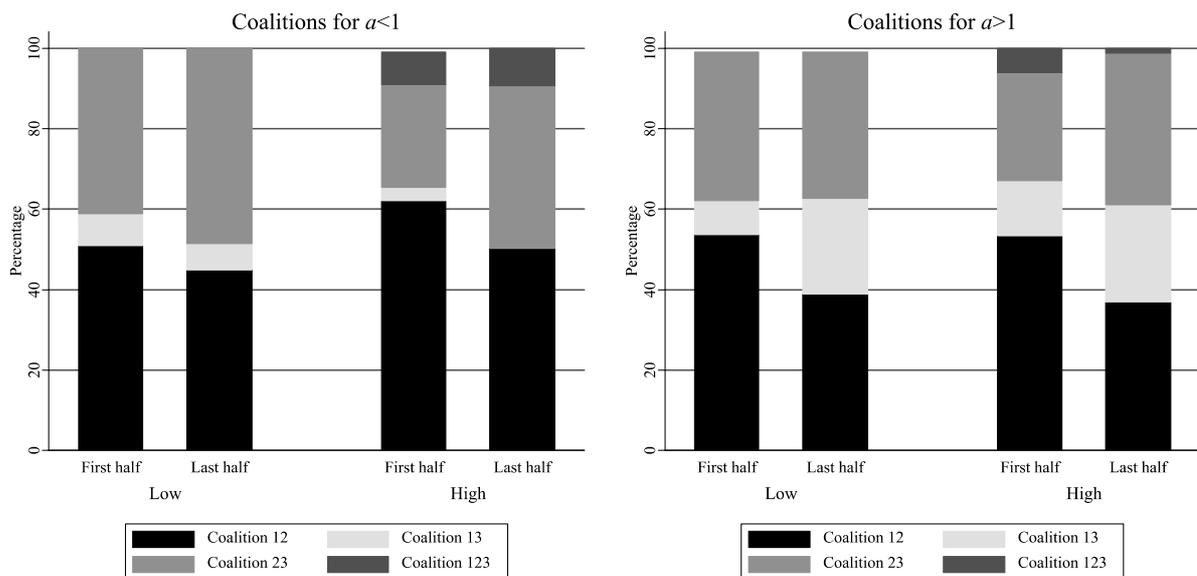


Notes: Bars show the fraction of real number outcomes that are within 0.05 of the outcome depicted on the horizontal axis. The horizontal axis gives the normalized outcome divided by a . The left panels show the distribution for $a < 1$, the right for $a > 1$. The top panels show the distribution for the first half (first 12 periods), the bottom for the last half (last 12 periods).

Strikingly, almost all real number outcomes lie between $-a/2$ and $a/2$, with $-a/2$ being one of the most frequently chosen outcomes. Note that $-a/2$ equalizes payoffs between players 1 and 2, but is a rather unfair outcome for player 3; in fact worse than the median preference. It seems that players 1 and 3 in many cases demand some part of the ‘surplus’ in a coalition with player 2. However, player 2 does not give more than the fair split to player 1. Furthermore, player 3 does not obtain a better outcome than $a/2$, since player 2 probably feels that she can certainly obtain $-a/2$ in a coalition with player 1. Such considerations of intra-coalitional fairness yield real number agreements increasing in a , even for weak levels of polarization, as we observe. Note, however, that as a increases it becomes more costly to the median player to give her coalition partner a ‘fair share.’

Figure 6 also shows that there is a strong learning effect: the distribution of outcomes in the first half (first 12 periods) is very different from those in the last half (last 12 periods). In the first half, intra-coalitional fairness considerations seem to play an important role, certainly within coalitions of players 1 and 2. Furthermore, coalitions tend to consist of players 1 and 2, in particular in *High* (see figure 7).

FIGURE 7: COALITIONS (LEARNING)



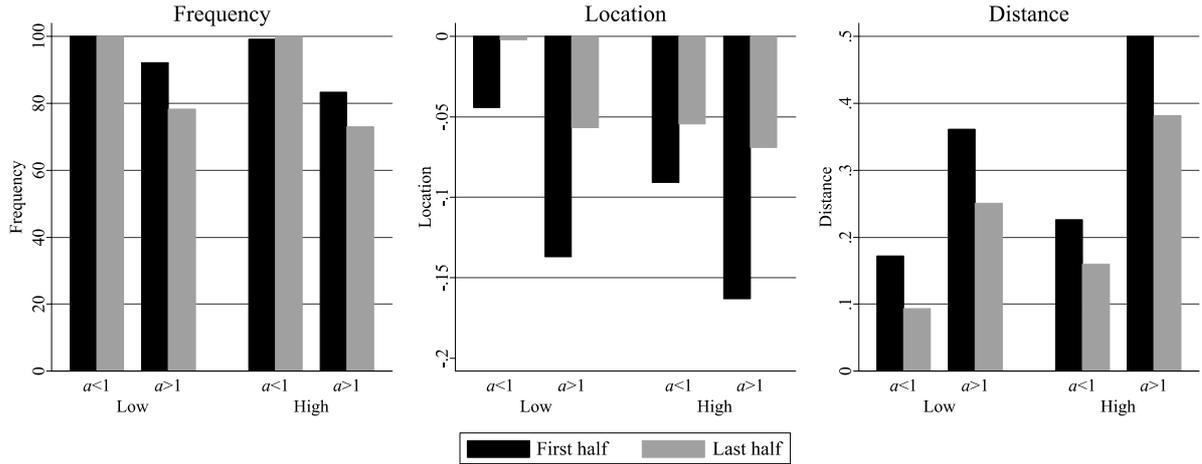
Notes: Stacked bars show the distribution of distinct coalitions. A coalition ij is defined as an outcome proposed by i and accepted by j or vice versa. A coalition ijk is an outcome with 2 yes votes (only possible in High).

In the course of the experiment, inter-coalitional competition becomes more important. In the second half, more coalitions arise of players 2 and 3 than in the first half ($p=0.03$ (pooled), $p=0.46$ (*Low*), $p=0.05$ (*High*)), resulting in a more even spread of positive and negative agreements. Furthermore, for $a > 1$ more coalitions are formed in the second half than in the first half between players 1 and 3 ($p=0.03^*$ (pooled), $p=0.03^*$ (*Low*), $p=0.21$ (*High*)).

To investigate the consequences of this increased inter-coalitional competition, figure 8 splits the data depicted in figures 3-5 and shows the frequency of real number agreements, their location and distance separately for the first and last half. Note that having the viable ‘outside option’ of a coalition with player 3 means that the median player can offer less to player 1. Median players appear to realize this remarkably well in the second half of the experiment. Agreements between players 1 and 2 tend to be

closer to 0 in the last half (see the center panel of figure 8). In particular, the number of fair 1-2 compromises drops considerably ($p < 0.01^*$ (pooled), $p = 0.04^*$ (Low), $p = 0.05$ (High)) with an accompanying increase in the number of outcomes at the median preference ($p = 0.01^*$ (pooled), $p = 0.03^*$ (Low), $p = 0.14$ (High)).

FIGURE 8: FREQUENCY, LOCATION & DISTANCE (LEARNING)



Notes: The left chart shows the percentage of outcomes that were real numbers. The middle chart shows the average location of real number agreements and the right chart the average distance between real number outcomes and the median preference.

Figure 8 also shows that the greater number of coalitions between players 1 and 3 results in a significantly lower frequency of real number agreements in the second half when $a > 1$ ($p = 0.04^*$ (pooled), $p = 0.03^*$ (Low), $p = 0.25$ (High)). The more equal spread of 1-2 and 2-3 coalitions also results in an average location closer to zero ($p = 0.04^*$, (pooled), $p = 0.12$ (Low), $p = 0.18$ (High)). The strongest learning effect, however, is that the distance of real number outcomes to the median preference becomes significantly lower ($p < 0.01^*$). Hence, it is the median player who benefits most from the increase in inter-coalitional competition.

PERFORMANCE OF SOLUTION CONCEPTS

Finally, we address the question of how polarization and structure affect the performance of the various solution concepts. Table 5 shows two measures of performance. The first measures the fraction of outcomes that were attracted (within a region of 0.05) by a prediction. The second is the average distance between an

outcome and a (set of) prediction(s) in the pay-off space. (The first measure is more intuitive, the second parameter-free and more powerful.)

TABLE 5: ACCURACY OF THE PREDICTIONS

			Fraction of outcomes attracted			Distance to the set/point		
	Point/Set	Periods	<i>Low</i>	<i>High</i>	Diff.	<i>Low</i>	<i>High</i>	Diff.
$a < 1$	0 = Core, UCS, SPE(H)	First half	0.22	0.19	0.03 (0.52)	0.34	0.40	0.06 (0.90)
		Last half	0.58	0.28	0.20 (0.02*)	0.17	0.27	0.10 (0.04*)
		Difference	-0.36 (0.03*)	-0.09 (0.14)		0.17 (0.03*)	0.13 (0.05*)	
$a > 1$	UCS = {0, 1-a, δ } ⁽¹⁾	First half	0.46	0.33	0.12 (0.25)	0.33	0.42	0.09 (0.20)
		Last half	0.60	0.33	0.27 (0.09)	0.17	0.32	0.15 (0.04*)
		Difference	-0.14 (0.09)	0.00 (0.20)		0.16 (0.05*)	0.10 (0.07)	
	SPE (H) ⁽²⁾	First half		0.17			0.84	
		Last half		0.31			0.59	
		Difference		-0.14 (0.11)			0.25 (0.04*)	

Notes: Numbers in columns 4 and 5 show the median fractions of normalized outcomes within 0.05 of the point depicted in the 2nd column. (For each independent datapoint (=matching group) the fraction was calculated and the median of these fractions is reported). Outcomes are grouped according to polarization (1st column) and predictor (2nd column). UCS=uncovered set; SPE (H) = non-cooperative equilibrium for *High*. Columns 7 and 8 show the median of the average distance between predicted and realized outcome in the payoff space. More specifically, for some prediction z and observation \hat{z} , this distance is $d(z, \hat{z}) = \sum_{i \in N} (u_i(z) - u_i(\hat{z}))^2$. For the uncovered set, we take the distance to the set:

$\min \{d(0, \hat{z}), d(\delta, \hat{z}), d(1-a, \hat{z})\}$. Numbers in parentheses denote the p -value for the (Wilcoxon or Mann-Whitney) tests on the differences.

⁽¹⁾ $1-a$ includes $a-1$ when $a=b$;

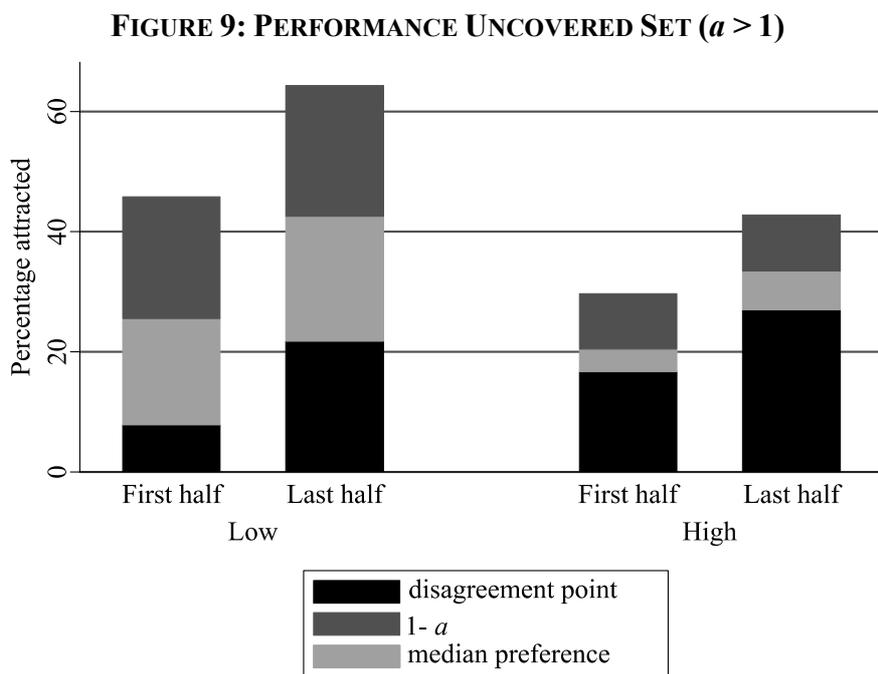
⁽²⁾ the point prediction varies with the player chosen to make the first offer (and is therefore not applicable to *Low*).

Recall from table 4 that for $a < 1$, the core of Γ and the SPE of Γ_T^H are both equal to the median preference 0. For $a > 1$, the core is empty, the uncovered set is $\{0, 1-a, \delta\}$ and the SPE of Γ_T^H High is a (typically) unique point. The set of SPE of Γ_T^L is $[-a, b] \cup \delta$ for any level of polarization we used.

Table 5 shows no treatment effects in the first half, and, except for the uncovered set in *Low*, no solution concept seems to predict very well. In the last half, however, this drastically changes. 0 attracts 58% of the outcomes in *Low* and 28% of the

outcomes in *High*; the uncovered set attracts 60% of the outcomes in *Low* and 33% of the outcomes in *High*; the SPE of $\Gamma_T^H(G)$ attracts 33% of all outcomes in *High*.

The core and uncovered set perform significantly better in the last half, in particular in *Low*. In *High*, the SPE of Γ_T^H also performs significantly better in the last periods. Most striking, though, is the treatment effect in the last half: the core and the uncovered set both perform significantly better in *Low* than in *High*. As figure 9 shows, this is due to the better performance of the predictions 0 and $1-a$ in *Low*.



Notes: the bars represent the percentage of outcomes attracted by each point. (Small differences with table 5 are due to the fact that the table reports medians, because the non-parametric tests are median tests.)

6. CONCLUSION

In this paper, we have studied the impact of structure on the outcome of a three-player bargaining game. We have compared a low-structure bargaining process –where players can freely make and accept proposals– to a high-structure process –where agenda-setting and voting is regulated by a Baron-Ferejohn alternating offers scheme. The two processes, which we model as non-cooperative games, differ only in bargaining structure and correspond to the same bargaining problem, which we have modeled as a cooperative game. An important assumption in this game is that the

disagreement point is not on the same single dimension as other possible outcomes. As a consequence, the emptiness or non-emptiness of the core varies with the parameters of our model. This is important, because core-existence may be crucial for the relevance of the bargaining structure. The variation in parameters also allows us to study polarization, which is relevant in its own right. We find that both polarization and structure matter.

In the classic median voter setting, the median preferred outcome is the unique core element irrespective of the divergence of players' interests (polarization). In addition, even if wing players would want to take standard fairness (or spite) considerations into account, there is no non-median outcome on which a majority of them could coordinate. Hence, polarization in a group of decision makers is theoretically expected to be of no importance to the bargaining outcome and its stability, which often seems at odds with reality.

Our modified median voter game is based on the observation that in many situations where players have to agree on an outcome that can be represented on a single dimension, the disagreement point will lie outside of the line. For a median voter setting with an exterior disagreement point, polarization turns out to be highly relevant. With weak polarization, the median preference remains the singleton core. When preferences are characterized by strong polarization, the disagreement point is the singleton core. For moderate levels of polarization, the core is empty and the uncovered set consists of three or four points. The subgame perfect equilibrium outcome for high structure converges (as the number of rounds increases) to the core, if this exists. It need not lie in the uncovered set for moderate polarization, however. Still, both this equilibrium and the uncovered set predict that the median player is worse off for moderate than for weak polarization when structure is high.

In our experiment, we find that polarization indeed matters. In particular, polarization hurts the median player. As predicted by theory, the median player is significantly worse off at moderate than at weak levels of polarization. In contrast to what theory predicts, more polarization hurts the median player even when her preference remains the core. The data appear to be the result of a mix between considerations of coalitional fairness and intercoalitional competition. Coalitional fairness favors a fair split between the median player and her partner in the coalition,

whereas intercoalitional competition favors the median preference. The first observation explains why even weak polarization hurts the median player: as polarization increases, giving the other player a fair share becomes more costly to the median player. With experience, intercoalitional competition becomes more dominant, however. The number of fair compromises between player 1 and 2 decreases significantly over time and the number of outcomes at the median preference increase significantly. Competition can also harm the median position, however. When polarization becomes sufficiently strong, experienced players 1 and 3 start to coordinate on the disagreement point, thereby undermining the median player's profits.

All in all, our theoretical analysis and experiment give an explanation of why polarization can matter in group decision making (such as that taking place in legislatures, coalitions or boards), even if the outcome set is (close to) unidimensional.

Turning to structure, theory does not shed much light on its influence. Cooperative solution concepts do not take structure into account. But in our environment non-cooperative game theory could not provide a clear answer either. The low-structure environment is so strategically rich that the subgame perfect equilibrium, which is impervious to further refinements, supports a wide range of points as an outcome. This provides an important motivation to run an experiment.

Our experiment shows that structure matters, even when there is a unique core element. For starters, the median player in our experiment is significantly better off with low structure. Outcomes in this treatment are significantly more often the median preference and significantly less often the fair compromise between players 1 and 2. It appears that the lower degree of structure gives median player more flexibility to exploit her superior bargaining position. This result supports the armchair observation that players in a better bargaining position prefer less regulation of negotiations. In particular, this is relevant for issues of institutional choice, where legislatures have to decide on a bargaining *procedure* before they decide on the outcome itself.

Furthermore, both the core and the uncovered set perform significantly better with low than with high structure. In the first half of the experiment, the difference

between the treatments is small (and insignificant). In the second half, however, the core/uncovered set attract around 60% of all outcomes with low structure and just 30% of all outcomes with high structure. This finding shows that structure is relevant, even if there is a core.

We see three important implications of the finding that structure matters. First, our results provide empirical support for the intuition that cooperative solution concepts are better suited for environments with lower degree of structure. Second, the fact that structure influences the payoffs of certain players and the performance of specific predictions means that ‘neutral’ simplifying assumptions (i.e., assumptions that do not favor any player *prima facie*) made to obtain tractable results in bargaining problems need not be as innocuous as is often assumed.

Finally, at a more applied level, understanding the influence of structure is relevant for studying institutional choice and parliamentary procedure. Even if the degree of structure may seem like a neutral parameter, it can significantly influence the bargaining outcome; and hence parties may have preferences for a specific degree. For instance, parties in the center of a political spectrum may prefer to prolong backroom discussions until agreement has been reached. Our results point to the more general idea that parties in a superior bargaining position will prefer institutions that impose less structure on the bargaining procedure, as it gives them more flexibility to exploit their bargaining position.

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