Information and Strategic Voting

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Abstract

I study voter behavior in a setting characterized by majority rule and mandatory voting, where voters choose from three options. Because of the possibility of Arrow's cycle, strategic behavior may emerge. In particular, I focus on how information contained in election poll results affects voter's choices. By normalizing the utilities of the best and worst options, each voter's preferences across the three options are characterized by the relative value attributed to the intermediate option. I also test how this relative importance impacts voting behavior and how this interacts with information. Quantal response equilibrium analysis is used to analyze the game and proves to be a good predictor for the experimental data. The main results are: (i) sincere voting increases as the importance of the intermediate option decreases; (ii) information yields coordination in favor of the candidate with the highest level of support (i.e., the 'majoritarian' candidate); (iii) voters who have the 'majoritarian' candidate as the second preferred option vote strategically for this candidate if the relative benefit is high enough.

JEL Classification: C92, D72, D83

Key words: Voting Behavior; Experimental Economics; Quantal Response

1 Introduction

Do people vote strategically in elections? Do polls influence how people vote in elections? An answer to this type of question is crucial for our understanding of the way voters make

their decision in modern democracies.[1] I investigate these questions in the specific context of multi-candidate and mandatory voting.

Previous studies have addressed these and related questions in different ways and under different contexts. Since most countries in Europe and the United States have voluntary voting, a significant part of the literature considers models adapted to this reality (e.g. Persson and Tabellini, 2002; Bendor, Diermeier and Ting, 2003; Feddersen and Sandroni, 2006; Palfrey, 2006). In a typical setting the situation is modeled as a participation game with two candidates, in which voters are either partisans or swing voters and must decide whether or not to incur the cost of voting and cast a ballot for one of the candidates. Note that with two candidates, if deciding to vote, all voters should vote sincerely since voting for one's least preferred candidate is a dominated strategy if voting is costly. Therefore a voter decides between either participating and voting sincerely or abstaining. As a consequence participation games focus mostly on voter turnout, in particular on the so-called 'voter paradox'.

The addition of a third-candidate gives rise to situations that allow for strategic behavior, as indicated, for example, by Persson and Tabellini (2002, chapter 2). With more than two candidates in the running voting sincerely might not be a (weakly) dominant strategy. Therefore, strategic behavior may emerge and equilibria in which voters are not sincere might be observed.

One strand in the literature focuses on a specific situation in which there is a Condorcet loser (i.e., a candidate that would lose any pairwise vote against another candidate) supported by a minority and a majority who is divided between two candidates (Palfrey, 1989; Myerson and Weber, 1993; Forsythe et al., 1993, 1996; Fey, 1997). The main question is whether this majority can coordinate on one candidate and avoid a victory of the Condorcet loser. Such an equilibrium is in line with Duverger's Law (see e.g. Riker, 1982) which states that under plurality rule only two candidates should be expected to receive votes. Myerson and Weber (1993) show that three equilibria can exist: two Duvergian, in which the majority coordinates on one of the candidates and one non-Duvergian, in which the majority splits their vote and the Condorcet Loser wins. Fey (1997) shows that this non-Duvergian equilibrium is not stable while Palfrey (1989) proves that when the size of the electorate increases only Duvergian

[1] Although not the main focus of this study, similar questions are also very relevant for committee voting. The results from this study can easily be used to better understand this environment as well.

equilibria can persist.

Experimental studies by Forsythe et al. (1993, 1996) (summarized by Palfrey, 2006) show that in the absence of any coordination device (polls, shared history, past elections, ballot position) the Condorcet Loser wins a significant proportion of the elections, in contrast with Duverger's Law. These experimental studies have also applied voluntary, non-binding polls. As supported by theoretical predictions by Fey (1997), the information provided by the poll helps the majority to coordinate on a candidate and win the election.

In this study I consider a different situation in which Arrow's cycles can easily emerge. In the absence of a Condorcet winner (and a Condorcet Loser) any candidate has an a priori chance of winning the election and each voter faces an a priori symmetric strategic problem. I also study the impact of information, conveyed by polls. The self reporting poll mechanism employed has previously been shown to give rise to strategic reporting. However, to avoid this additional layer of strategic behavior, I opt for a situation in which an opinion poll truthfully reveals the electorate's preferences. In this sense the information mechanism used in this paper is closer to Grosser and Schram (2008), who study the effect of polls on turnout in a voluntary voting game with only two candidates.

As a real world counterpart, this kind of information can be seen as a 'noiseless' version of an early opinion poll, reflecting more the actual preferences than the intended voting behavior. As shown by McKelvey and Ordeshook (1986); Gelman and King (1993); Brown and Jr. (1999); Erikson and Wlezien (2001) and Campbell (2004) early opinion polls are not necessarily a good predictor of the actual election outcome. They also show that polls closer to election day are better predictors. They argue that polls are a function of beliefs and preferences. In the context of this study, we can therefore plausibly argue that in an early poll, without information about others' preferences each voter would tend to act sincerely and reveal his true preference.[2] Opinion polls closer to the election would tend to catch more strategic behavior and aim basically at anticipating the election outcome.

The system of mandatory voting I focus on is empirically important. Figure 1 indicates the countries where some degree of mandatory voting for public offices is used. Note that a large minority of the countries have electoral system with some kind of mandatory turnout.

[2]It is shown later in this paper that in a setting without information voters tend to act mostly sincerely, i.e., vote for their most preferred alternative. Therefore a poll taken in such a setting would capture most of the structure of preferences.

Analytically, the assumed mandatory voting allows me to focus on strategic voter choices without needing to correct for the interaction with the turnout decision. It is important to note, however, that it is not the element of mandatory voting that creates the potential for strategic voting, but the contest between multiple candidates. The mandatory rule makes strategic voting more salient, since voters are obliged to face the voting decision.

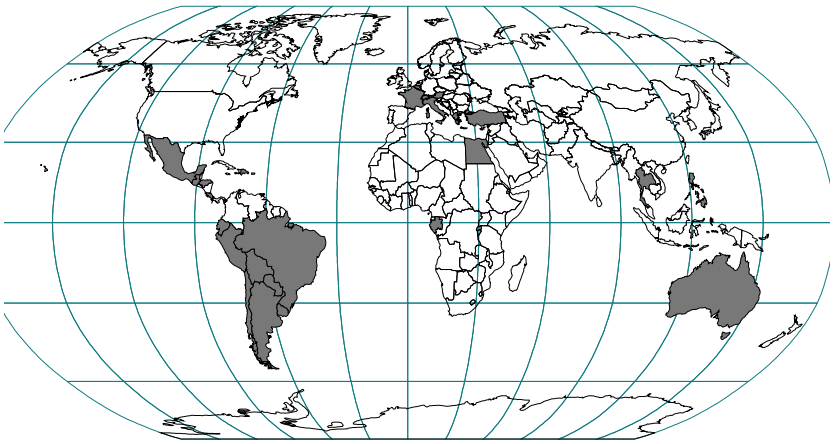


Figure 1: Countries with mandatory voting. Data Source: Gratschew (2001).

The situation modeled in this paper will typically lead to Arrow's cycle, giving any candidate an a-priori equal chance of winning. Perfect information about the other voters' preferences will in some treatments be made available before the election. Therefore the strategic problem faced by each voter with or without information is not trivial, and predictions will be complex. These will depend on the preference ordering and on how voters who rank the majoritarian candidate (i.e., the candidate with the largest support) as second best decide to act. Voters from this group, which will be named compromisers, might decide to be sincere and defend their most preferred candidate or compromise and opt to vote strategically, supporting the majoritarian candidate. Assuming voters will vote either for their most preferred (sincere voting) or second most preferred candidate (strategic voting), this decision will depend on the expected payoff difference between these two options. The expected payoff, in turn, depends on the extent of information available and on the relative importance of each voter's second most preferred alternative. If this importance is low, voter's will be more likely to vote sincerely, while strategic behavior may emerge if this relative importance is high enough.

To analyze the model theoretically and derive behavioral predictions, a bounded rationality model is used. A Quantal Response analysis with the logit specification is applied, showing that without information all voters vote symmetrically and mostly sincerely while with information,

different types of voters act distinctly. Moreover, with information, if the relative importance of the second most preferred option is high enough compromisers will vote mostly strategically.

Given the characteristics of the model under study I opted for an experimental approach. The experimental environment allows, among others, for induced preferences, controlled information and direct comparative statics analyses. In contrast to field data, an experimental setting gives precise information on a voter's preference, allowing one to observe if the vote is sincere or not. Also, exogenous opinion polls can be added without noise, making it possible to observe their effects in isolation. Similarly, the relative importance of each alternative can be manipulated in order to answer the research questions.

My experimental results are in line with the Quantal Response predictions. Specifically I observe (i) a higher frequency of sincere voting the lower is the relative importance of a voter's second most preferred option; (ii) information yielding coordination on the victory of the majoritarian candidate; (iii) with information, compromisers voting mostly sincerely if the relative importance of the second most preferred candidate is low and mostly strategically if it is high.

The remainder of the paper is organized as follows: Section 2 presents the model, its solution and behavioral predictions while section 3 introduces the experimental design. Section 4 analyzes the results and section 5 presents concluding remarks. A set of appendices compliments the paper.

2 The Model

Each of N voters must choose from three alternatives, A,B and C. Each voter $i = 1, \ldots, N$ has a strict preference ordering over these alternatives. Since I am interested in mandatory voting rules, each voter will be required to cast exactly one vote for one of these three alternatives. Voters maximize expected utility, which is determined by the elected alternative.

Majority rule determines the winner, with ties broken by an equal probability random draw. The utility of each voter is set according to the rank of the elected alternative. If his preferred, intermediate or least preferred alternative is elected he receives u^{best}, $u^{intermediate}$ or u^{worst} respectively. Normalizing u^{best} and u^{worst}, each voter's preferences are characterized by the utility attributed to the intermediate option. Finally, I assume that utility is independent

of individuals and options, i.e., $u^{intermediate}$ is the same for every voter. Hence, only the ordering of options distinguishes voters.

Before each election, all voters' preferences are determined randomly, independently of previous preferences and other voter's draws. The own preferences are revealed to the voter by nature at the beginning of the period. The extent of information about the electorate's preferences is a variable in the model. The setting can be, either uninformed, in which case voters (aside from their own preference ordering) know only the prior probability distribution of preferences, or informed, in which case they know the ex-post realized distribution of preferences for the election concerned. This variable is meant to capture the possible publication of (noiseless) pre-election polls, as described in the introductory section.

An electorate is, therefore, characterized by the number of voters, the utility of the intermediate option and the extent of pre-election information.

Because I am not interested in specific characteristics of the distinct options, I will focus on a game in which every voter has an a-priori symmetric problem regardless of his preference ordering. Without loss of generality, I therefore restrict the possible preferences to $\{(A,B,C);(B,C,A);(C,A,B)\}$, in which the listed order represents the preference ordering. Preferences are independently randomly drawn from this set with equal probability for each voter. These preferences will typically form an Arrow's cycle potentially giving rise to strategic behavior.

Voting games have the typical problem of multiple equilibria. Consider a preference distribution denoted by the vector $(N_{A,B,C}, N_{B,C,A}, N_{C,A,B})$ such that $N_{A,B,C}+N_{B,C,A}+N_{C,A,B}=N$. $N_{A,B,C}$ denotes the number of voters with preference ordering (A,B,C) and similarly for $N_{B,C,A}$ and $N_{C,A,B}$. Denote the election outcome by a vector (v_A, v_B, v_C) such that $v_A+v_B+v_C=N$, where v_k denotes the number of votes for option k. Take, for example, a situation in which $N=12$ and each preference ordering is equally represented with full information, i.e., preference groups are distributed as $(4,4,4)$. In this case all situations in which exactly two groups vote sincerely (i.e., vote for the most preferred alternative) and the remaining votes strategically (i.e., vote for the second most preferred alternative) will be a Nash equilibrium. The voting outcome would be, for example, $(0,8,4)$ and since nobody is pivotal no one can benefit from deviating. A similar voting outcome would be observed if only one group voted sincerely,

with the other two voting strategically. Again, nobody is pivotal in such a situation, creating a Nash equilibrium. Sincere voting by everyone can also be an equilibrium. Such voting behavior leads to an expected payoff of $\frac{u^{best}+u^{intermediate}+u^{worst}}{3}$. Each voter is pivotal, however. Voting strategically will lead to a payoff equal to $u^{intermediate}$. If $u^{intermediate} < \frac{u^{best}+u^{intermediate}+u^{worst}}{3}$, all voting sincerely is a Nash Equilibrium.

It is also possible to find equilibria in which members of the same group act differently. Take for example, in the same example above, two groups voting sincerely. If the remaining group has half of its members voting sincerely and the remaining voting for their second most preferred alternative the voting outcome would be, for example, $(4, 2, 6)$. Once again nobody is pivotal, and this is an equilibrium.

One could try to simplify the analysis by assuming that indifferent (non-pivotal) voters vote sincerely. This may yield an empty set of equilibria, however, due to discontinuities in the best response correspondence. Take the same example as before. In any given equilibrium all voters could be pivotal; some, but not all, could be pivotal; or no voter could be pivotal. First note that in any equilibrium all pivotal voters must be voting sincerely. If this were not the case, i.e., some are voting strategically, at least one of these voters could deviate and decide the election in favor of his most preferred candidate. On assumption all non-pivotal voters also vote sincerely. This implies that the only possible equilibrium is one where all vote sincerely, a case in which all are pivotal in the example under consideration. If the condition presented above for the existence of this equilibrium is not satisfied, this would lead to an empty set of equilibria. Note that this reasoning also rules out all mixed strategy equilibria. For these reasons I do not assume sincere voting by non-pivotal voters.

To tackle the multiple equilibria problem one must employ an equilibrium selection device. In order to solve this model I apply a Quantal Response approach (McKelvey and Palfrey, 1995), with the logit specification. It is useful to first more formally define sincere and strategic voting:

Definition 1 ***Sincere Voting:*** *Voting for the most preferred alternative.*

Note that voting for one's least preferred alternative is a weakly dominated strategy. I will therefore assume that voters never play this strategy.[3] I can, then, define strategic voting:

[3]This is not a crucial assumption. If it is relaxed, the probability of choosing this strategy very quickly converges to zero in the Quantal Response correspondence.

Definition 2 ***Strategic Voting:*** *Voting for the second most preferred alternative.*

Since I assume people do not play weakly dominated strategies, each voter's strategy is fully characterized by the variable π_i, indicating the probability of voting sincerely. The assumption made above then imply that the probability of voting strategically is given by $1 - \pi_i$. Define $\bar{u}_i(\pi_{-i})$ as the expected payoff difference from voting sincerely in comparison to voting strategically, as a function of all other voters' strategies π_{-i}. Following McKelvey and Palfrey (1995), in a Quantal Response equilibrium we have :

$$\forall i = 1, \ldots, N, \ \pi_i = \frac{\exp(\lambda \bar{u}_i(\pi_{-i}))}{\exp(\lambda \bar{u}_i(\pi_{-i})) + 1}$$

where λ indicates the 'rationality' parameter. This rationality parameter is general enough to capture different sources of noise, as for example, distractions, perception biases, miscalculations or limited computational capability (Goeree and Holt, 2005).

When $\lambda = 0$ players purely randomize. For any positive value of λ it is possible to compute Quantal Response equilibria. We call this collection of Quantal Response equilibria and correspondent λ values the 'Quantal Response Correspondence'. When $\lambda \to +\infty$, the Quantal Response equilibria approaches the Nash Equilibria of the game (McKelvey and Palfrey, 1995).

Except for the case $\lambda = 0$, there is not necessarily a unique equilibrium. It is possible, however, to identify a unique branch of the Quantal Response Correspondence that starts at $\lambda = 0$ and continuously converges to a unique Nash Equilibrium as $\lambda \to +\infty$ (McKelvey and Palfrey, 1995, Theorem 3, item 3). We call this branch the 'Principal Branch' and this unique Nash Equilibrium the 'limiting Logit Equilibria' of the game. Except for very special cases, the principal branch needs to be computed numerically. In order to trace it I use the Homotopy Approach as outlined by Turocy (2005, 2006).

Using the Quantal Response model, with the logit specification, has several advantages. Among others: (i) it includes a selection procedure by selecting the Nash equilibrium that is the limiting logit equilibrium along the principal branch of the Quantal Response Correspondence; (ii) it gives a good base for observed behavior when considering bounded rationality and (iii) the principal branch will always have the intuitive characteristic that players of the same type play symmetric strategies.

In all cases of $\lambda > 0$ the Quantal Response function is increasing in the expected payoff

difference. The expected payoff difference will vary with the extent of information and realized distribution[4], but it will always include the probability of being pivotal, since for every non-pivotal situation the payoff difference will be 0.

For the determination of the Quantal Response Equilibria I will use the parameterization $N = 15, u^{best} = 10, u^{worst} = 1$. I will have two values for the intermediate option, one reflecting a low importance ($u^{intermediate} = u = 3$) and another reflecting a high importance ($u^{intermediate} = u = 8$) relative to the best outcome. This parameterization corresponds to the one I used in the experimental design (cf. section 3).

In order to further structure the game a few more definitions are useful:

Definition 3 ***Majoritarian Set:*** *The set of alternatives with the highest number of votes if all voters vote sincerely.*

Definition 4 ***Majoritarian Candidate:*** *The alternative(s) with the highest number of votes if voting is restricted to the Majoritarian set and all voters vote sincerely.*

Note that if the Majoritarian Set is singleton it equals the Majoritarian Candidate. If it contains two elements (i.e., two options receive equal sincere support, the third receives less) than the Majoritarian Candidate is the option from the Majoritarian Set that gives highest utility to the supporters of the third option. In the present game, except for the case when all preferences are equally represented, the Majoritarian Candidate is unique. Having defined the Majoritarian Candidate it is possible to define three types of voters used in the analysis of the game:

Definition 5 *Types:*

(Majoritarian) Supporter: *Voter with the majoritarian candidate as his most preferred alternative.*

(Majoritarian) Opposer: *Voter with the majoritarian candidate as his least preferred alternative.*

(Majoritarian) Compromiser: *Voter with the majoritarian candidate as his second most preferred alternative.*[5]

[4] Appendix A shows the details of the computations.

[5] In case of a 3-way tie the Majoritarian Set equals the Majoritarian Candidate and is composed of all three candidates. I therefore define all voters as Supporters.

Observe that depending on the extent of information, realized distribution and the value of the intermediate option the decision on whether to vote sincerely or strategically may be non-trivial. Conditional on being pivotal there might be situations in which voting sincerely leads to a lower expected payoff than voting strategically.

2.1 Uninformed Setting

Consider the situation without information. The voter knows only the prior distribution of probabilities, the electorate size, the value of the intermediate option and his own preference. Knowing his own preference he can update the probability distribution using Bayes' rule and use this to calculate the probability of being pivotal given others' strategies and compute his expected payoff difference between voting sincerely and strategically.

This rather complicated computation is easier understood by an example. Consider the case in which $N = 15$. For a given voter, the most likely distributions among the other voters are $(4, 5, 5)$, $(5, 4, 5)$ and $(5, 5, 4)$, where the first number indicates the number of other voters with the same preference, and the other two the number in the remaining groups. If he believes that all others are voting sincerely this voter considers himself to be pivotal in all three situations. In the first situation his own vote would either create a three-way tie (if voting sincerely) or give the victory to the second most preferred candidate. Voting sincerely may be profitable, depending on the value of the intermediate option[6]. For the other two situations voting sincerely is always profitable, since he would be decisive in favor of his most preferred candidate. Considering only these 3 situations voting sincerely is more likely to be most profitable. Indeed, considering all other possible pivotal situations with their respective probabilities it can be shown that voting sincerely is more profitable than voting strategically. If all players have similar prior beliefs and reasoning, all should vote sincerely, confirming beliefs and forming a Bayesian-Nash Equilibrium. Hence, regardless of the intermediate preference parameter, there is a Bayesian-Nash equilibrium in which all vote sincerely.

Figure 2 shows the principal branch of the Quantal Response Correspondence for the game without information for the two different values of the intermediate option. Note that both

[6]In this particular situation, voting sincerely will be strictly profitable if $u^{intermediate} < \frac{u^{best}+u^{worst}}{2} = 5.5$, where the equality comes from a normalization $u^{best} = 10$ and $u^{worst} = 1$. Therefore if the intermediate option is low enough, creating a tie is profitable. If it is high enough, voting strategically is more profitable.

branches indeed converge to the sincere voting equilibrium just described.

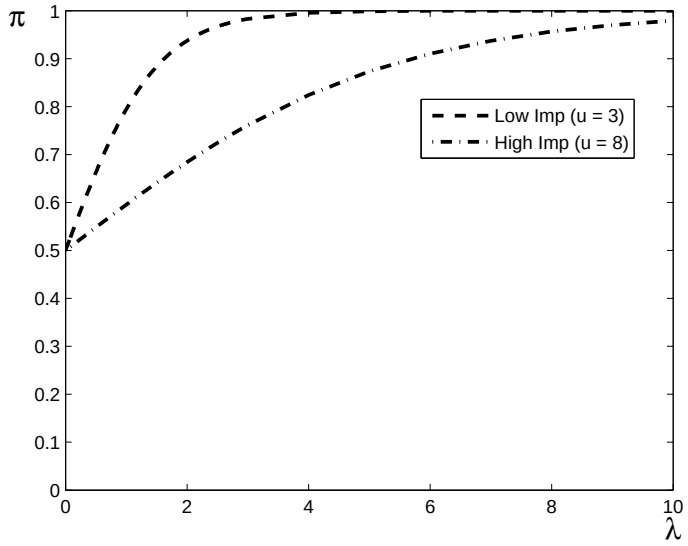


Figure 2: Principal Branch of the Quantal Response Correspondence. Uninformed Setting.

In a given parameterization, since all voters have the same prior, and symmetric information, they are predicted to act the same. Moreover, with a low value of the intermediate option learning is faster, and convergence to the Bayesian-Nash Equilibrium is faster. The interpretation is that with a lower importance of the intermediate option the expected payoff difference is larger, and therefore the incentive to play the sincere strategy is stronger.

Based on this Quantal Response analysis I derive two behavioral predictions for the game without information.

Behavioral Prediction 1 *Without information about others' preferences the probability of voting sincerely is decreasing in the importance of the intermediate option.*

Behavioral Prediction 2 *Without information about others' preferences, for a given level of the intermediate option, all voter types will vote sincerely with the same probability.*

2.2 Informed Setting

Consider next the game with full information. Now each voter knows the realized distribution and therefore his own type (supporter, opposer or compromiser). Take as example the case with equal share, i.e., $(N_{A,B,C}, N_{B,C,A}, N_{C,A,B}) = (5, 5, 5)$, which can serve as a comparison to the a-priori expected uninformed situation. Figure 3 plots the principal branch of the

Quantal Response correspondence and adds curves from figure 2 for comparison. In this case all voters' circumstances are perfectly symmetric. In comparison to the previous case, however, the information removes the uncertainty. For the case of low importance of the intermediate option, all voting sincerely is still a Nash equilibrium, and therefore it converges faster than the high importance case. For the case of high importance of the intermediate option all voting sincerely is not a Nash equilibrium under perfect information. The limiting logit equilibria converges to a mixed strategy Nash equilibrium in which all voters vote sincerely with a probability of approximately 0.896.

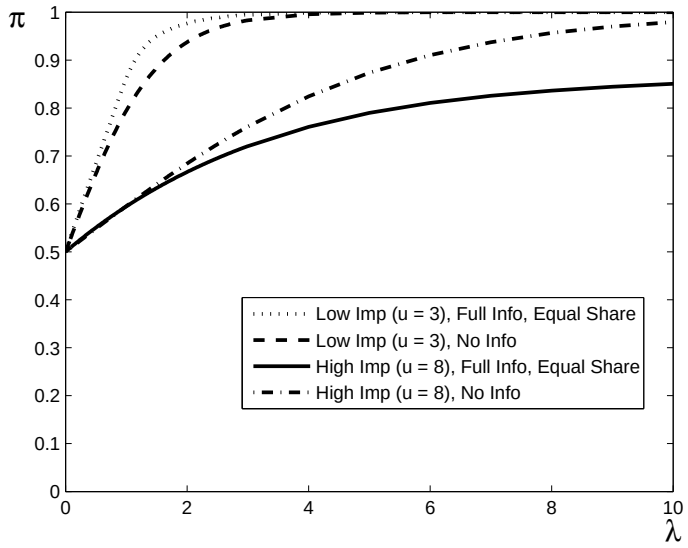
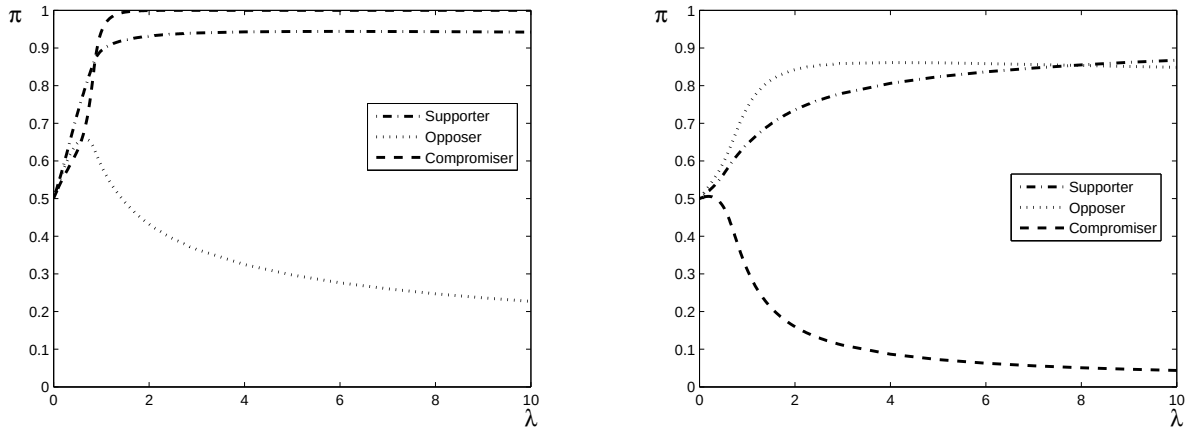


Figure 3: Principal Branch of the Quantal Response Correspondence. Informed Setting with Equal share vs. Uninformed Setting.

Given the random allocation of preferences, the resulting distribution can be described by a multinomial distribution. The distribution of types takes a slightly different form, however. Since for each potential situation other than the equal share it is possible to identify a unique majoritarian candidate, different distributions in terms of preferences will have the same types distribution. For example, consider the cases in which preferences $(N_{A,B,C}, N_{B,C,A}, N_{C,A,B})$ are (6,5,4) or (4,6,5). In terms of types they will represent the same situation: 6 supporters, 5 opposers and 4 compromisers.[7] Therefore the equilibrium analysis will be exactly the same in both cases.

[7]In first case the Majoritarian Candidate is A. Therefore voters with preference ordering (A, B, C), (B, C, A) and (C, A, B) are Supporters, Opposers and Compromisers, respectively. Similarly in the second case the Majoritarian Candidate is B. Therefore Supporters, Opposers and Compromisers are voters with preference ordering (B, C, A), (C, A, B) and (A, B, C), respectively.

It turns out that the random allocation of preferences yields a situation with 6 supporters, 5 opposers and 4 compromisers with a probability of 13.18%. As an illustration, figure 4 plots the principal branch of the Quantal Response correspondence for this case.



(a) Low Imp ($u = 3$) (b) High Imp ($u = 8$)

Figure 4: Principal Branch of the Quantal Response Correspondence. Informed Setting: 6 supporters, 5 opposers and 4 compromisers.

The most interesting feature of these two graphs is the fact that, as the importance of the intermediate option increases, the compromisers qualitatively change their behavior from voting mostly sincerely to mostly strategically. Supporters always vote mostly sincerely.[8] Appendix B shows similar graphs for all situations. Figure 5 plots the weighted average across informed situations. Qualitatively, overall behavior is similar to the situation just described.

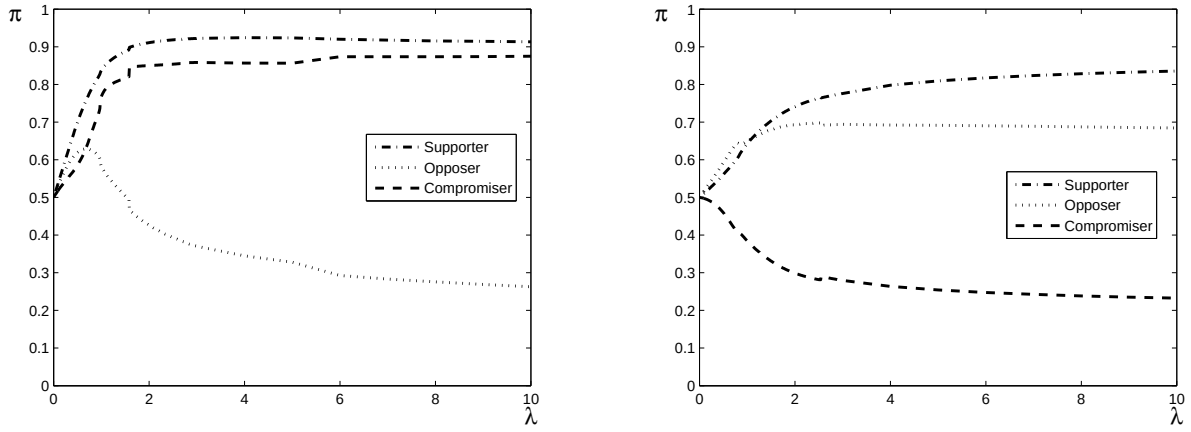
Based on this Quantal Response analysis I can now formulate three more behavioral predictions:

Behavioral Prediction 3 *With full information the overall probability of sincere voting is decreasing in the importance of the intermediate option.*

Behavioral Prediction 4 *With full information types should act distinctly, in contrast to the symmetric behavior predicted without information.*

Behavioral Prediction 5 *With full information compromisers vote mostly sincerely if the intermediate value is low and mostly strategically if it is high.*

[8]Opposers' behavior can be interpreted technically and economically. Economically they are close to indifference and can be easily swayed either way. In case of low importance, compromisers are voting mostly sincerely. Since opposers support the compromiser's candidate as their second most preferred option, they tend to vote slightly strategically even though they are very unlikely to be pivotal. For similar reason, with high importance, they will vote more sincerely. Technically, opposer's choice can be seen as a device that strikes the equilibrium of the model.



(a) Low Imp ($u = 3$)

(b) High Imp ($u = 8$)

Figure 5: Principal Branch of the Quantal Response Correspondence. Informed Setting: Weighted Average across all possible situations.

3 Experimental Design

Two pilot sessions were run at Fundação Getúlio Vargas, São Paulo, Brazil in August 2007. 30 undergraduate subjects from the programs in Economics, Business and Law participated. This allowed for 2 electorates, varying information within a session and the importance of the intermediate option across sessions, as described above. In addition to a show-up fee of R$ 5.00 subjects were paid R$ 0.03 per experimental point[9]. Average earnings were R$ 19.85, including the show-up fee, and each session lasted about one and a half hours. The experiment was computerized using Z-tree (Fischbacher, 2007). Translation of the instructions can be found in Appendix C.

The experimental design aims at studying the impact of the relative importance of the intermediate option and the extent of information on voting behavior. Therefore, 4 treatments are needed. Relative importance is varied across-subjects while the comparison of no information against information is made within subjects.

The electorate is fixed during a session and consists of 15 voters. There are 3 preference orderings, $\{(A, B, C); (B, C, A); (C, A, B)\}$, which are assigned with equal probability to each subject. There is a new draw before every election. Draws are independent across subjects and elections. All this is common knowledge. In every election each subject is required to cast one vote for either A, B or C. Majoritary rule determines the winner, with ties broken by equal probability random draw. Subjects are paid in each round according to the rank of the winner

[9]As of 27th of August, 2007, the exchange rate was R$ 1.95 for one U.S. dollar

in their own preference ordering. If the winner is the highest ranked option a subject is paid 10 points and for the lowest ranked 1 point. The value of the intermediate option is constant for a given electorate and is set to be either 3 or 8 according to treatment. Every individual is informed about his or her own preferences before each election.

In any session there are 80 elections. In the first 40 rounds subjects are uninformed about other voters' preferences. In the last 40 rounds they are informed about these preferences. In these informed rounds, participants know the aggregate induced preferences of all other voters in the electorate. After each election, the aggregate voting outcome is shown to all subjects. Table 1 summarizes the design.

	Information	
Intermediate option	Low Importance ($u = 3$), Uninformed	Low Importance ($u = 3$), Informed
	High Importance ($u = 8$), Uninformed	High Importance ($u = 8$), Informed

Table 1: Experimental Design.

4 Results

4.1 Uninformed Treatment

The uninformed setting is expected to be the treatment with least noise. In this treatment each repetition of the game is exactly the same except for the preference orderings attributed to each individual voter. Nevertheless, all preferences are equivalent and pose the exactly same strategic situation to the subjects. For this reason, and because the uninformed setting was played first, I use it to estimate the noise parameter λ in the Quantal Response model and use this estimate to predict the remaining parts of the experiment.

Figure 6 shows the experimental data for the uninformed rounds along with the group averages. Note that, in accordance with behavioral prediction 1, more sincere voting is observed in the treatment with lower importance of the intermediate option[10].

Figure 7 plots the observed mean frequencies over the Quantal Response correspondence from figure 2. Observe how the Quantal Response model helps to understand why observed behavior may differ in two situations with the same limiting logit equilibrium. For the same estimated noise level, the predicted extent of sincere voting differs substantially. The $\hat{\lambda}$ that

[10]Since I have only pilot data I do not run statistical tests.

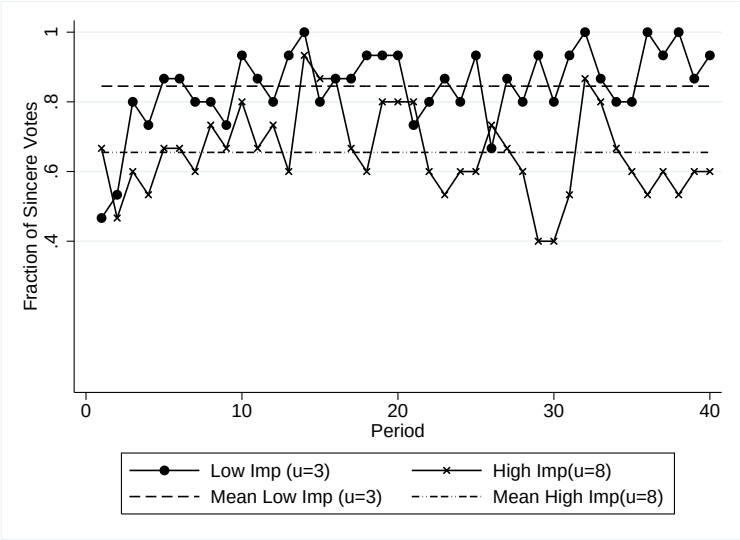


Figure 6: Experimental Data, Uninformed Setting.

minimizes the mean squared error (MSE) of the prediction to observation across the two treatments is $\hat{\lambda} = 1.35$ (with a MSE equal to 0.0005). From the figure, it appears that the resulting predictions for the fraction of sincere voting ($\tilde{\pi} = 0.6275$ for high importance and $\tilde{\pi} = 0.8613$ for the low importance) are quite close to the observed mean fractions ($\hat{\pi} = 0.6550$ for high importance and $\hat{\pi} = 0.8450$ for low importance).

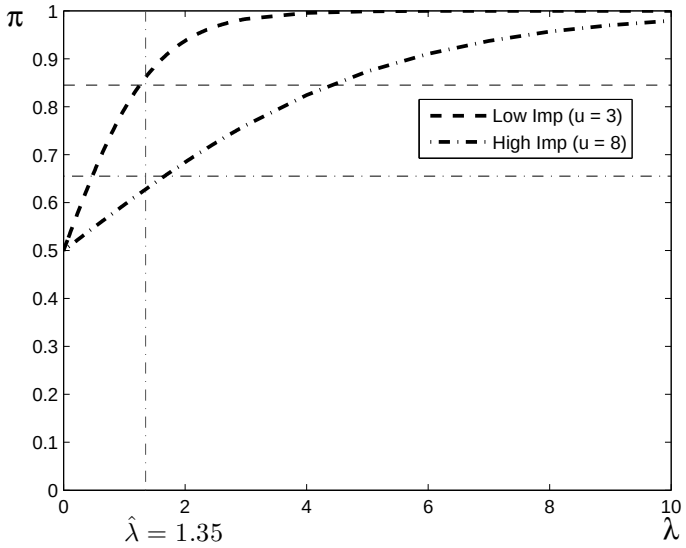


Figure 7: Quantal Response Correspondence with the Average of the Experimental Data, Uninformed Setting.

4.2 (Uninformed vs.) Informed Treatment

Using the estimated $\hat{\lambda}$ I predict aggregate behavior in the informed treatment. Figure 8 shows the observed data along with the predicted values which yield an MSE of 0.0018 (see, also, table 2). As predicted by behavioral prediction 3 the extent of sincere voting is higher when the importance of the intermediate option is lower. Note, by comparing figures 6 and 8, that information reduces the frequency of sincere voting in the case when the intermediate option has low importance. This is as expected, since with information, more strategic behavior should emerge.

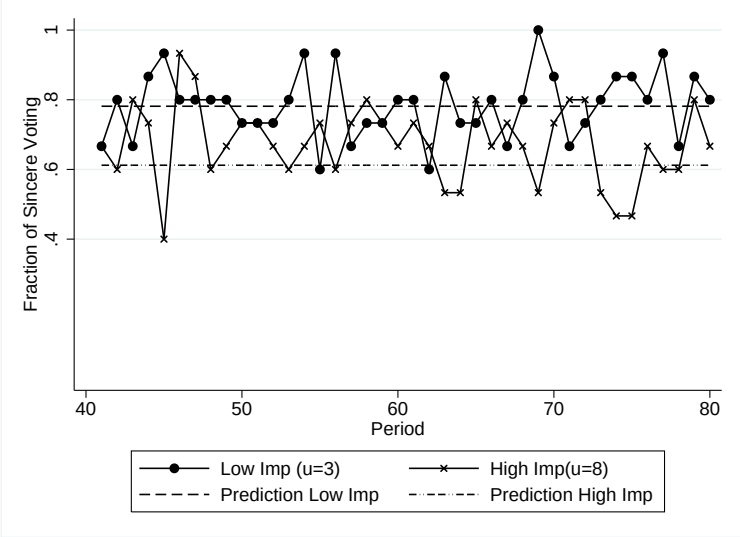


Figure 8: Experimental Data, Aggregate Behavior, Informed Setting.

Figure 9 shows the experimental data grouped by type. The first thing to notice from eyeballing this graph is that the data support behavioral prediction 4 that types act symmetrically when uninformed but distinct patterns evolve in the informed treatment. The next subsection explores these patterns in more detail.

4.3 Types' behavior in the Informed Treatment

Table 2 summarizes observed and predicted frequencies of sincere voting by type, with perfect information. The MSE of the prediction is 0.0189 for $u = 3$ and 0.0417 for $u = 8$. Note that in terms of comparative statics and qualitatively the predictions point in the right direction. Specifically, supporters vote mostly sincerely, opposers are close to random behavior and compromisers vote mostly sincerely under low importance of the intermediate option and mostly strategically under high importance.

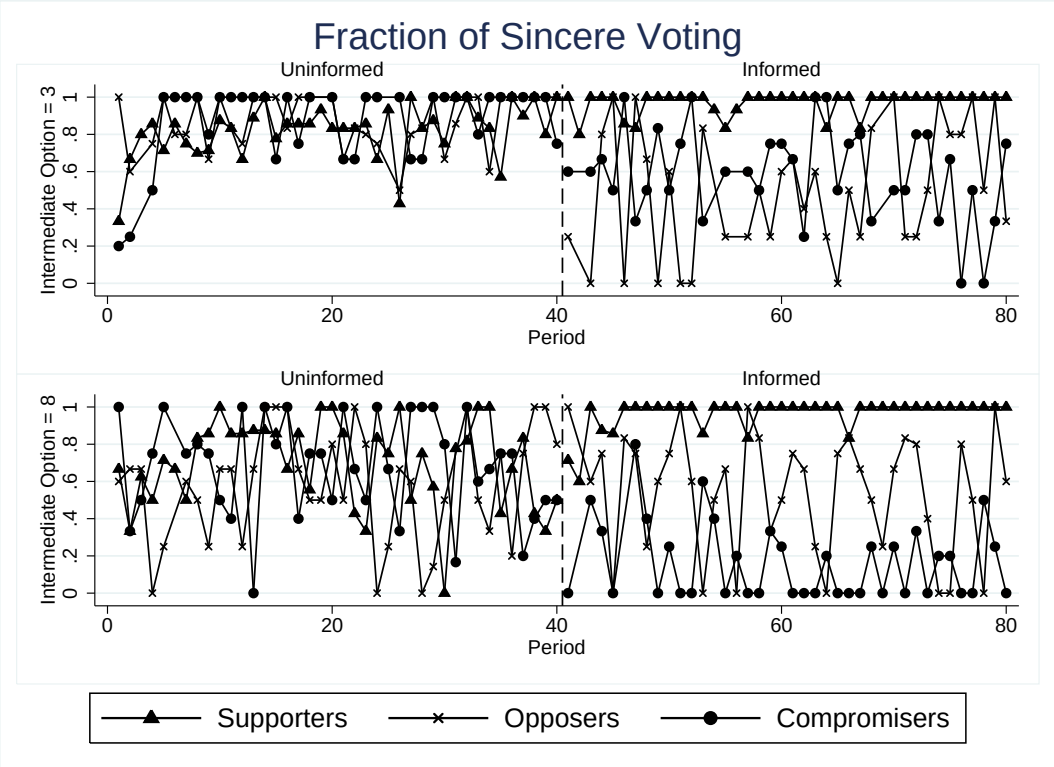


Figure 9: Experimental Data, Sincere Voting by Type.

	Prediction	Observed Data
Low Importance ($u = 3$)		
Supporters	0.8774	0.9714
Opposers	0.5242	0.4981
Compromisers	0.8093	0.5916
Total	0.7801	0.785
High Importance ($u = 8$)		
Supporters	0.6877	0.9642
Opposers	0.6722	0.5444
Compromisers	0.3447	0.1644
Total	0.6121	0.6733

Table 2: Predicted and Averaged Observed Sincere Voting, Informed Treatment.

To see how the various voter types are affected by information, figure 10 shows the full information data on sincere voting as depicted in figure 9, but now grouped by type. This clearly shows support for behavioral prediction 5 that the compromisers are the group who actually change behavior across treatments.

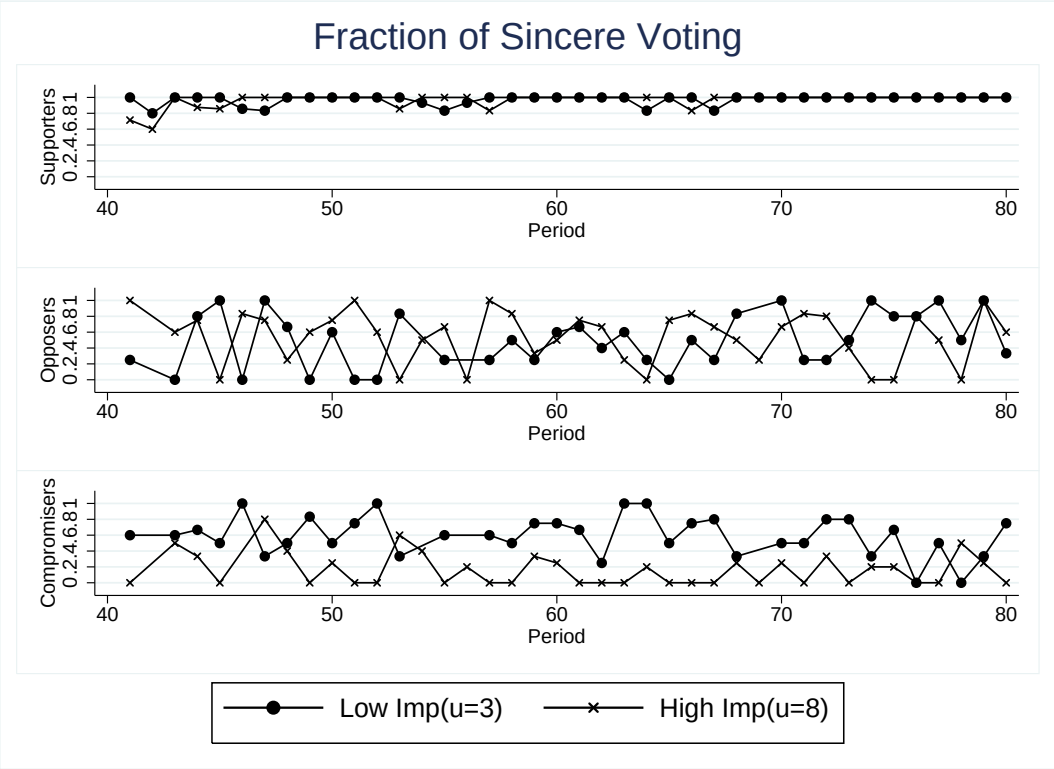


Figure 10: Experimental Data, Sincere Voting by Type.

4.4 Election Winner

Finally, figure 11 shows the outcome of the elections, before tie breaking.[11] Without information the majoritarian candidate wins more elections in the treatment with lower importance of the intermediate option. This is as expected from the behavioral predictions, since in this treatment more sincere voting is expected, and this should lead to a higher victory rate for the majoritarian candidate.

Also as indicated by previous studies (cf. section 1), information leads to strong coordination around the majoritarian candidate. As expected from the behavioral analyses, with higher

[11]Each election period yields a single winner. However, in case of ties, tie breaking procedure would randomly select a winner among the top voted candidates. Since my interest lies on voting behavior, I plot the voting outcome before the tie breaking procedure takes place. Since in each period I can identify the Majoritarian, Opposer and Compromiser candidates (i.e, the most preferred candidate of each group), I can therefore categorize type of voting outcome.

importance of the intermediate option, the majoritarian candidate almost always wins, while with lower importance the winning rate is still very high.

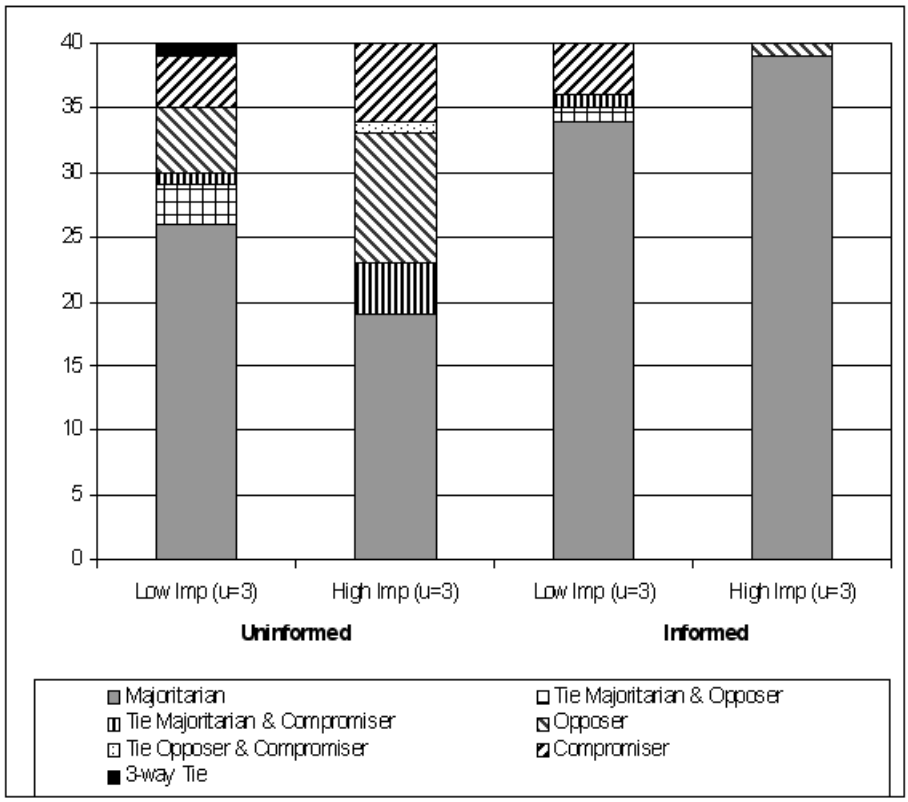


Figure 11: Type of Winner. Number of Elections.

5 Concluding remarks

In this paper I study a voting environment in which Arrow's cycle can easily emerge and voters are faced with the decision of voting sincerely or strategically. Voters know their preference, but may or may not have information about the actual preference distribution. Moreover, the preferences themselves are not the most important, but the relative position of the majoritarian candidate in one's preference ordering.

Summarizing the results from the pilot, all behavioral predictions derived from the logit equilibrium are supported, revealing the Quantal Response model as a good predictor. In terms of precision the Quantal Response is very good at predicting differences in the uninformed treatment and aggregate behavior in the informed treatment. It is not as precise in predicting behavior dissagregated by type in the informed treatment, though it does accurately predict the comparative statics across and within types.

In fact, in terms of comparative statics all the results are as predicted. In accordance with behavioral predictions 1 and 3 I observe more sincere voting when the importance of the intermediate option is lower. Also, in line with behavioral predictions 2 and 4 I observe types acting symmetrically when uninformed and differently when informed. Very notably, in accordance with behavioral prediction 5 compromisers are the type who change their behavior from voting mostly sincerely to mostly strategically when the importance of the intermediate option increases. Finally, information appears to function as a coordination device around the victory of the Majoritarian candidate.

Therefore, answering the main research question, information does seem to matter, coordinating voters around the majoritarian candidate. People indeed vote strategically, but the extent of this decision is guided by the relative importance of the intermediate candidate, which makes voting sincerely more or less attractive. To conclude, the logit specification of the Quantal Response model seems to capture most of the boundedly rational behavior not captured by standard models.

A Appendix: Computation of the Expected Payoff Difference

I denote by $\bar{u}_i(\pi_{-i})$ the expected payoff difference from voting sincerely in comparison to voting strategically, as a function of all other voters' strategies π_{-i}. This computation follows the elaborations by Palfrey (1989) and Fey (1997), adjusting for the finite number of voters and the tie breaking procedure.

The expected payoff difference function uses the fact that the payoff difference is only non-zero in pivotal situations. There are 9 potentially pivotal situations, where a voter either breaks or creates a tie. Define by $p_{(l),(j)}$ the probability that voting sincerely yields outcome l while voting strategically gives j.[12] Define $d_{(l),(j)}$ as the corresponding (expected) payoff difference.[13] I supress the explicit dependence of $p_{(l),(j)}$ on π_{-i} for notational convenience. Therefore, for a

[12] Both l and j can depict single outcomes or, in case of a tie, multiple outcomes.

[13] In case l and j are both single outcomes the payoff difference involves no risk. However if either is a multiple outcome the random draw tie breaking procedure will induce uncertainty. I assume voters are expected utility maximizers and consider the expected payoff of each situation.

voter with preference ordering (A, B, C):

$$\begin{aligned}\bar{u}_i(\pi_{-i}) &= p_{(A),(B)}d_{(A),(B)} + p_{(A),(A,B)}d_{(A),(A,B)} + \\ & p_{(A),(A,C)}d_{(A),(A,C)} + p_{(A),(A,B,C)}d_{(A),(A,B,C)} + \\ & p_{(C),(B,C)}d_{(C),(B,C)} + p_{(A,B),(B)}d_{(A,B),(B)} + \\ & p_{(A,C),(C)}d_{(A,C),(C)} + p_{(B,C),(B)}d_{(B,C),(B)} + \\ & p_{(A,B,C),(B)}d_{(A,B,C),(B)}\end{aligned}$$

Similar expressions can be written for the other preference orderings.

Computation of $d_{(l),(j)}$ is straightforward, since it is just the expected difference from the payoff received by the voter under situation l or j. For example:

$$d_{(A),(A,B)} = u^{best} - \frac{u^{best} + u^{intermediate}}{2} = \frac{u^{best} - u^{intermediate}}{2}$$

On the other hand, computation of $p_{(l),(j)}$ is more laborious. Denote the outcome of the voting of the other $(N-1)$ voters by a vector (h_A, h_B, h_C) such that $h_A + h_B + h_C = N - 1$, where h_k denotes the number of votes for option k from the other $(N-1)$ players. Different outcome vectors might yield the same pivotal situation. For example, both (7,7,0) and (6,6,2) would pose a voter with preference (A, B, C) with the pivotal situation in which sincere voting leads to a victory of A while strategic voting leads to victory of B.

Preference distribution of the other $(N-1)$ players can be denoted by the vector $(M_{A,B,C}, M_{B,C,A}, M_{C,A,B})$ such that $M_{A,B,C} + M_{B,C,A} + M_{C,A,B} = N - 1$. $M_{A,B,C}$ denotes the number of voters with preference ordering (A, B, C) and similarly for $M_{B,C,A}$ and $M_{C,A,B}$. Moreover, denote by superscripts *sincere* and *strategic* the number of voters with that preference ordering voting sincerely and strategically, respectively. Note that $h_A = M^{sincere}_{A,B,C} + M^{strategic}_{C,A,B}$ and similarly for h_B and h_C. Since $M_{A,B,C} = M^{sincere}_{A,B,C} + M^{strategic}_{A,B,C}$ any sincere voting vector $(M^{sincere}_{A,B,C}, M^{sincere}_{B,C,A}, M^{sincere}_{C,A,B})$ will have a correspondent outcome vector (h_A, h_B, h_C). This correspondence is not one to one, however. For example, if the M voters' preferences is given by $(4, 5, 5)$, both sincere voting vectors $(4, 5, 4)$ and $(3, 4, 3)$ correspond to the outcome vector $(5, 5, 4)$, which in turn represents to a voter with preference (A, B, C) the pivotal situation in which sincere voting leads to a victory of A while strategic voting leads to victory of B.

Considering only quasi-symmetric situations[14], denote by $\pi_{A,B,C}$ the probability of sincere voting by a voter with preference ordering (A, B, C) and similarly $\pi_{B,C,A}$ and $\pi_{C,A,B}$. Therefore the probability of observing a specific vector of sincere voting can be writen as a composition of binomial probabilities:

$$\begin{pmatrix} M_{A,B,C} \\ M^{sincere}_{A,B,C} \end{pmatrix} \pi^{M^{sincere}_{A,B,C}}_{A,B,C} (1-\pi_{A,B,C})^{M_{A,B,C}-M^{sincere}_{A,B,C}}$$
$$* \begin{pmatrix} M_{B,C,A} \\ M^{sincere}_{B,C,A} \end{pmatrix} \pi^{M^{sincere}_{B,C,A}}_{B,C,A} (1-\pi_{B,C,A})^{M_{B,C,A}-M^{sincere}_{B,C,A}}$$
$$* \begin{pmatrix} M_{C,A,B} \\ M^{sincere}_{C,A,B} \end{pmatrix} \pi^{M^{sincere}_{C,A,B}}_{A,B,C} (1-\pi_{C,A,B})^{M_{C,A,B}-M^{sincere}_{C,A,B}}$$

Then, each $p_{(l),(j)}$ is composed of the sum of the probabilities of each sincere voting vector that pose the voter with the same corresponding pivotal situation. As it can be noticed by the above exposition, the mapping of sincere voting vectors into outcome vectors depends on the distribution of M. In the case of perfect information the distribution is given. In the case without information for each possible realized distribution of M[15], a separate computation must be made and a weighted average then taken. The probability of a distribution of M is given by the multinomial expression:

$$\left(\frac{1}{3}\right)^{M} \frac{M!}{M_{A,B,C}!M_{B,C,A}!M_{C,A,B}!}$$

Matlab codes of the computations can be made available upon request.

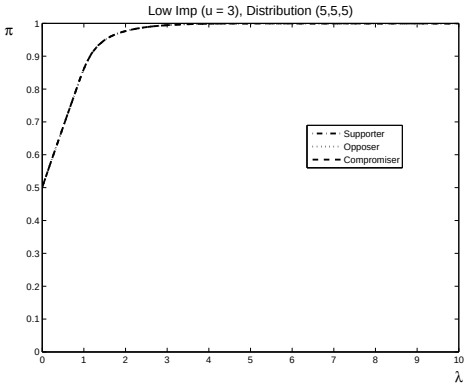
[14] A situation is quasi-symmetric when players of the same type act the same.

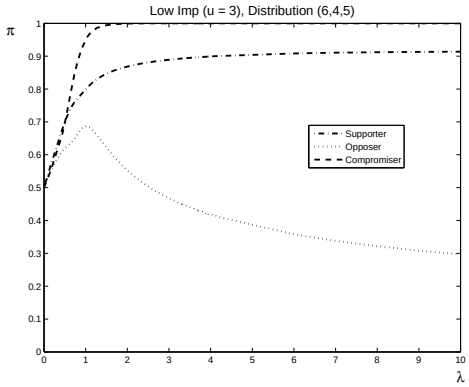
[15] The probability distribution is computed from the perspective of a specific voter who knows his own preference and therefore can update the probability distribution of other voters' preferences.

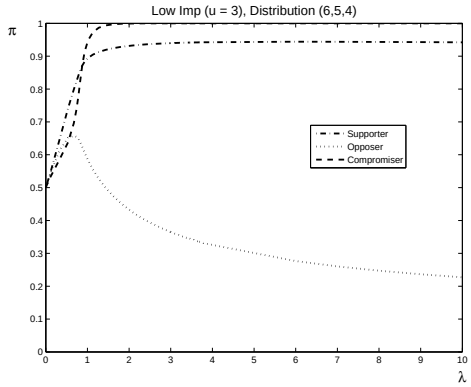
B Appendix: Principal Branch of the Quantal Response Correspondence for the Informed Setting

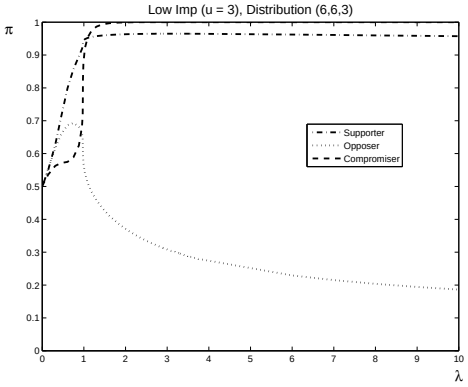
In this appendix I present all graphs of the Principal Branch of the Quantal Response Correspondence from $\lambda \in [0, 10]$, grouped by importance of the intermediate option. The distribution in parentheses indicates the number of Supporters, Opposers and Compromisers.

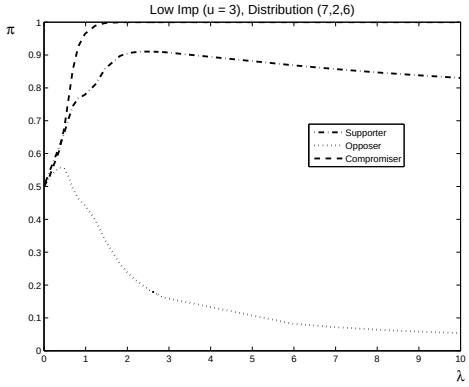
B.1 Low Importance of the Intermediate option (u=3)

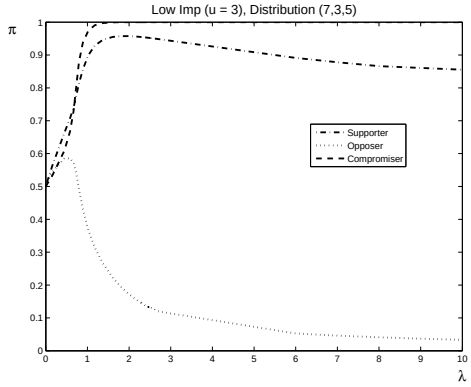


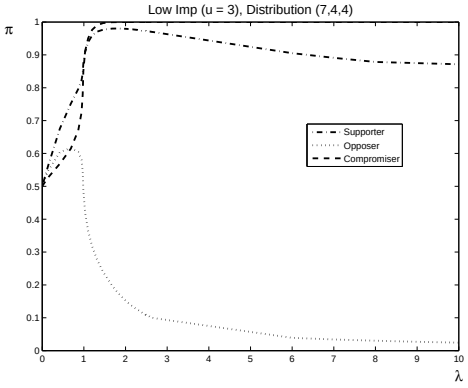


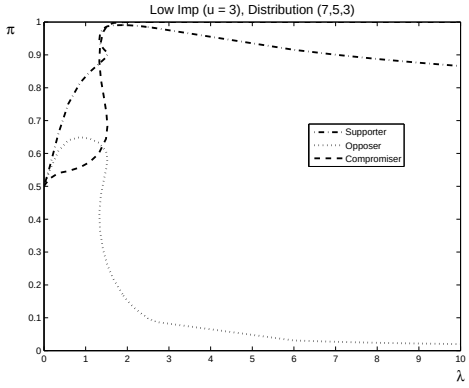


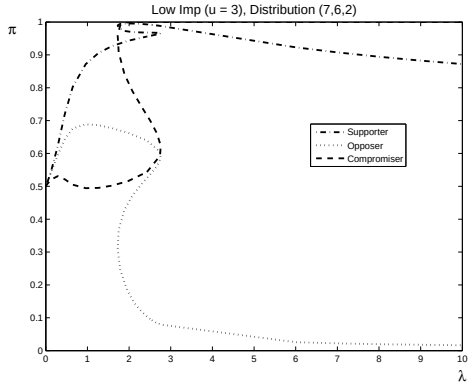


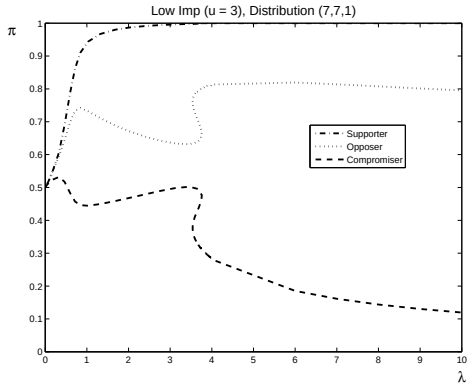


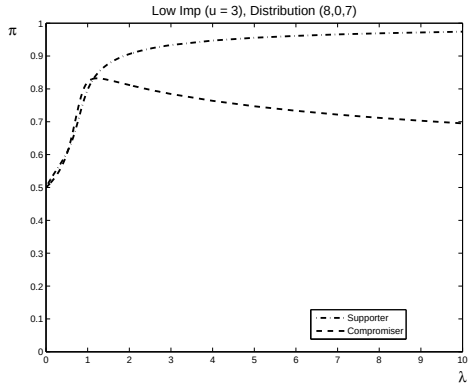


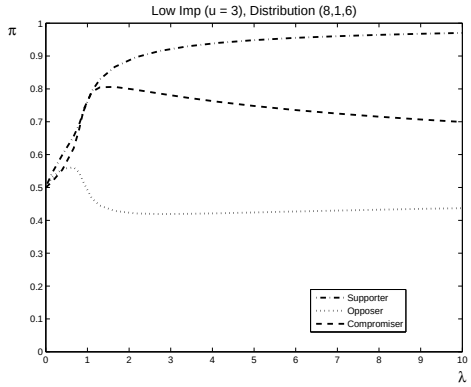


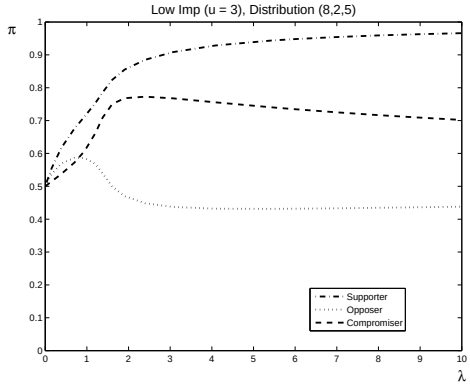


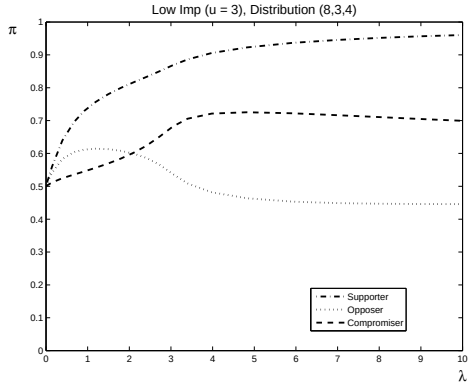


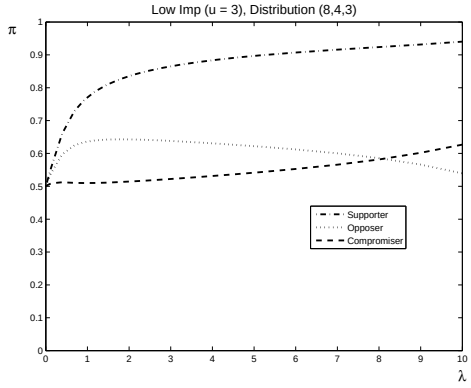

Low Imp (u = 3), Distribution (7,7,1)
π
λ
Supporter
Opposer
Compromiser


Low Imp (u = 3), Distribution (8,0,7)
π
λ
Supporter
Compromiser


Low Imp (u = 3), Distribution (8,1,6)
π
λ
Supporter
Opposer
Compromiser


Low Imp (u = 3), Distribution (8,2,5)
π
λ
Supporter
Opposer
Compromiser


Low Imp (u = 3), Distribution (8,3,4)
π
λ
Supporter
Opposer
Compromiser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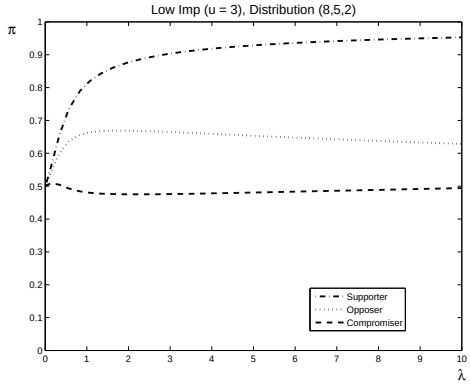
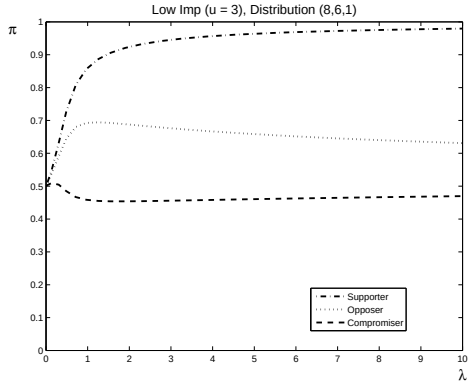
π
λ
Supporter
Opposer
Compromiser


Low Imp (u = 3), Distribution (8,5,2)
π
λ
Supporter
Opposer
Compromiser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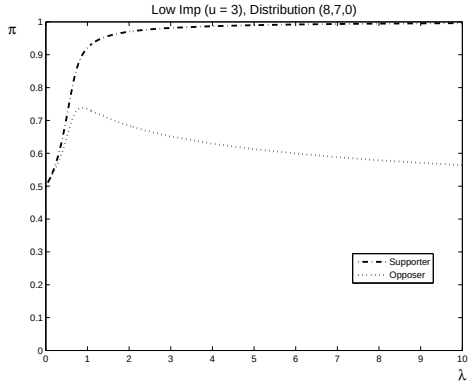
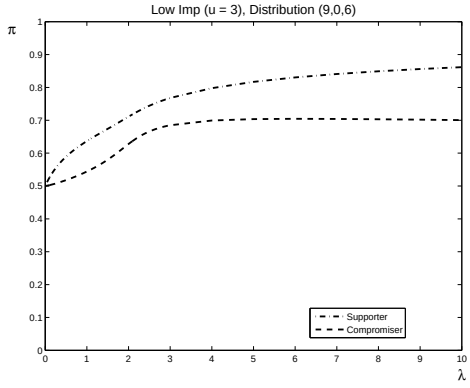
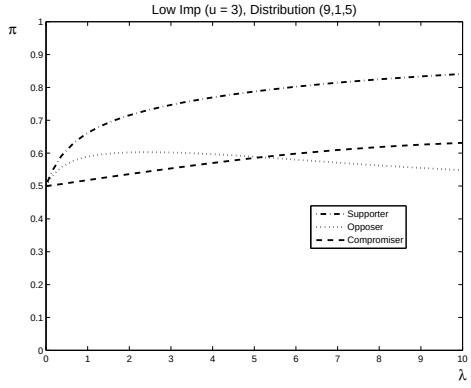
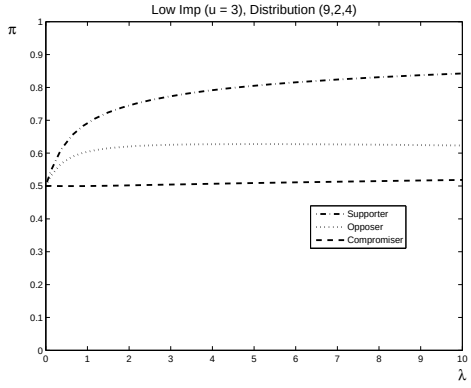
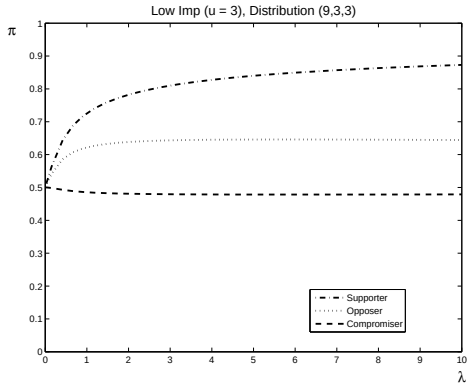
π
λ
Supporter
Opposer
Compromiser

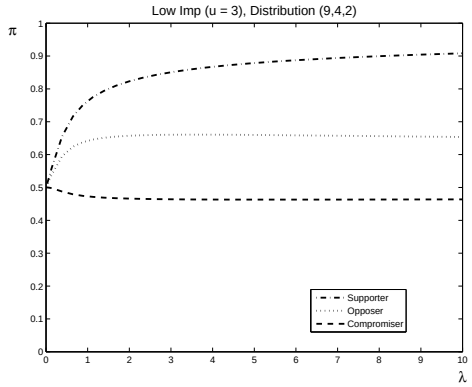

Low Imp (u = 3), Distribution (8,7,0)
π
λ
Supporter
Opposer


Low Imp (u = 3), Distribution (9,0,6)
π
λ
Supporter
Compromiser


Low Imp (u = 3), Distribution (9,1,5)
π
λ
Supporter
Opposer
Compromiser


Low Imp (u = 3), Distribution (9,2,4)
π
λ
Supporter
Opposer
Compromiser


Low Imp (u = 3), Distribution (9,3,3)
π
λ
Supporter
Opposer
Compromiser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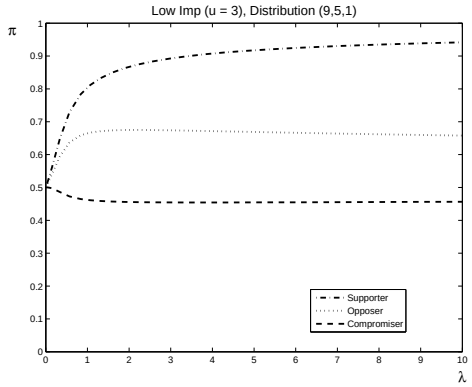
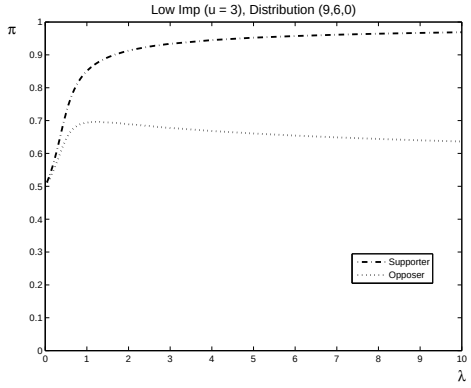
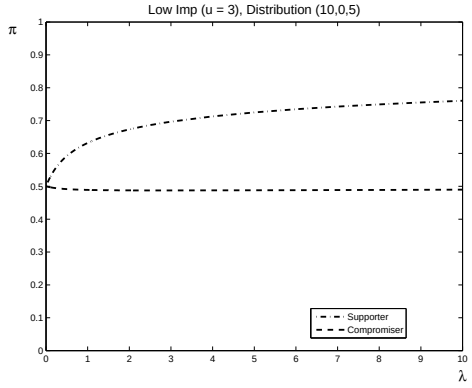
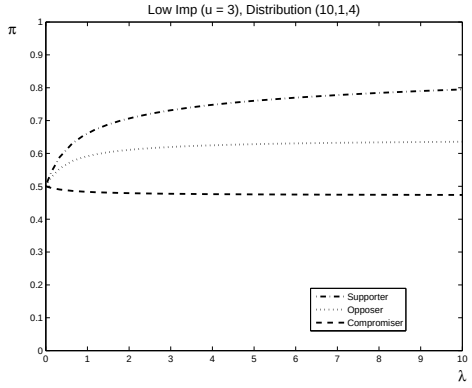
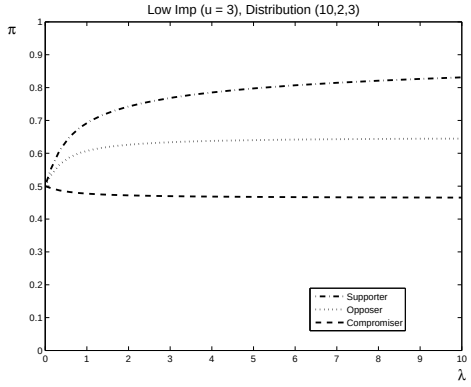
π
λ
Supporter
Opposer
Compromiser

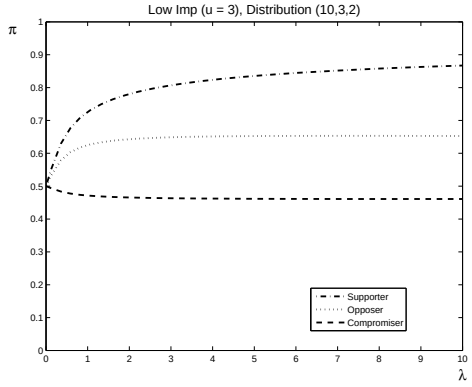

Low Imp (u = 3), Distribution (9,5,1)
π
λ
Supporter
Opposer
Compromiser

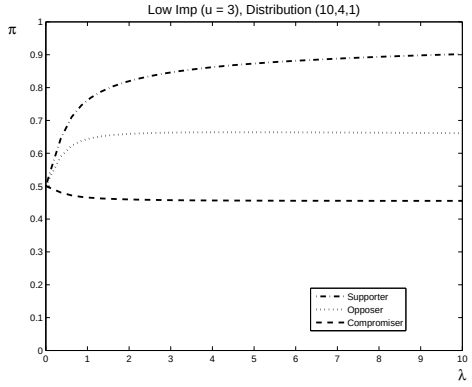

Low Imp (u = 3), Distribution (9,6,0)
π
λ
Supporter
Opposer

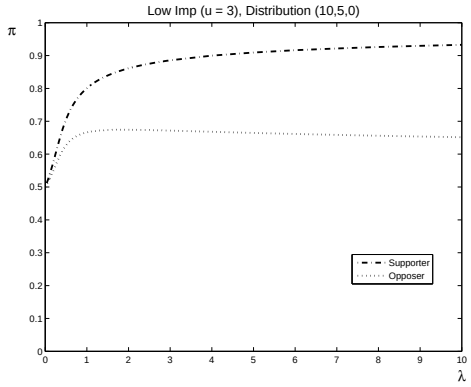

Low Imp (u = 3), Distribution (10,0,5)
π
λ
Supporter
Compromiser

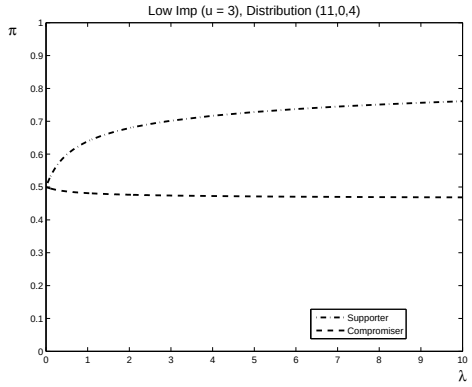

Low Imp (u = 3), Distribution (10,1,4)
π
λ
Supporter
Opposer
Compromiser

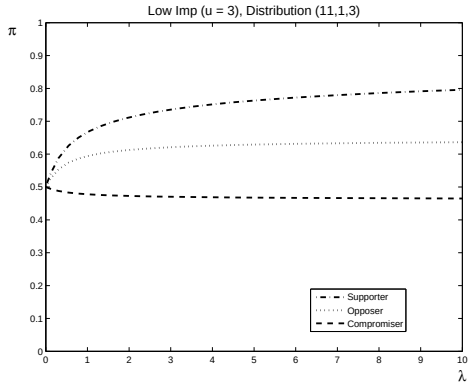

Low Imp (u = 3), Distribution (10,2,3)
π
λ
Supporter
Opposer
Compromiser

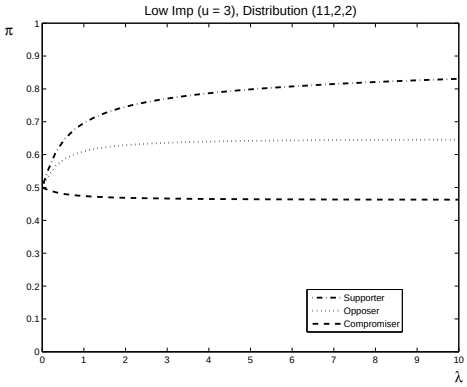

Low Imp (u = 3), Distribution (10,3,2)
π
λ
Supporter
Opposer
Compromiser

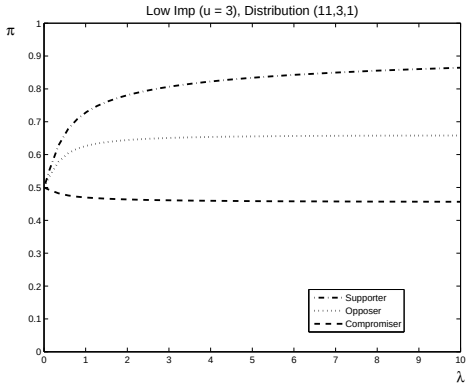

Low Imp (u = 3), Distribution (10,4,1)
π
λ
Supporter
Opposer
Compromiser

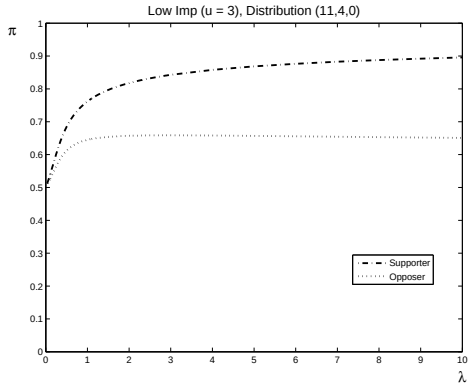

Low Imp (u = 3), Distribution (10,5,0)
π
λ
Supporter
Opposer

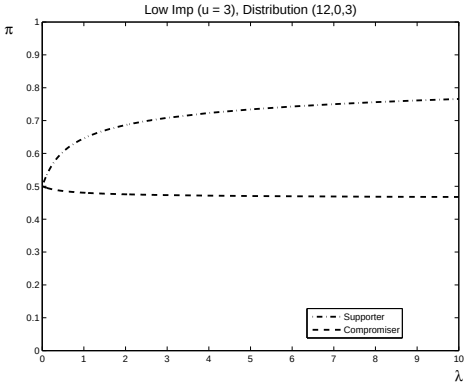

Low Imp (u = 3), Distribution (11,0,4)
π
λ
Supporter
Compromiser

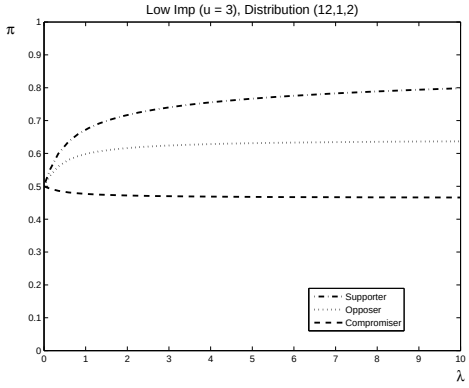

Low Imp (u = 3), Distribution (11,1,3)
π
λ
Supporter
Opposer
Compromiser

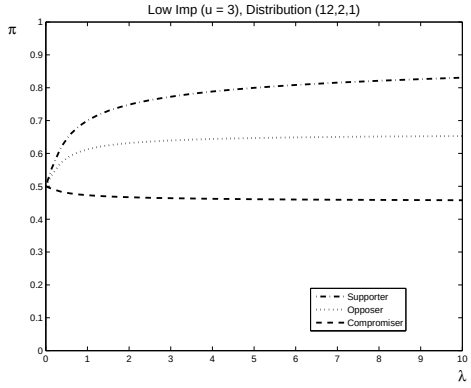

Low Imp (u = 3), Distribution (11,2,2)
π
λ
Supporter
Opposer
Compromiser

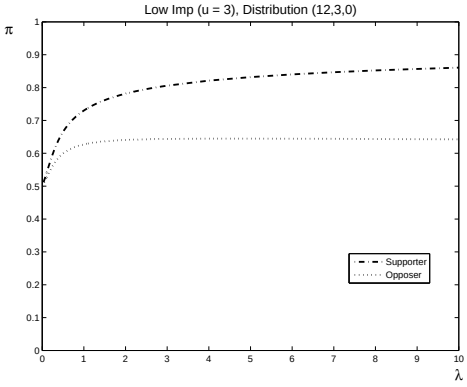

Low Imp (u = 3), Distribution (11,3,1)
π
λ
Supporter
Opposer
Compromiser

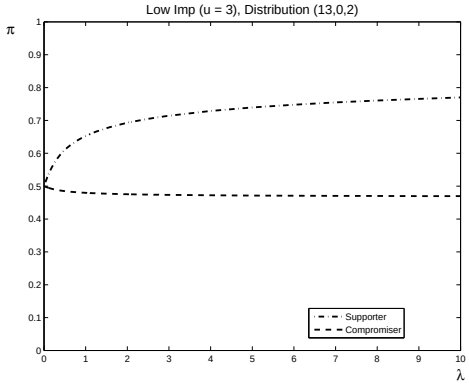

Low Imp (u = 3), Distribution (11,4,0)
π
λ
Supporter
Opposer

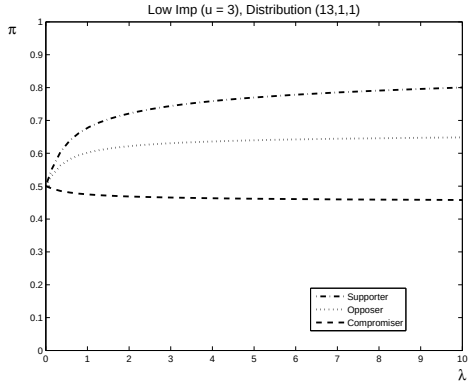

Low Imp (u = 3), Distribution (12,0,3)
π
λ
Supporter
Compromiser

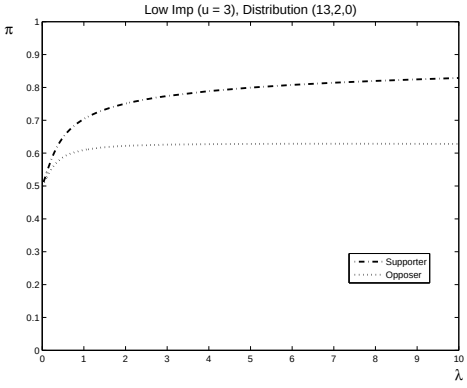

Low Imp (u = 3), Distribution (12,1,2)
π
λ
Supporter
Opposer
Compromiser

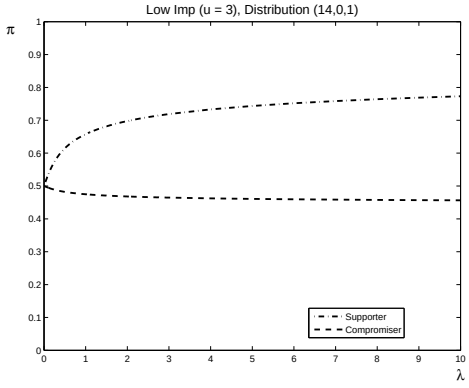

Low Imp (u = 3), Distribution (12,2,1)
π
λ
Supporter
Opposer
Compromiser

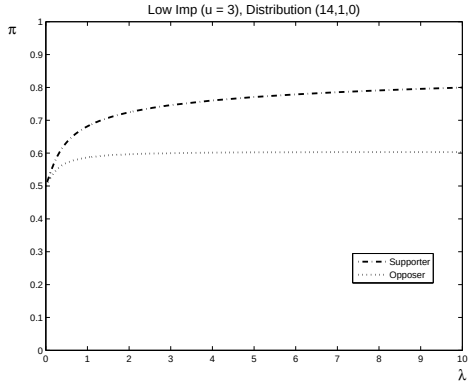

Low Imp (u = 3), Distribution (12,3,0)
π
λ
Supporter
Opposer

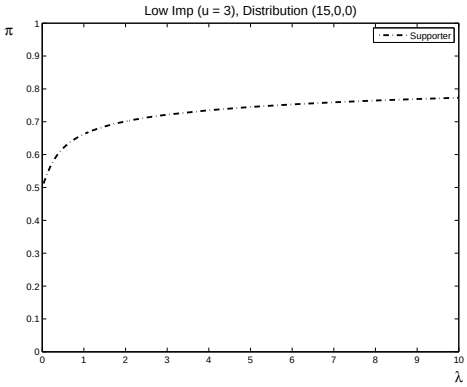

Low Imp (u = 3), Distribution (13,0,2)
π
λ
Supporter
Compromiser


Low Imp (u = 3), Distribution (13,1,1)
π
λ
Supporter
Opposer
Compromiser

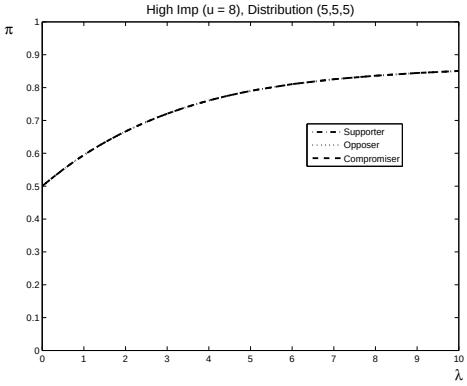

Low Imp (u = 3), Distribution (13,2,0)
π
λ
Supporter
Opposer

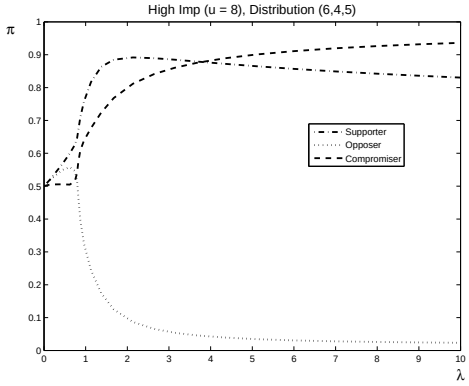

Low Imp (u = 3), Distribution (14,0,1)
π
λ
Supporter
Compromiser

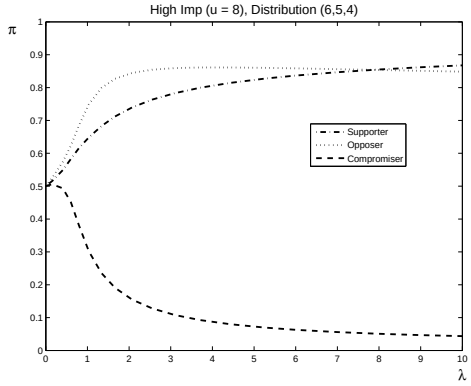

Low Imp (u = 3), Distribution (14,1,0)
π
λ
Supporter
Opposer

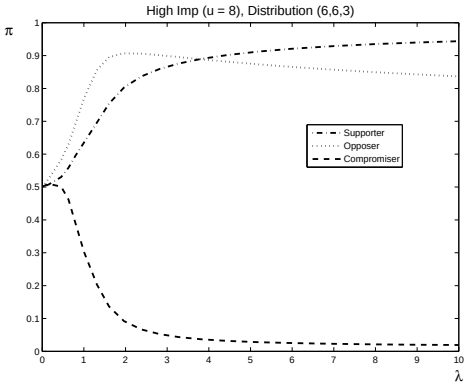

Low Imp (u = 3), Distribution (15,0,0)
π
λ
Supporter
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
1
2
3
4
5
6
7
8
9
10

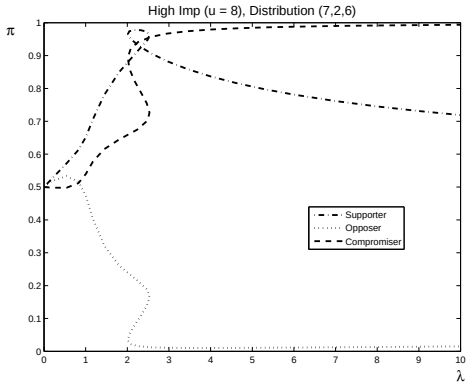
B.2 High Importance of the Intermediate option (u=8)

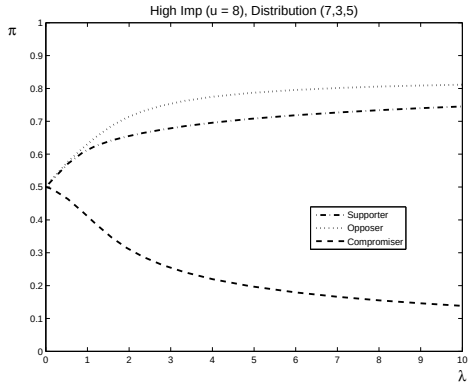


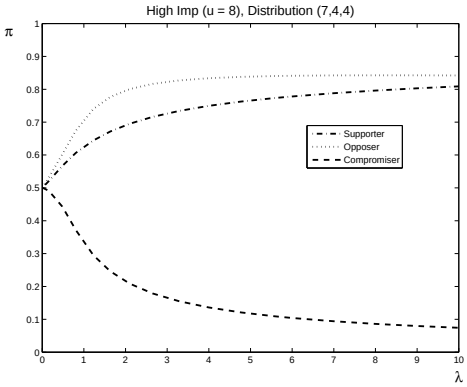


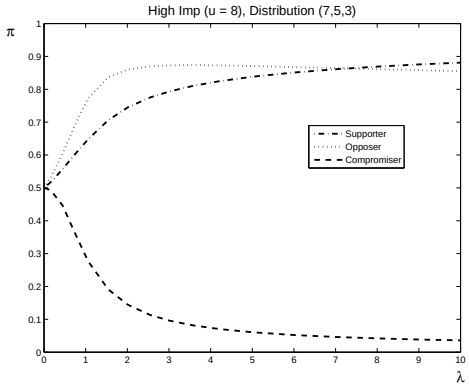


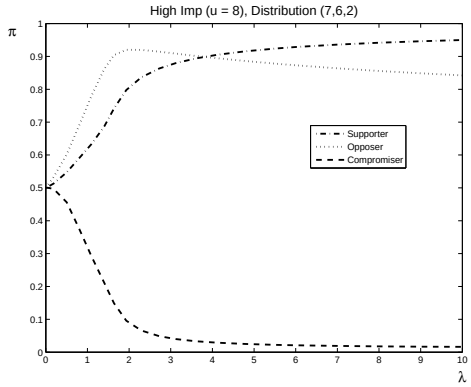


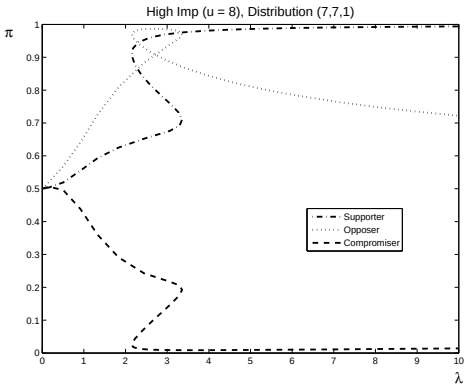


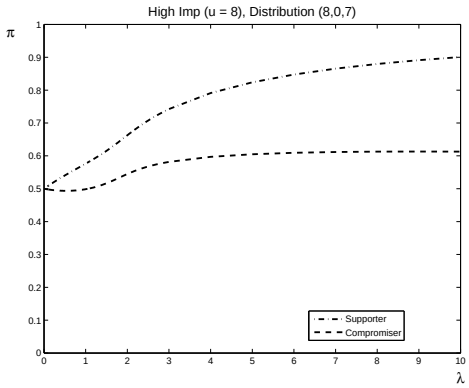


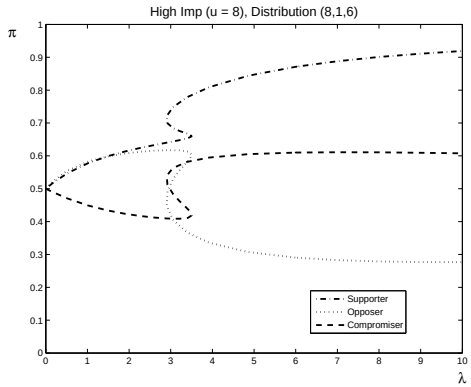


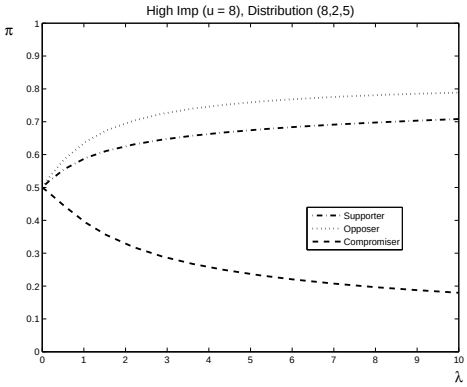


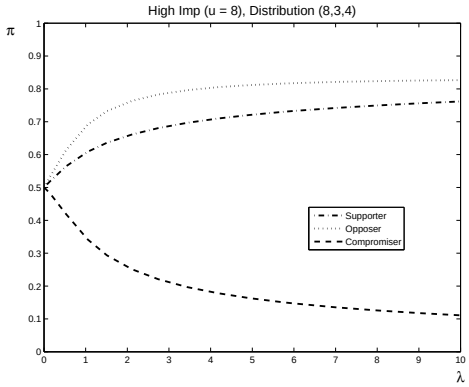


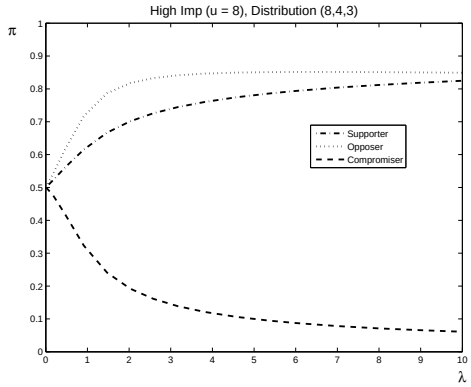


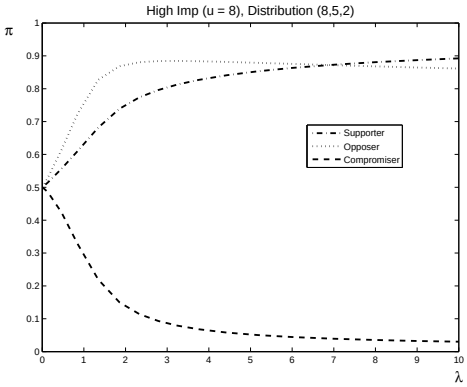


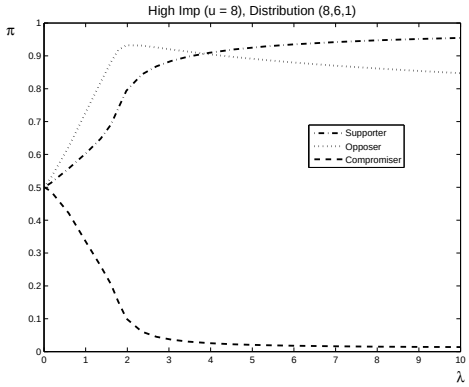


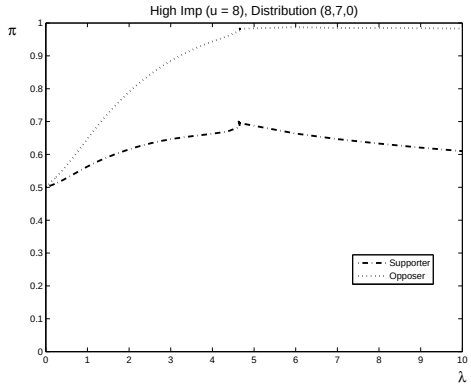

High Imp (u = 8), Distribution (8,2,5)
π
λ
Supporter
Opposer
Compromiser

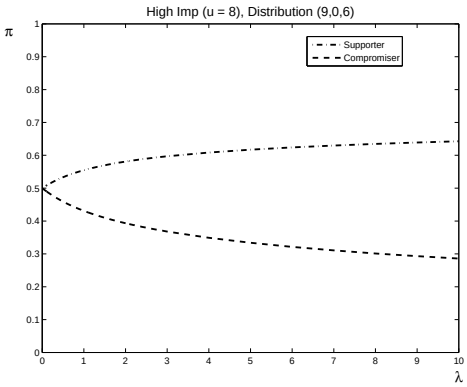

High Imp (u = 8), Distribution (8,3,4)
π
λ
Supporter
Opposer
Compromiser

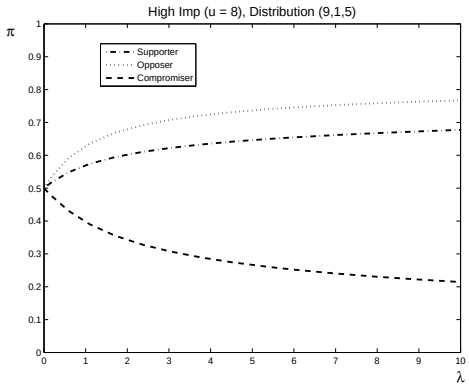

High Imp (u = 8), Distribution (8,4,3)
π
λ
Supporter
Opposer
Compromiser

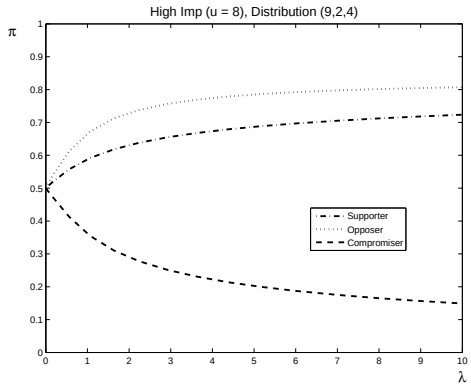

High Imp (u = 8), Distribution (8,5,2)
π
λ
Supporter
Opposer
Compromiser

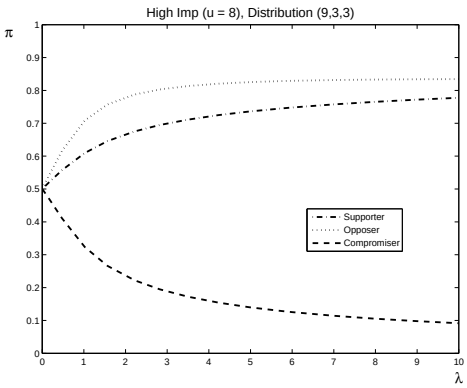

High Imp (u = 8), Distribution (8,6,1)
π
λ
Supporter
Opposer
Compromiser

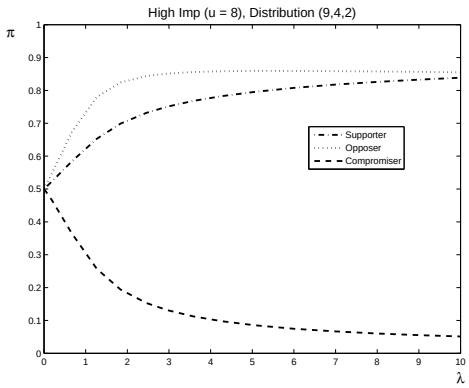

High Imp (u = 8), Distribution (8,7,0)
π
λ
Supporter
Opposer

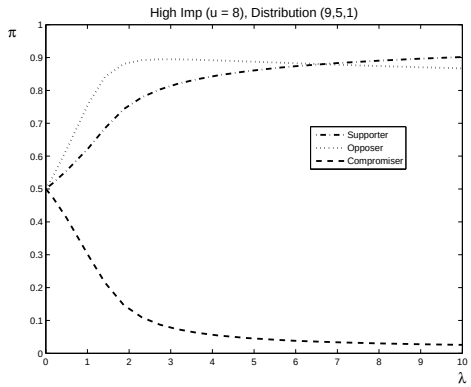

High Imp (u = 8), Distribution (9,0,6)
π
λ
Supporter
Compromiser

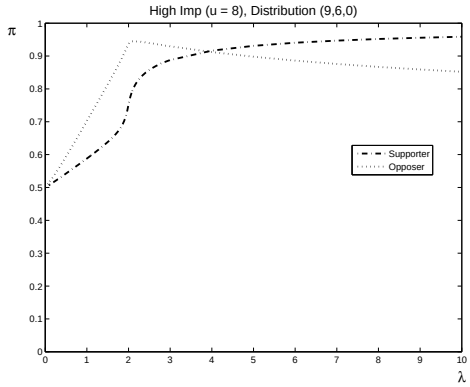

High Imp (u = 8), Distribution (9,1,5)
π
λ
Supporter
Opposer
Compromiser

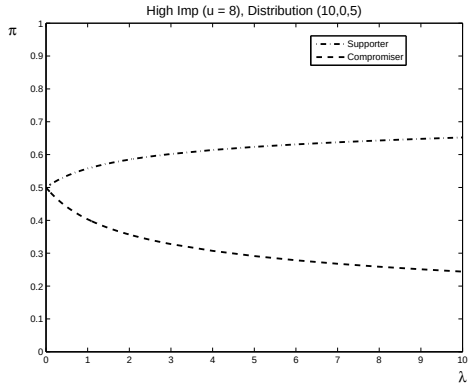

High Imp (u = 8), Distribution (9,2,4)
π
λ
Supporter
Opposer
Compromiser

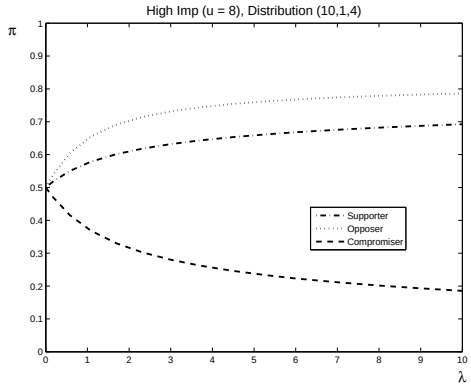

High Imp (u = 8), Distribution (9,3,3)
π
λ
Supporter
Opposer
Compromiser

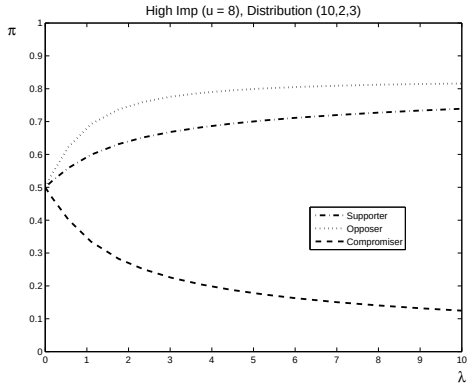

High Imp (u = 8), Distribution (9,4,2)
π
λ
Supporter
Opposer
Compromiser

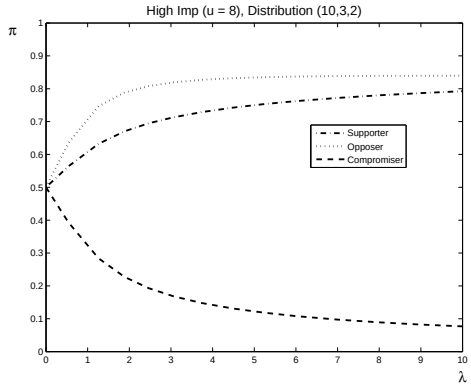

High Imp (u = 8), Distribution (9,5,1)
π
λ
Supporter
Opposer
Compromiser

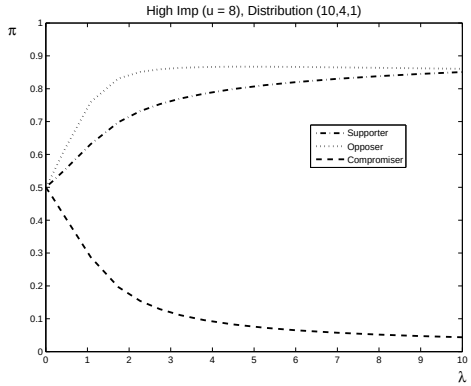

High Imp (u = 8), Distribution (9,6,0)
π
λ
Supporter
Opposer

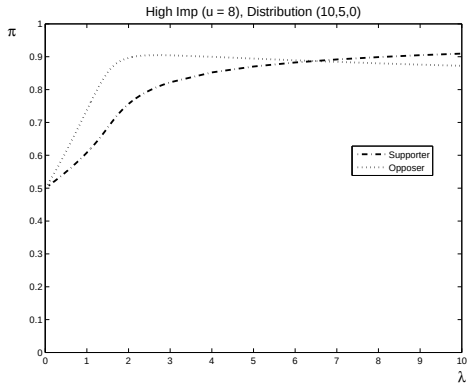

High Imp (u = 8), Distribution (10,0,5)
π
λ
Supporter
Compromiser


High Imp (u = 8), Distribution (10,1,4)
π
λ
Supporter
Opposer
Compromiser


High Imp (u = 8), Distribution (10,2,3)
π
λ
Supporter
Opposer
Compromiser


High Imp (u = 8), Distribution (10,3,2)
π
λ
Supporter
Opposer
Compromiser


High Imp (u = 8), Distribution (10,4,1)
π
λ
Supporter
Opposer
Compromiser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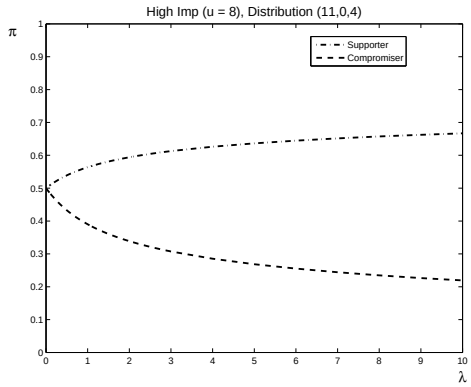
Supporter
Opposer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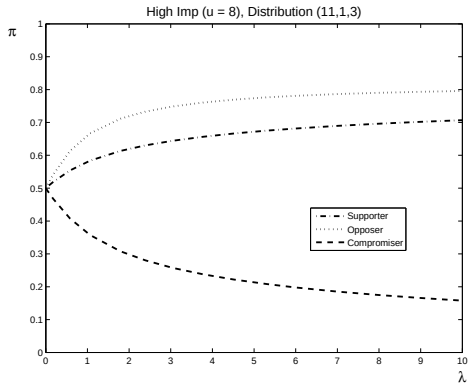
λ
Supporter
Compromiser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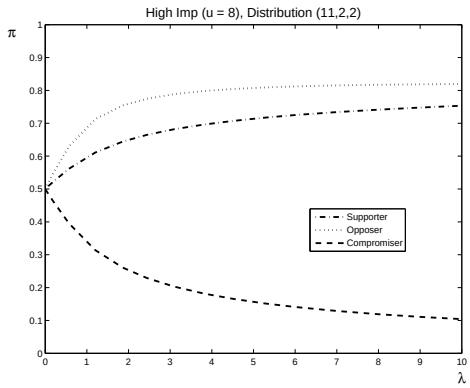
π
λ
Supporter
Opposer
Compromiser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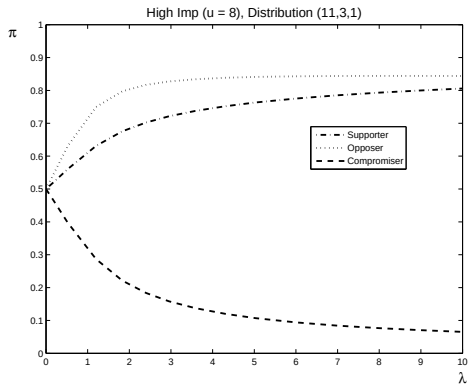
λ
Supporter
Opposer
Compromiser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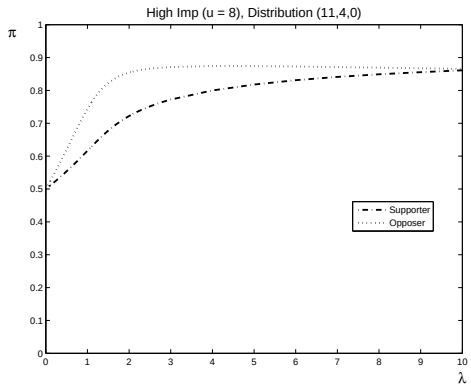
π
λ
Supporter
Opposer
Compromiser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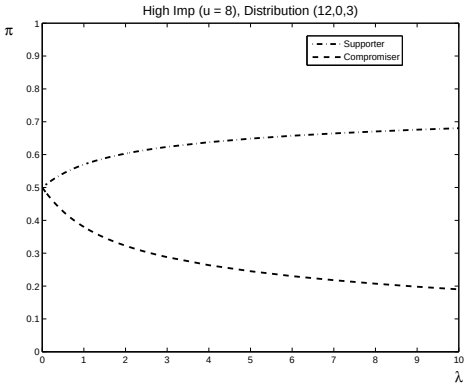
π
λ
Supporter
Opposer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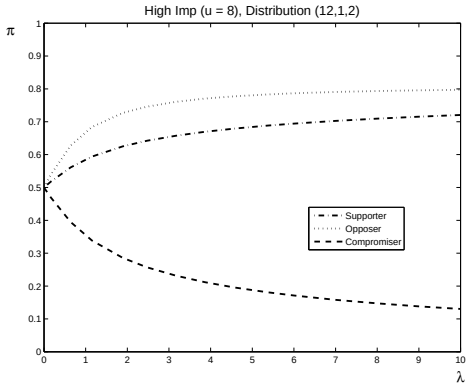
λ
Supporter
Compromiser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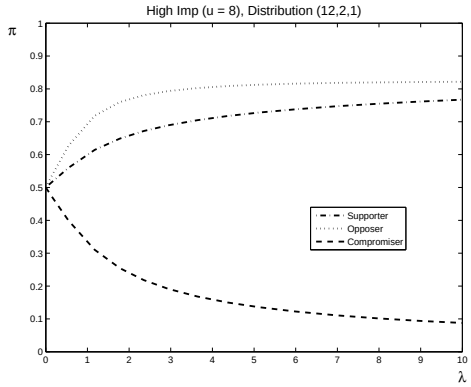
π
λ
Supporter
Opposer
Compromiser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π
λ
Supporter
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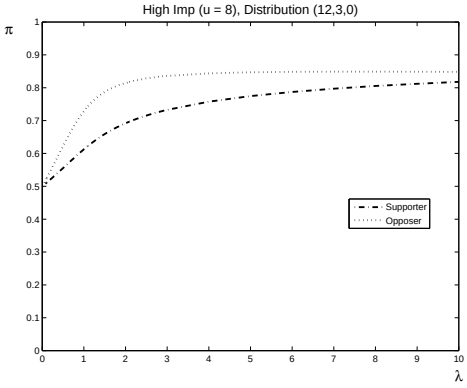
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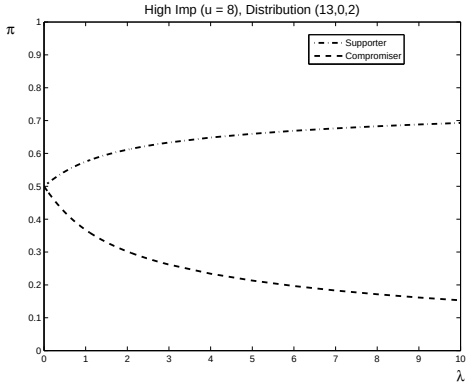
π
λ
Supporter
Opposer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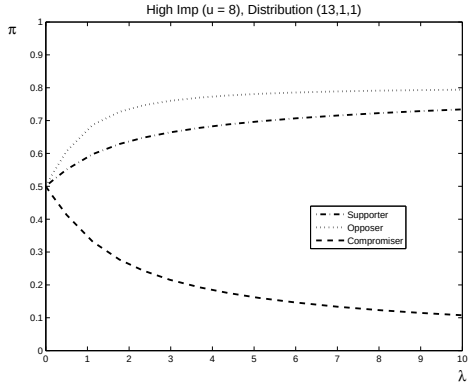
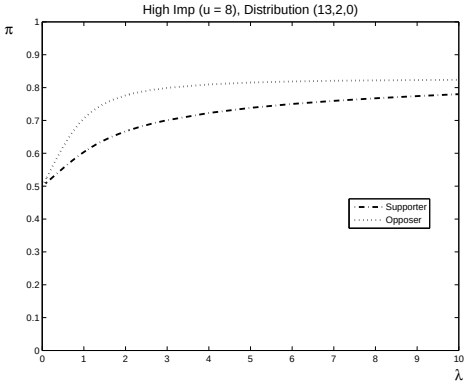
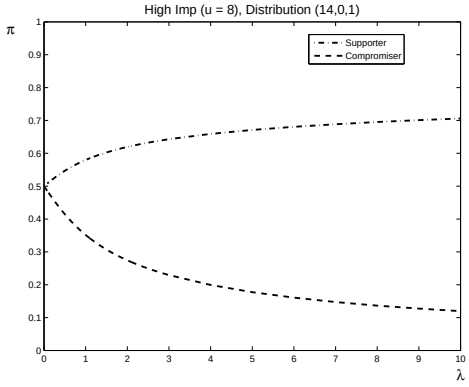
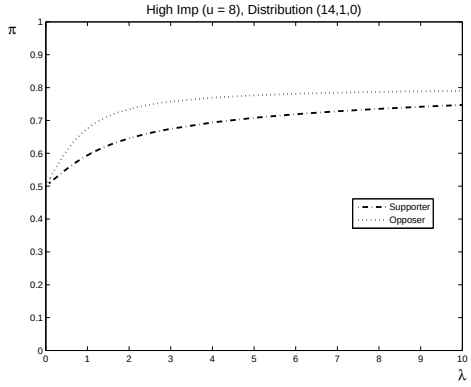
π
λ
Supporter
Compromiser


High Imp (u = 8), Distribution (13,1,1)
π
λ
Supporter
Opposer
Compromiser


High Imp (u = 8), Distribution (13,2,0)
π
λ
Supporter
Opposer


High Imp (u = 8), Distribution (14,0,1)
π
λ
Supporter
Compromiser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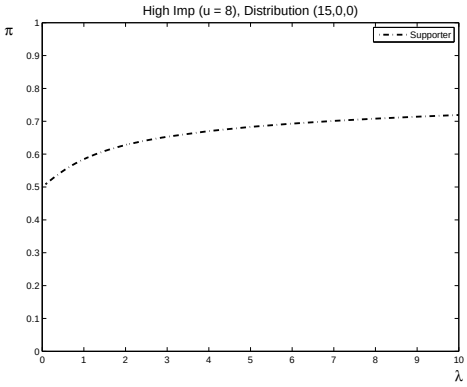
π
λ
Supporter
Opposer


High Imp (u = 8), Distribution (15,0,0)
π
λ
Supporter

C Appendix: Translation of the Experimental Instructions

Here I include the translation of the instructions given to subjects in the sessions with high importance of the intermediate option. In the sessions with low importance of the intermediate option they were the same, with the only difference that the points received in case of victory of the second most preferred candidate was 3 instead of 8. Instructions were shown both on screen and as a printed copy.

Welcome to this experiment on decision making. Read these instructions carefully.
You will also receive a printed copy.

In this experiment you may earn money, which will be paid to you later this week.
Your earnings will be announced privately after the experiment.
Your earning will depend on your decisions as well as on the decisions of your colleagues.

In this experiment one decision will be requested in each round.
The number of rounds will not be revealed.

You decision consists in voting for one of the options: A, B or C.

In each round you will be assigned a preference ordering which will determine your earnings according to the winner of the vote.

Your preference ordering, and the preference ordering of your colleagues,
can be one of the following:
A B C
B C A
C A B

In each round, each of the 3 preference orderings will be attributed
to each person with equal chance.

In case the elected option is the option listed first you will receive 10 points;
in case the elected option is the option listed second you will receive 8 points;
in case the elected option is the option listed last you will receive 1 point.

The electorate consists of 15 people, whose identities will not be revealed.
This electorate will be kept fixed during the whole experiment.

The elected option is the one receiving the highest number of votes.
In case of a tie, one of the options with the highest number of votes
will be randomly selected with equal chance.

In this experiment there are 2 blocks:

In one of the blocks you will not receive any additional information besides your own preference.

In the other block you will receive additional information about the distribution of preferences in that round before you cast your vote.

There is a trial round at the start of the experiment.

You receive R$ 5.00 for your participation.
In addition, each experimental point will be converted to R$ 0.03.

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