Firms, networks and markets: A survey of recent research

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Abstract

This paper presents a survey of recent research on the formation of networks between firms. The paper considers networks in two economic environments. We first consider networks of research collaboration between horizontally related firms. We then study networks between vertically related firms (such as buyers and sellers). We examine the architecture of strategically stable networks and the relation between individual incentives and social welfare in networks.

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1 Introduction

The traditional model of interaction among firms is built on two basic assumptions: anonymity of the individual firm and centralized interaction among the firms. This perspective also informs the way we view markets, as a setting where a set of buyers and sellers can freely interact with each other while their activities are coordinated by prices. There is a large body of empirical work which suggests that interaction among firms is built on a variety of formal and informal relationships. This work has motivated the development of theoretical models of these relationships. In this paper, we survey recent research on this subject.

We shall focus on the recent research on this subject which uses the approach of strategic games of network formation. In this approach, the firms are nodes while the arcs between the nodes are the relationships between firms. We shall study incentives of firms to form pair-wise links with other firms and the implications of these link formation decisions for the overall architecture of the network. We shall also explore the effects of network formation for the functioning of markets and the level of social welfare generated.

The organization of economic activity has been a field of extensive research in economics as well as in sociology. The present survey is quite selective in the material it aims to discuss and it is important that we place the above approach in context. In economics, there are two strands of the literature which are closely related. The first strand we would like to mention is transaction cost economics and the related large literature in contract theory which addresses some of the issues. See e.g., Hart (1995), Nooteboom (1999), and Williamson (1985). The focus here has been on the scope of the market versus the firm. In this context, networks can be thought of as an intermediate level of organization between firms and markets. The second strand is the extensive literature on coalitions; for a survey of this work, see Bloch (1997). Perhaps the distinctive feature of the networks approach is that it allows for non-exclusive relations between players. In the earlier work on coalitions, attention has been restricted to a consideration of exclusive relations, in other words a player can be a member of one and only one coalition. By contrast, networks allow for the following pattern of relations: firms 1 and 2 have a research joint venture, firms 2 and 3 have a research joint venture, while there is no such link between firms 1 and 3. It is important to develop models which allow such collaboration patterns for two reasons: one, because these patterns are observed in practice and two, because these patterns have output, welfare and profit implications which are quite different from the standard models of coalitions. Finally, we would like to mention that there is a substantial literature in economic sociology which addresses issues relating to embeddedness of firms. For an introduction to issues that are central to this line of work, see Granovetter (1985). Perhaps the distinctive feature of the work surveyed here is the individual incentives approach to the emergence of embeddedness.

The present paper presents a survey of the recent research on firms' networks in economics. The rest of the paper is organized as follows. In section 2 we study networks of collaboration among firms that compete in product markets. We start with a brief discussion of the different forms of collaboration observed in practice and also present a summary of the broad empirical patterns. We then study theoretical models of two forms of collaboration: R&D cooperation and market sharing agreements. In section 3, we study buyer-seller networks. We start with a brief discussion of empirical patterns. We then present theoretical models of buyer-seller networks. We also study the trade-offs between networks and vertically integrated firms. Section 4 concludes.

2 Networks of collaboration among horizontally related firms

In this section we will study the incentives of competing firms to enter into collaborative arrangements with each other. We will consider two types of collaboration: R&D collaboration and market sharing agreements. It is useful to first introduce the general framework for analysis.

2.1 Networks: Preliminaries

Let $N = \{1, 2, ..., n\}$ denote a finite set of ex-ante identical firms. We shall assume that $n \geq 3$. For any $i, j \in N$, the pair-wise relationship between the two firms is captured by a binary variable, $g_{i,j} \in \{0, 1\}$; $g_{i,j} = 1$ means that a link is established between firms i and j while $g_{i,j} = 0$ means that no link is formed. A network $g = \{(g_{i,j})_{i,j \in N}\}$, is a formal description of the pair-wise collaboration relationships that exist between the firms. We let $\mathcal{G}$ denote the set of all networks. We note that the set of all undirected networks on n vertices is equivalent to the set $\mathcal{G}$. Let $g + g_{i,j}$ denote the network obtained by replacing $g_{i,j} = 0$ in network g by $g_{i,j} = 1$. Similarly, let $g - g_{i,j}$ denote the network obtained by replacing $g_{i,j} = 1$ in network g by $g_{i,j} = 0$.

A *path* in g connecting firms i and j is a distinct set of firms $\{i_1, \ldots, i_n\}$ such that $g_{i,i_1} = g_{i_1,i_2} = g_{i_2,i_3} = \cdots = g_{i_n,j} = 1$. We say that a network is *connected* if there exists a path between any pair $i, j \in N$. A network, $g' \subset g$, is a *component* of g if for all $i, j \in g'$, $i \neq j$, there exists a path in g' connecting i and j, and for all $i \in g$ and $j \in g$, $g_{i,j} = 1$ implies $g_{i,j} \in g'$. We will say that a component $g' \subset g$ is *complete* if $g_{i,j} = 1$ for all $i, j \in g'$. Finally, let $N_i(g) = \{j \in N \backslash \{i\} | \; g_{i,j} = 1\}$ be the set of firms in $N \backslash \{i\}$ with whom firm i has a link in the network g, and let $\eta_i(g) = |N_i(g)|$. A symmetric network is one in which for every $i \in N$, $\eta_i(g) = k$, with $k \in \{1, 2, .., n\}$. The number k will be referred to as the *degree* of network g. We note that if the number of firms is even, then there always exist a set of links l such that the resulting network is symmetric of degree k, where $k = 0, 1, ..., n - 1$.

We now define some networks that play a prominent role in our analysis. The *complete* network, g^c, is a network in which $g_{i,j} = 1$, $\forall i, j \in N$, while the *empty* network, g^e, is a network in which $g_{i,j} = 0$, $\forall i, j \in N$, $i \neq j$. The *dominant group* architecture, g^{dk}, is characterized by one complete non-singleton component with $k \geq 2$ firms and $n - k$ singleton firms. Thus, there is a set of firms $N' \subset N$ with the property that

$g_{i,j} = 1$ for every pair $i, j \in N'$ while for any $k \in N \backslash N'$, $g_{k,l} = 0$, $\forall\ l \in N \backslash \{k\}$. An *inter-linked* star is defined as follows. Let $h(g) = \{h_1(g), h_2(g), \ldots, h_m(g)\}$ be a partition of the firms with the feature that $\eta_i(g) = \eta_j(g)$, if and only if $i, j \in h_l(g)$ for some $l \in \{1, 2, ..., m\}$. The groups are numbered in ascending order of links. An inter-linked star is a partition with two features: (i) $\eta_i(g) = n - 1$ for all firms $i \in h_m(g)$ and (ii) $\eta_j(g) = |h_m(g)|$, for all firms $j \in h_1(g)$. A firm in the former group is referred to as a *central* firm, while a firm in the latter group is referred to as a *peripheral* firm. The *star* is a special case of this architecture, in which there are only two groups, and $|h_m(g)| = 1$ and $|h_1(g)| = n - 1$.

2.2 R&D Collaboration

Research collaboration among firms takes on a variety of forms. Perhaps the most prominent ones are joint ventures and joint R&D contracts.

1. Joint ventures and research corporations: A combination of at least two separate companies into a 'distinct' company; profits and losses are usually shared according to equity investments.
2. Joint R&D and technology exchange agreements: Agreements regulate technology and R&D sharing and transfer between two or more firms. Joint R&D refers to agreements such as joint research pacts which establish joint undertaking of research projects with shared resources and joint product development agreements. On the other hand, technology exchange agreements cover technology sharing agreements, cross-licensing and mutual second-sourcing.

In a recent paper, Hagedoorn (2002) provides a survey of empirical work on R&D collaboration among firms. This survey covers a period of almost 40 years starting in the 1960's and is a very useful source of information on overall trends. The main findings are as follows. The *first* finding is that there has been a significant increase in the number of R&D collaboration arrangements between firms since the 1960's. This number rose sharply in the 1990's and remains high in recent years. The *second* finding is that the relative importance of joint ventures in this activity has gone down dramatically in this period. In the early years, most collaboration took the form of joint ventures, while in recent years almost 90% of the collaborations take the form of joint R&D and technology sharing agreements. The *third* finding pertains to sectoral pattern of R&D partnerships. Over the years, the share of high technology sectors, such as pharmaceuticals, information technology, aerospace and defence sectors, has increased at the expense of medium technology sectors, such as instrumentation and medical equipment, consumer electronics, and chemicals. In the 1970's, the medium technology sectors accounted for over 50% of the total number of collaborations. By the late 1990's, the share of high technology industries has increased to over 80%. In particular, the information technology sector accounted for over 50% of all collaborations in this period. Similarly, the share of pharmaceuticals has increased steadily and now accounts for about 30% of all R&D collaborations.

These patterns are one motivation for studying incentives of firms to collaborate on research and the implications of these collaborations for the functioning of markets. We will consider the following general model: in an oligopoly, prior to competing in the market, firms have an opportunity to form *pair-wise* collaborative links with other firms. These pair-wise links involve a commitment to share R&D results and thus lead to lower costs of production of the collaborating firms. The interest is in the incentives of firms to form links and the architecture of the resulting collaboration network. Moreover, we also study the resulting network from the perspective of welfare maximization. We start with a model in which the extent of cost reduction from any link is exogenously fixed. Later we will consider a model where firms choose R&D effort levels and this in turn defines the extent of cost reduction due to the formation of a link.

2.2.1 Exogenous Cost Reduction

A collaboration link can be interpreted as an agreement to jointly invest in cost-reducing R&D activity. We will suppose that a collaboration link requires a fixed investment, given by $f > 0$, from each firm. We shall further assume that initially firms are symmetric with zero fixed costs and have identical constant returns-to-scale cost functions.

Collaborations lower marginal costs of production. A network g, therefore, induces a marginal cost vector for the firms which is given by $c(g) = \{c_1(g), c_2(g), ..., c_n(g)\}$. We assume that firm i's marginal cost in network g is a linearly decreasing function of the number of collaboration links. Formally,

$$c_i(g) = \gamma_0 - \gamma\eta_i(g) \ , \quad i \in N. \tag{1}$$

where $\gamma_0 > 0$ represents a firm's marginal cost when it has no links, while $\gamma > 0$ is the cost reduction induced by each collaboration link maintained by a firm. To keep matters simple, we shall assume that $\gamma_0 - (n-1)\gamma > 0$, that is, firms' marginal costs remain positive in every network.

Given this cost vector (1), and the specification of the demand function in the product market, the firms compete in the second stage in the market. For every network g, we assume there is a well-defined Nash equilibrium in the second stage product market game. The profits of firm i, *gross* of the cost of forming links are given by $\pi_i(g)$.

We incorporate the fixed costs of forming links f in the payoffs as follows. Fix a network g. The *net* profit of each firm $i \in N$ is given by $\Pi_i(g) = \pi_i(g) - \eta_i(g)f$. Given a network g, let g_{-i} denote the network in which all of firm i's links are deleted. We can now define a stable network as follows.

Definition 2.1 *Let f be the fixed cost of link formation. A network g is stable if the following conditions hold:*

1. *For $g_{i,j} = 1$, $\pi_i(g) - \pi_i(g - g_{i,j}) \geq f$, $\pi_j(g) - \pi_j(g - g_{i,j}) \geq f$*

2. *For* $g_{i,j} = 0$, $\pi_i(g + g_{i,j}) - \pi_i(g) > f \implies \pi_j(g + g_{i,j}) - \pi_j(g) < f$
3. *For every* $i \in N$, $\pi_i(g) - \eta_i(g)f \geq \pi_i(g_{-i})$.

This notion of stability is based on the notion of pair-wise stability introduced in Jackson and Wolinsky (1996). In words, the first condition requires that, in a stable network, any firm that is linked to another has no incentive to sever the link; the second condition requires that any two firms that are not linked should have no incentive to establish a collaboration link. These two conditions constitute a "marginal" check for stability. The third condition is an "aggregate", or "global", check for stability and requires that a firm should find it preferable to maintain its collaboration links in the network rather than severing them altogether. This condition can be seen as an individual rationality condition for participation in the network.

We study networks of collaboration in the familiar context of a market with homogeneous products and price and quantity competition, respectively. We assume the following linear inverse market demand function:

$$p = \alpha - \sum_{i \in N} q_i, \quad \alpha > 0. \tag{2}$$

We start with the case of Bertrand competition. Given a network g, what are the *gross* payoffs of different firms under Bertrand competition? Standard considerations (exploiting the idea of a finite price grid) allow us to state that there exists an equilibrium, and that in this equilibrium a firm will make strictly positive profits only if it is the unique minimal cost firm in the market. In other words:

$$\pi_i(g) = 0, \text{ if } \; c_i(g) \geq c_j(g), \quad \text{for } i \neq j; \tag{3}$$

$$\pi_i(g) > 0, \text{ if } \; c_i(g) < c_j(g), \forall\, j \neq i. \tag{4}$$

Since g is arbitrary, the above expression allows us to specify the payoffs for all possible networks. In this setting we can ask: What are the stable networks of collaboration in this setting of extreme competition? The following result, due to Goyal and Joshi (2002a), provides a complete answer to this question.

Proposition 2.1 *Suppose there is price competition among the firms. If marginal cost satisfies* (1) *and demand satisfies* (2) *then the empty network,* g^e*, is the unique stable network.*

The intuition behind this result is simple. Suppose g is a non-empty network and that firm i has a link in this network. Then i is either the unique minimum cost firm, in which case its collaborators (of whom there must be at least one) have an incentive to delete their links or, on the other hand, firm i is not the unique minimum cost firm and then it has an incentive to delete its links altogether. Thus a network g in which firm i has a link cannot be stable. It is straightforward to show that the empty network is indeed stable. These arguments are very general; in particular, we do not make use of the linear structure of the demand or the cost function. This suggests that the absence of collaborative links is likely to obtain in very competitive markets.

We next examine the case of Cournot competition. Given any network g, the Cournot equilibrium output can be written as:

$$q_i(g) \;=\; \frac{(\alpha-\gamma_0)+n\gamma\eta_i(g)-\gamma\sum_{j\neq i}\eta_j(g)}{(n+1)}, \quad i \in N. \tag{5}$$

In order to ensure that each firm produces a strictly positive quantity in equilibrium, we will assume that $(\alpha-\gamma_0)-(n-1)(n-2)\gamma > 0$. The second stage Cournot gross profits for firm $i \in N$ are given by $\pi_i(g) = q_i^2(g)$. The following result, due to Goyal and Joshi (2002a), offers a complete characterization of stable networks.

Proposition 2.2 *Suppose that marginal cost satisfies (1), demand satisfies (2) and that firms compete in quantities. Then a stable network has the dominant group architecture, in which the size of the dominant group varies with the cost of forming links.*

We outline the steps in the proof of this result. The first step shows that (gross) marginal profits of a firm are increasing with respect to the number of links it has in a network. The second step exploits this property to show that if two firms have any links at all in any stable network, then they must also be linked with each other. This property immediately tells us that if a stable network is symmetric then it must be either empty or complete. If the network is asymmetric, then it can have at most one non-singleton component and this component must have the dominant group architecture.

We now make some remarks relating to the above result. *First,* note that the cost of forming collaboration links has a significant impact on the structure of the collaboration network. In particular, for low costs, the complete network is uniquely stable, for moderate costs only networks with relatively large dominant groups are stable, for high costs, only medium size dominant groups are stable (small and large groups are not sustainable), while for very high costs, the empty network is uniquely stable. Hence, the effect of R&D costs on the size of the dominant group is *non-monotonic.* The intuition for this pattern is as follows: when costs are low, the incentive constraint of the isolated firm to form a link is binding. The marginal payoff to an isolated firm from an additional link is declining in the size of the dominant group. Hence, as the costs of forming R&D collaboration links increase, smaller groups are sufficient to discourage the isolated firm from forming a link. However, beyond a certain cost level, the incentive constraint for a firm in the dominant group to retain its links is binding. The returns from links to a firm in a dominant group are non-monotonic in the size of the dominant group: they are increasing for group sizes until some critical value k^*, and then declining. This implies that for high cost levels, small and large dominant groups are not stable.

The property of increasing returns noted above in the context of Cournot competition suggests that firms with many links may have an incentive to make money transfers to firms who are poorly linked to induce them to form links. These considerations motivate an analysis of the nature of stable networks when transfers are allowed across firms.

We need some additional notation to present the results here. Let $t_i = \{t_i^1, ..., t_i^n\}$ be the transfers offered by firm i to other firms. We shall suppose that $t_i^j \geq 0$, for all $i, j \in N$, and that $t_i^i = 0$, for all $i \in N$. We modify the concept of stability above to accommodate the possibility of transfers as follows.

Definition 2.2 *A network g is stable against transfers if:*

1. *For all $g_{i,j} = 1$, $[\pi_i(g) - \pi_i(g - g_{ij})] + [\pi_j(g) - \pi_j(g - g_{i,j})] > 2f$.*
2. *For all $g_{i,j} = 0$, $[\pi_i(g + g_{i,j}) - \pi_i(g)] + [\pi_j(g + g_{i,j}) - \pi_j(g)] < 2f$.*
3. *There exist transfers $t_i \in R^n$, $i = 1, 2, ..., n$ such that*

$$\pi_i(g) - \eta_i(g)f + \sum_{j \in N_i(g)} (t_j^i - t_i^j) \geq \pi_i(g_{-i}). \tag{6}$$

Condition 1 requires any existing collaboration in a stable network to yield joint gains to the collaborators that exceed the joint cost of maintaining such collaboration. Likewise, condition 2 requires that if a link between a pair of firms does not exist in a stable network, this link must yields joint gains to the parties that are lower than the total cost of forming such link. Finally, condition 3 requires that a firm payoff net of transfers and link costs is larger than what this firm would make in isolation. The following result due to Goyal and Joshi (2002a), characterizes the set of networks that are stable with respect to transfers.

Proposition 2.3 *Suppose that marginal cost satisfies (1), demand satisfies (2) and that firms compete in quantities. Suppose g is stable against transfers. (i). If g is symmetric then it must be either empty or complete. (ii) If g is asymmetric and connected then it is an inter-linked star. (iii) If g is asymmetric and unconnected then it has at most one non-singleton component. This component is either an inter-linked star or it is complete.*

Part (i) of the proof follows directly from the increasing returns property derived above. Part (ii) uses the following implication of increasing returns: suppose that g is stable with respect to transfers. If firm i has a link with firm j then it must also have a link with every firm k which has as many links as j in the network $g - g_{i,j}$. This property provides a simple "marginal" check for stability against transfers by examining the incentive of two firms in a network g to be jointly better off by forming a link. The proof now proceeds as follows: take some firm i which is minimally linked. It must have at least one link, since the network is connected. Note next that this firm must only be linked to firms in the maximally linked group. Suppose it is linked with some firm j which is not maximally linked. Then from the above property it follows that firm j is linked to all firms, and thus is maximally linked, a contradiction. Thus a minimally linked firm is linked to the maximally linked firms only. Moreover, from the above property, it follows that this maximally linked firm is linked to all firms. Thus the network is an inter-linked star.

We now turn to part (iii), asymmetric networks which are unconnected. We start by noting another implication of increasing returns from links: there can be at most one non-singleton component in a network which is stable with respect to transfers. What is the architecture of this non-singleton component? A straightforward adoption of the arguments behind Proposition on inter-linked stars above, yields us the fact that if the non-singleton component is incomplete then it must be an inter-linked star. This leaves out only one possibility: the complete component. But an unconnected network with one complete non-singleton component is simply the dominant group network.

The property of increasing returns implies that if a link is formed between a firm in the dominant group and an isolated firm, then the former gains more from the link as compared to the latter. This suggests that the firm in the dominant group has an incentive to subsidize the formation of the link. Thus we should expect that transfers will sometimes eliminate some dominant groups that are stable in the absence of transfers.

A special case of inter-linked stars is the star network and it can be shown that the star is stable once transfers are allowed. The following result, due to Goyal and Joshi (2002a) derives the conditions on the parameters under which the star is stable with respect to transfers.

Proposition 2.4 *Let $n \geq 4$. Suppose that marginal cost satisfies (1), demand satisfies (2) and that firms compete in quantities. Then there exist F_H and F_L, where $0 < F_L < F_H$, such that the star architecture is stable against transfers if and only if $F_L < f < F_H$.*

We note that the peripheral firms are symmetric above and yet they do not form a link. Given the property of increasing returns, it follows that the returns from linking with the central firm must be less than the gross rewards from doing so. Hence every link in a star network is subsidized by the central firm. Moreover, in the construction of the transfers the peripheral firms are forced to their outside option, which is to be without a link and therefore isolated. One implication of this is that the profits of the central firm are strictly larger than the profits of the peripheral firms. This follows from noting, first, that from the stability of the star it follows that the central firm is better off in the star network (and the corresponding transfers) as compared to the empty network, and, second, that $\pi_i(g^s) - f + t_n^i = \pi_i(g^s - g_{n,i}) < \pi_n(g^e)$, where i is a 'peripheral' firm and n is the central firm.

The above results also suggests that non-empty networks under price competition are possible once transfers are allowed. The construction in the above result suggests that a star can also be sustained under price competition, with the central firm subsidizing each of the peripheral firms exactly to compensate them for the cost of forming links. This argument illustrates the role of transfers in generating market power; recall that in the absence of transfers the unique stable network under price competition is the empty network.

We next compare individual incentives with socially desirable outcomes. For any network g, aggregate welfare, $W(g)$, is defined as the sum of consumer surplus and aggregate profits of the n firms. We shall define the social welfare (gross of costs of forming links) from a network g as follows:

$$W(g) \;=\; \frac{1}{2}Q^2(g) \;+\; \sum_{i \in N} q_i^2(g). \tag{7}$$

We shall say that a network g^* is *efficient* if $W(g^*) \geq W(g)$, for all $g \in \mathcal{G}$.

In this discussion, we will restrict ourselves to low costs of forming links. To the best of our knowledge, characterization of efficient networks for general cost of forming links is still an open problem.

We first examine the architecture of efficient networks under price competition. Let $\underline{c}$ be the minimum cost attainable by a firm in any network; this is achieved when a firm has $(n-1)$ links. The following result, due to Goyal and Joshi (2002a), provides a complete characterization of efficient networks under price competition, when the costs of forming links are negligible.

Proposition 2.5 *Suppose that there is price competition among the firms. If marginal cost satisfies* (1) *and demand satisfies* (2) *then an efficient network g is an inter-linked star, in which there are two central firms while the others have just two links each.*

The arguments underlying this result are as follows. The first step is to show that social welfare is decreasing in the cost level of the minimum cost firm. Thus it is maximized when some firm attains minimum costs, with $(n-1)$ links. If there is unique firm with these costs then this firm will certainly charge a price above its costs to make profits and it can be shown that this will lower social welfare. Hence social welfare is maximized if two or more firms have minimum possible costs. The final step is to note that since there are small but positive costs to forming links, a network with exactly two firms having $(n-1)$ links is uniquely optimal. We note that the optimal network corresponds to an inter-linked star architecture with 2 centers.

We next consider the nature of efficient networks under quantity competition. Let $c(k)$ denote the marginal cost of a firm with k links. To ensure that all firms produce a strictly positive output in the Cournot equilibrium corresponding to any network, we will maintain the restriction that $\alpha - nc(0) + (n-1)c(n-2) > 0$. The following result, due to Goyal and Joshi (2002a), provides a complete characterization of efficient networks under quantity competition, when the costs of forming links are negligible.

Proposition 2.6 *Suppose there is quantity competition among the firms and the costs of forming links are negligible. If marginal cost satisfies* (1) *and demand satisfies* (2) *then the complete network is the unique efficient network.*

The proof of this result consists of three inter-related arguments. First, take an arbitrary network g, the corresponding cost vector $\{c_1(g), c_2(g), ..., c_n(g)\}$ and the Cournot equilibrium output, $q_i(g)$, for $i \in N$. Note that if we now link up every pair of firms but keep the level of output for each firm as under g, then the aggregate profits will increase. Second, share the aggregate output $Q(g) = \sum q_i(g)$ equally among the n firms. Since aggregate output is unchanged, consumers surplus is unchanged. Similarly, since every firm has the same costs, the aggregate profits are also unchanged as compared to the situation in step 1. Therefore aggregate welfare is unchanged as a result of the reallocation of the output $Q(g)$. The third step notes that aggregate output for network g is given by $[n\alpha - \sum c_i(g)]/(n+1)$ and so it is maximized in the complete network. Clearly, then consumers surplus is maximized in the complete network as well. It then follows that the complete network maximizes aggregate welfare.

We now turn to a discussion of some other issues that have been explored in this literature. *First*, we discuss the issue of link formation. In the model discussed above we have assumed that a link is only formed if both firms agree to make investments in forming a link. This seems like a reasonable assumption in the context of R&D collaboration. In some contexts, however, it is possible that a firm can learn from other competing firms by making unilateral investments. A model of networks in oligopoly with one-sided links has been explored in Billand and Bravard (2002). They cite examples of economic intelligence and patent citations to motivate the idea of one-sided links. They consider a set-up which is similar to the model presented above except for this difference in link formation. Given that links are one-sided, the network formation process is a non-cooperative game of link formation and can be solved using the notion of Nash equilibrium. The following results, due to Billand and Bravard (2002), characterize equilibrium networks under price and quantity competition, respectively.

Proposition 2.7 *Suppose link formation is one-sided. Assume (1) and (2) hold and firms compete in prices. Then, there is a number F such that if $f < F$ the star network in which the centre invests in links with all other firms is the unique equilibrium network, while if $F < f$ then the empty network is the unique equilibrium.*

The intuition for this result is quite straightforward: under Bertrand competition, a firm only wants to invest in link formation if it is the unique minimum cost firm. Thus the only possible equilibrium networks are the empty network and the star in which the central firm forms links with all other firms. The result then follows.

Proposition 2.8 *Suppose link formation is one-sided. Assume (1) and (2) hold and firms compete in quantities. There exist numbers f_1 and f_2 with $f_1 < f_2$ such that if $f < f_1$ then the complete network is the unique equilibrium network, if $f_1 < f < f_2$*

then a network in which some firms access all firms while some firms access no other firms is the unique equilibrium, while if $f > f_2$ then the empty network is the unique equilibrium.

The intuition for this result is as follows. First, note that when a firm forms a link which is one-sided it does not alter the costs of the other firms. Second, note that, under quantity competition, the returns from link formation to a firm are increasing in its own links and decreasing in the number of links of the other firms. The first observation implies that a firm will either form links with all other firms or form no links at all. When costs are low enough every firm will link with every other firm, yielding the complete (directed) network. On the other hand, if the costs are very high then clearly no firm wishes to form any links and this yields the empty network. In the intermediate cost case, we exploit the second observation to note that the market cannot support all firms linking with every other firm. This leads to asymmetric structures with some firms linked with all firms while others do not form any links at all.

The above results have been obtained in the context of very specific models with linear cost reduction and demand. In a recent paper, Goyal and Joshi (2002b) study general network formation games and show that in any network game, where the marginal returns from links to a player are increasing in the number of her links, a stable network has the dominant group architecture. This brings us to the issue of network formation process. The notion of pair-wise stability we have used is local and we do not have an explicit model of network formation. There are different ways to deal with this issue. One possibility is to have a model of proposals where a link is formed only when both involved players propose a link to the other player. For a discussion on this approach see Goyal and Joshi (2002a). A second possibility is to have a dynamic model of network formation with forward looking players; this is an open area for research. A third issue concerns the incentives of firms to write contracts which impose exclusive relationships; see Fella and Goyal (2002) for an attempt in this direction.

2.2.2 Endogenous cost reduction

In the model above, we assumed that every link induces a fixed exogenous level of cost reduction for the two firms involved. This corresponds to a situation where each firm commits a fixed amount of resources to the collaboration project. In many cases it seems that efforts of firms are not verifiable and this leads us to examine the influence of the network on incentives to do research. We consider the following three stage game. In the first stage, firms form pair-wise collaboration links. In the second stage, each firm chooses a level of costly R&D effort. This effort also helps reduce the costs of other firms with whom this firm collaborates. In stage three, the firms operate in the market, taking as given the costs of production.

As before, we are interested in the incentives to collaborate and the architecture of the resulting network. We also wish to examine the role of the type of market competition in shaping the networks of collaboration. We will first examine the case of weak market rivalry. As an extreme case of this, we will consider the case of independent producers. We will then examine the case of markets with strong rivalry; here we will consider a homogeneous product oligopoly market with quantity-setting firms. We will also examine the effects of collaboration links on the nature of market outcomes. Finally, we shall compare stable collaboration networks with efficient networks.

Given a network g, every firm chooses a R&D effort level $e_i(g)$. We assume that firms are initially symmetric, with zero fixed costs and identical constant marginal costs $\overline{c}$. Given a network g and the collection of effort levels $\{e_i(g)\}_{i \in N}$ the cost of firm i is given as follows:

$$c_i(\{e_i(g)\}_{i \in N}) = \overline{c} - \sum_{k \in N_i(g)} e_k - \sum_{l \notin N_i(g)} e_l \tag{8}$$

We shall assume that R&D effort is costly. Given a level e_i of effort, the cost of effort is $Z(e_i) = \gamma e_i^2$, $\gamma > 0$. Under this specification, the cost of R&D effort is an increasing function and exhibits decreasing returns. The parameter γ measures the curvature of the cost of effort function. Throughout, we shall assume γ to be sufficiently large so that the firms' decision problems have interior solutions. It is worth noting that in the above formulation, every firm chooses a single level of effort and this is shared with every collaborator. In some settings, it may be more natural to assume that a firm has the choice of a level of effort in each joint project it undertakes. In addition the firm may also be able to carry out independent work on private projects. In a recent paper, Goyal, Konovalov and Moraga (2002) examine a model with these features.

A network of collaboration g leads to a vector of R&D efforts $\{e_i(g)\}_{i \in N}$, which in turn defines the firms' marginal costs $\{c_i(g)\}_{i \in N}$. Given these costs, firms operate in the market by choosing quantities $\{q_i(g)\}_{i \in N}$. As above, demand is assumed to be linear and given by $Q = \alpha - p$, $\alpha > \overline{c}$. Let $\pi_i(g)$ be the net profits attained by firm i in network g.

The definitions of stability and efficiency here are as follows:

Definition 2.3 *We shall say that a network g is stable if and only if for all $i, j \in N$:*

1. *If $g_{ij} = 1$, then $\pi_i(g) \geq \pi_i(g - g_{ij})$ and $\pi_j(g) \geq \pi_j(g - g_{ij})$*
2. *If $g_{ij} = 0$ and $\pi_i(g + g_{ij}) > \pi_i(g)$ then $\pi_j(g + g_{ij}) < \pi_j(g)$.*

Likewise, for any network g social welfare is defined as the sum of consumer surplus and producers' profits. We shall say that a network g^* is *efficient* if $W(g^*) \geq W(g)$, for all $g \in \mathcal{G}$.

Independent markets case: Each firm faces demand $q_i = \alpha - p$ and the profits of firm i in collaboration network g are $\pi_i(g) = [\alpha - q_i - c_i(g)]\, q_i - \gamma e_i^2(g)$. Standard derivations show that equilibrium market quantities and profits are

$$q_i(g) = \frac{\alpha - c_i(g)}{2}, \pi_i(g) = \left[\frac{\alpha - c_i(g)}{2}\right]^2 - \gamma e_i^2(g). \tag{9}$$

In the second stage of the game, firms choose their R&D effort levels to maximize the reduced-form profits. This equilibrium level of efforts is given by:[1]

$$e(g^k) = \frac{(\alpha - \overline{c})}{4\gamma - k - 1}. \tag{10}$$

We can use this to compute the cost levels and profits as a function of the degree of the network.

$$c(g^k) = \overline{c} - \frac{(\alpha - \overline{c})\,(k+1)}{4\gamma - k - 1}. \tag{11}$$

The profits attained by the representative firm in a symmetric network of degree k can be obtained by plugging (10) into the profits expression above.

$$\pi(g^k) = \frac{(\alpha - \overline{c})^2 \gamma (4\gamma - 1)}{(4\gamma - k - 1)^2}. \tag{12}$$

The above computations show that if firms operate in independent markets, then individual R&D effort, cost-reduction and profits are increasing in the level of collaborative activity.

We next examine the nature of stable networks. We first note that for all $k = 0, 1, ..., n-2$, any pair of firms can increase their profits by linking to each other. This statement can indeed be made general by noting that for any network g, whether symmetric or asymmetric, the R&D efforts of the (linked) firms are strategic complement variables. Thus, no firm would reduce its effort should two other firms form a link with each other, which implies that the unique stable network when firms operate in different markets is complete. We next note that when firms operate in independent markets, social welfare is given by

$$W(g) = \sum_{i=1}^{N} (q_i^2(g)/2 + \pi_i(g)). \tag{13}$$

It can be verified that welfare is clearly increasing in the number of collaborations k. Taken along with our earlier observation that efforts are strategic complements allows us to state the following result in the independent markets case.

Proposition 2.9 *Suppose firms operate in independent markets. The complete network is the unique stable and efficient network.*

[1] We shall assume that $\gamma > n/4$ in the discussion of the independent markets case.

The above results show that if firms are operating in independent markets then the complete network maximizes research effort, cost reduction and profits, is uniquely stable and also welfare maximizing. We now examine the effects of market competition by considering a market with homogenous products and quantity-setting firms.

Homogenous Product Markets: In the market competition stage, for a given cost configuration of firms, $\{c_i(g)\}_{i \in N}$, the equilibrium quantity of firm i in a homogeneous product oligopoly is given by:

$$q_i(g) = \frac{\alpha - nc_i(g) + \sum_{j \neq i} c_j(g)}{n+1}, \tag{14}$$

and the profits of the Cournot competitors are given by

$$\pi_i(g) = \left(\frac{\alpha - nc_i(g) + \sum_{j \neq i} c_j(g)}{n+1} \right)^2 - \gamma e_i^2(g). \tag{15}$$

With this profit expression in hand, we can derive the equilibrium level of R&D effort by firms in a symmetric network of degree k. They are given by:

$$e(g^k) = \frac{(\alpha - \overline{c})\,(n-k)}{\gamma(n+1)^2 - (n-k)(k+1)}. \tag{16}$$

The above expression tells us that if firms are competitors in a homogeneous product market then R&D effort of a firm is decreasing in the level of collaborative activity. The intuition behind this result is that even though a firm's research effort helps reduce its own production cost, it also lowers rivals' costs, which makes them tougher competitors. The imperfect appropriability of R&D effort allows a firm's collaborator to gain a competitive advantage. This detrimental effect is traded off against the incentives to conduct research to reduce own marginal costs in a way such that R&D effort declines with the number of collaborators. We next examine the relation between cost reduction and the level of collaborative activity. We note that as collaborative activity increases there are two conflicting effects: on the one hand, individual effort declines, while on the other hand, each firm now has access to the efforts of a larger number of firms. The following result, due to Goyal and Moraga-González (2001) clarifies the relative magnitudes of these effects.

Proposition 2.10 *Suppose firms are competitors in a homogeneous product market. Research effort is a declining function of the level of collaborative activity. However, the relationship between cost reduction and the level of collaborative activity is non-monotonic. Moreover, cost reduction is maximum when each firm is linked with roughly half of the other firms.*

This result shows that in low density networks the cost-reducing benefits of an extra collaboration are substantial as compared to the detrimental effects arising from the induced decrease in the R&D activity of the firms. When the network is relatively dense, the latter effect dominates and an increase in the level of collaborative activity results in an increase in the firms' operating costs. In other words, 'effective' R&D exhibits a non-monotonic relationship with respect to the density of the network.

The profits attained by a firm in a symmetric network of degree k can be obtained by substituting the equilibrium level of effort into costs and these into (15):

$$\pi(g^k) = \frac{(\alpha - \bar{c})^2 \left[\gamma^2(n+1)^2 - (n-k)^2\right]}{\left[\gamma(n+1)^2 - (n-k)(k+1)\right]^2}. \tag{17}$$

This profit expression allows us to state the following partial result, due to Goyal and Moraga-González (2001), on stable networks.

Proposition 2.11 *Suppose firms are competitors in a homogeneous product market and $\gamma = 1$. Then the empty network is not stable, while the complete network is stable.*

We next examine the relationship between aggregate performance and the level of collaborative activity among firms in an industry. The next result, due to Goyal and Moraga-González (2001), is about the social welfare aspects of collaboration networks.

Proposition 2.12 *Suppose firms are competitors in a homogeneous product market and $\gamma = 1$. Then there exists some intermediate level of collaborative activity $\widetilde{k}$ with $0 < \widetilde{k} < n - 1$ for which social welfare is maximized.*

Seen together, the different results obtained in the contexts of independent markets and Cournot oligopoly yield a number of observations. First, we note that firms generally have an incentive to collaborate, irrespective of the product market setting, so the empty network is never incentive-compatible. Second, while in the independent market case the firm incentives to undertake R&D effort increase with the number of collaborations each firm maintains, in the homogeneous product case these incentives decline. This is due to the fact that in the latter context, collaborations bring about detrimental business-stealing effects which dampen firms' incentives to invest in cost-reducing activities. This difference in firm behavior in the two contexts has important consequences from the point of view of social welfare. It turns out that the private incentives to collaborate are aligned with the social ones if firms operate in independent markets. In contrast, when firms are competitors, the private incentives may lead to an excessive degree of collaboration, as shown by the fact that the complete network is stable but never efficient. The complete network is not efficient because firms anticipate a substantial business-stealing effect and undertake very little effort in R&D.

The discussion of the endogenous R&D effort model suggests several topics for further research. First, the analysis of the effects of collaboration in the homogenous product case is restricted to symmetric networks and is therefore quite incomplete. The reason for this is that we do not have general methods for studying the spillovers of individual links. This also makes a general analysis of stability of n players games difficult. Thus a better understanding of how spillovers work through networks is necessary if we are to make progress in our understanding of how networks affect individual incentives. Second, we have used a very specific model of effort: each firm chooses an effort level the fruits of which are shared with all the linked partners. This formulation precludes the consideration of differential effort across projects. It also precludes a study of the trade-offs that arise when making decision on allocation of resources between own independent projects and joint projects; see Goyal, Konovalov and Moraga-González (2002) for an attempt in this direction.

2.3 Market Sharing Agreements

The study of collusion among firms has tended to focus on price-fixing agreements and quantity controls by cartels. Another mechanism used by firms is market sharing agreements: if there is a clear association between particular firms and markets then potential competitors can collude by agreeing not to operate in each other's territory. There is evidence to suggest that firms use this route to reduce competition and increase own profits in markets. For example, in 1990, the European Commission concluded that Solvay and ICI had been operating a market sharing agreement by confining their soda-ash activities to their traditional home markets, namely Western European markets for Solvay and the UK for ICI. For a discussion of examples on market sharing agreements, see Belleflamme and Bloch (2001).

We shall examine the incentives of firms to enter into this type of market sharing arrangements and the impact of this on the functioning of the market. We will consider a simple model with n ex-ante symmetric firms and associate with each firm i a homogeneous product market i. Before engaging in competition in these markets, the firms can form collaboration links with each other. A collaboration link between i and j is an agreement between the two firms to stay out of each other's market. If firm i has $\eta_i(g)$ links in a network g, then there are $n - \eta_i(g)$ active firms in market i who compete as Cournot oligopolists. The Cournot profits earned by a firm that is active in market k is given by $h(n - \eta_k(g))$. Therefore, the gross payoff to firm i in a network g is given by:

$$\pi_i(g) \;=\; h(n - \eta_i(g)) \;+\; \sum_{k;\, g_{i,k}=0} h(n - \eta_k(g)) \tag{18}$$

The marginal gross payoff to i from establishing a link with j is given by:

$$\psi(\eta_i(g), \eta_j(g)) \;=\; [h(n - \eta_i(g) - 1) - h(n - \eta_i(g))] - h(n - \eta_j(g)) \tag{19}$$

It is reasonable to assume that profits in market i are decreasing in the number of firms active in the market. We shall assume, in addition, that profits are also log-convex in the number of active firms in a market.

$$h(l) \leq h(l-1); \;\; [h(l+1) - h(l)]/h(l+1) \geq [h(l) - h(l-1)]/h(l). \tag{20}$$

With these assumptions in place, we can state the following characterization of pairwise stable networks, due to Belleflamme and Bloch (2001).

Proposition 2.13 *Suppose that individual firm profits are decreasing and log-convex in the number of firms in a market. Then a network is stable if and only if it consists of a group of isolated firms and distinct complete components of different sizes.*

The arguments for this result are as follows: the first step shows that if two firms have a link then they must have an equal number of links. The idea here is that if i and j form a link and $\eta_i(g) > \eta_j(g)$, then firm j is giving up more profits in market i than it is gaining in its home market, by forming a link with firm i. This means that the firm will not form this link. The second step is the claim that a component in a stable network must be complete. Why is this true? From the first step, it follows that every firm in a component must have the same number of links. Given this, the claim follows from the assumption of log-convexity. These two steps prove that a stable network must consist of complete components. Step two also suggests that the components cannot be of equal size, because then firms in the two components will have an incentive to form links. Thus in a stable network, distinct components must be of unequal size.

How do the stable networks compare with socially desirable outcomes? As before, we shall define social welfare as the sum of consumers surplus and producers profits. The following result then follows from standard arguments which favour competition.

Proposition 2.14 *Consider the linear homogenous product oligopoly with quanity competition. The empty network maximizes social welfare.*

3 Networks of buyers and sellers

In various industries, such as e.g. automobiles, clothing, electronics, and pharmaceuticals, manufacturers develop networks of exchange with their input suppliers. Moreover, in the last few decades the importance of spot exchange in input procurement has decreased in favor of other methods of input acquisition such as manufacturer-supplier long-term contracting and buyer-seller exchange networks. For example, from 1980 to 1990, the major car manufacturers reduced their number of direct input suppliers by more than 50 percent (Nooteboom, 1999). This trend is more prominent in Japanese automobile and electronic manufacturing, well-known for its tradition in collaborative subcontracting relations. The number of direct suppliers to Japanese car manufacturers in 1988 was roughly one half of what it was for American or European manufacturers, for similar volumes of production (Lamming, 1993).

For electronics and automobiles, Nishiguchi (1994) presents wide ranging evidence on the ways in which the Japanese industrial model has evolved from the traditional bargaining-oriented manufacturer-supplier relationship (arm's length markets) to the current problem-solving-driven strategic industrial outsourcing. Firms rely more and more on a subset of suppliers with whom they maintain closed business ties. For instance, in the period from 1980 to 1990, Fuji Electric Tokyo buys an additional 7 percent of its inputs from sub-contractors but it has reduced the number of principal subcontractors by 38 percent. On average, the number of regular customers of electronics subcontractors in Japan is 10; second-tier subcontractors have an average of 1.5 customers.

Similar findings appear in a study of network ties among "better-dress" firms in the New York apparel economy: Uzzi (1996) finds that about 25 percent of the manufacturers have tightly knit networks composed of 5 or fewer exchange partners; 30 percent have exchanges with 5 to 12 partners, while about 40 percent maintain business ties with more than 20 contractors. A similar pattern is observed for the contractors: roughly 34 percent have tightly knit networks of 3 manufacturers or less; about 45 percent have an average network size of 4 to 8 manufacturers; and about 20 percent have large networks of 9 or more exchange partners.

The evidence above motivates us to explore the incentives of buyers and sellers to form multiple relations which seem to lie somewhere in between markets and vertical integration. We would also like to examine the consequences of this link formation activity. In this survey, we shall focus on the recent papers by Kranton and Minehart (2000, 2001). The general setting is that of a market where buyers and sellers operate in complex environments and thus long-term contracting is not feasible. Buyers, prior to knowing their valuations for a product, have an opportunity to form links with sellers. These links possibly enable buyers to procure goods or inputs. Buyers trade-off expected gains from trade against costs of link formation.[2]

3.1 Buyer-seller networks: the purchasing view

In Kranton and Minehart (2001), the authors construct a theory to examine the emergence of networks between buyers and sellers; in this theory buyers have a purely *purchasing* motivation. In Kranton and Minehart (2000) the authors extend this theory to examine buyer-seller networks as compared to other forms of market organizations, in particular, spot exchange and vertical integration. In this work, sellers have an *outsourcing* motivation. We begin by presenting the purchasing view.

Consider an economy populated by buyers and sellers. Let $S = \{s_1, s_2, ..., s_n\}$ denote the set of identical sellers. Each seller j has the capacity to produce a single unit of a good at constant returns to scale. Normalize the marginal cost to zero. Let $B = \{b_1, b_2, ..., b_m\}$ be the set of buyers. Each buyer i derives utility v_i from the consumption of at most a single unit of the good. Let v_i be independently and identically distributed on the real line according to the continuous distribution function F. Assume that buyers' valuations are private information to them and that F is common knowledge.

Assume that a buyer can only obtain a good from a particular seller if the two of them maintain a link. In line with the notation above, for any $i \in B$ and $j \in S$, the

[2]For a related strand of the literature on buyer-seller networks with a different modelling approach – based on heuristic learning rules and random linking decisions – see Kirman, Weisbuch and Herreiner (2000) and Kirman and Vriend (2000).

pair-wise relationship between buyer i and seller j is captured by the binary variable $g_{i,j} \in \{0, 1\}$; $g_{i,j} = 1$ means that buyer i and seller j maintain a link while $g_{i,j} = 0$ means that no link between these two agents exists. The collection of links between buyers and sellers forms a network $g = \{g_{i,j}\}_{i \in B, j \in S}$.

In this model, the following questions can be asked. First, given a network between sellers and buyers g, what are the feasible allocations of goods? Second, can efficient allocations of goods be characterized? Third, is there a competitive mechanism which allocates goods efficiently in a network? Fourth, do self-motivated buyers form efficient networks? We address these questions in what follows.

We note first that in buyer-seller networks the link pattern g constrains the set of feasible allocations of goods. An allocation of goods is $A = \{a_{ij}\}_{i \in B, j \in S}$, where $a_{ij} = 1$ when buyer b_i is allocated a good from seller s_j and $a_{ij} = 0$ otherwise. In a feasible allocation a buyer gets at most one unit of a good, and this unit can only be provided from one of the sellers with whom he maintains links; likewise, a seller can only allocate her good to a single buyer and this buyer must be among those who keep links with her.

The answer to the first question above can be given by employing *the marriage theorem*, a well known result from the mathematics of combinatorics. This result helps us establish whether a particular allocation of goods is feasible or not in a network g. This theorem says that for a subset of sellers $\widetilde{S}$ and a subset of buyers $\widetilde{B}$ there is a feasible allocation of goods, i.e., every buyer in $\widetilde{B}$ obtains a good from a seller in $\widetilde{S}$, if and only if every subset $\widetilde{B}'$ in $\widetilde{B}$ containing k buyers is linked collectively to at least k sellers in $\widetilde{S}$, for each k, with $1 \leq k \leq \widetilde{B}$ in network g.

Among the set of feasible allocations, we are interested in characterizing efficient ones. We now present welfare (gross of links costs) in a network of exchange g. For a given realized vector of buyer valuations $v = \{v_1, v_2, ..., v_m\}$, and a feasible allocation of goods A, realized gross economic welfare is given by the sum of the valuations of those buyers who secure goods. Let $w(v, A)$ denote this gross surplus. Then $w(v, A) = \sum_{i \in B} \sum_{j \in S} v_i a_{ij}$. Of the set of feasible allocations, an efficient allocation A^* is that which yields the highest possible welfare that can be derived from exchange in the network. For a given network of exchange g, let $GW(g)$ denote the maximal gross economic welfare that can be obtained. Since valuations are random we have:

$$GW(g) = E_v[w(v, A^*(v; g))], \tag{21}$$

where the expectation operator is taken over all possible realizations of buyers' valuations.

Efficient buyer-seller networks balance the expected gains from exchange with the links costs. Let $W(g)$ denote the net welfare derived from network g. From the definition of gross welfare above we have

$$W(g) = GW(g) - c \sum_{i \in B} \sum_{j \in S} g_{i,j} \tag{22}$$

where $c > 0$ is the cost of forming a link between a buyer and a seller. As above, network g is efficient if $W(g) > W(g')$ for all $g' \neq g$.

We now explore the architecture of efficient networks. When link costs are small, an efficient network should have enough links so that the buyers with the highest valuations can all obtain goods. If, for instance, there are three buyers and two sellers, any set of two buyers should all be able to obtain goods. A network is said *allocatively complete* if and only if for every subset of buyers there is an allocation such that every buyer obtains a good. An allocatively complete network which minimizes in link-costs is said to be a *least-link allocatively complete network.* These are networks that achieve all the economies of sharing with the minimal number of links. The following result, due to Kranton and Minehart (2001), uses the marriage theorem to characterize least-link allocatively complete networks; moreover, when cost of links are small, efficient networks and least-link allocatively complete networks coincide.

Proposition 3.1 *In a least-link allocatively complete network, each seller is linked to exactly $m - n + 1$ buyers and each buyer has from 1 to n links. Moreover, if link costs are sufficiently small, then least-link allocatively complete networks are the efficient networks.*

We now turn to examine whether there exists a competitive mechanism that allocates goods efficiently in a network. Then, we discuss the connection between efficient networks and equilibrium networks. Given a network of sellers and buyers, we need a competitive mechanism to allocate goods from the former to the latter. Competition in a network can in principle take several forms. One possible, perhaps natural, way would be one in which sellers compete in prices to attract the linked buyers. Since sellers are capacity constrained, a buyer would decide which seller to buy from taking into account not only the prices charged by his set of linked sellers but also the link pattern and the probability of being rationed. This friction would give market power to the sellers and prices would presumably be well beyond marginal cost, like in Burdett *et al.* (2001). Another possible way in which competition can take place in a network is through direct buyer-seller bargaining, like in Corominas-Bosch (1999). Kranton and Minehart present a simpler, but more abstract, model of competition for goods in a network. As we shall see below, this model has the desirable property that the allocation of goods is efficient. They consider a generalized auction mechanism by which goods are allocated from sellers to buyers. In their abstraction of a competitive model, all sellers hold simultaneously ascending-bid auctions, with the going price being common across sellers. As the price increases, buyers decide whether to drop out of each of its linked sellers' auctions. The price goes up until a sufficient number of buyers have exited the market so that there is a subset of sellers for whom demand equals supply. These auctions then clear at the current price. The rest of the sellers continue raising the price until they sell their goods. In this setting, it is a weakly dominant strategy for a seller to hold an auction since it earns nothing if it does not. Likewise, it is an equilibrium (following iterated elimination of weakly dominated strategies) for each buyer to remain in the auction of each of its linked sellers until the price reaches his/her valuation. Since the price paid by a buyer in the auction is equal to the valuation of the next best bidder, the buyer has no incentives not to bid truthfully.

With this market mechanism at hand, it is easy to calculate the payoffs to the different agents. Given a network g, let $V_i^b(g)$ denote the expected payoff to buyer i in the generalized auction, and let $V_j^s(g)$ be seller j's expected payoff. These payoffs can be computed using the order statistics of the distribution F. The following result, due to Kranton and Minehart (2001), establishes that this auction mechanism for competition in a network yields efficient outcomes.

Proposition 3.2 *Given a network g, for each realization of buyers' valuations v (i) the generalized auction mechanism described above allocates goods efficiently and (ii) the equilibrium allocation and prices are pairwise stable.*

The proof of this result shows that in equilibrium the highest valuation buyers obtain the goods whenever they are not constrained by the network (feasibility). Moreover, note that every buyer who secures a good in the generalized auction pays a price equal to the valuation of the next best bidder and that this price equals the social opportunity cost of obtaining the good. The price is high enough so that no competing buyer will offer a seller a higher price.

We are now ready to examine whether buyers' incentives to form links with sellers are aligned with the social incentives. For this purpose, consider the following two-stage game: In the first stage, buyers simultaneously choose to form costly links with the sellers. Each link costs $c > 0$ for a buyer. The set of links of buyer i in network g is denoted g_i. In the second stage, buyers learn their valuations and participate in the auctions of those sellers with whom they maintain links.

The interest is on whether equilibrium networks are similar to efficient networks. The private incentives are governed by buyers' payoffs since links are initiated by these agents. The net payoff to buyer i in g is $\pi_i^b(g) = V_i^b(g) - c\sum_{j=1}^S g_{ij}$. A seller final payoff is $V_j^s(g)$. The equilibrium notion pertinent to this game is that of perfect Bayesian equilibrium (PBS). A link profile g^* is a PBS if and only if $g_i^* = \arg\max_{g_i} V_i^b(g_i, g_{-i}^*) - c\sum_{j=1}^S g_{ij}$, where g_{-i} denotes the network that result from deleting all the links of buyer i. The following result, due to Kranton and Minehart (2001), establishes the connection between efficient and equilibrium networks.

Proposition 3.3 *For any link cost $c > 0$, each efficient network is an equilibrium of the network formation game. Moreover, if the link cost is sufficiently small then only efficient link patterns are equilibrium networks.*

This result is intimately related to the efficient manner in which goods are allocated in a network as a result of the competitive process outlined above. For a given link pattern g, the allocation mechanism leads to the achievement of maximal surplus. The price paid by any buyer is equal to the social opportunity cost of obtaining the good and thus buyers' payoffs are equal to the contributions of their links to economic welfare. In other words, if one of the links of a buyer is severed, the loss in his gross payoff is exactly equal to the loss in gross economic surplus. This result holds true for any other competitive process that yields an efficient allocation of goods and in which buyers' gross payoffs are equal to the marginal surplus from exchange.[3]

The previous argument suggests that efficient networks are necessarily equilibrium networks. The second part of the proposition shows that profit-motivated buyers cannot form networks that are not efficient provided that the costs of link formation are sufficiently small. This is because for low link costs, in a network that is not least-link allocatively complete some buyer has an incentive to add a new link or to break an existing one.

In summary the theoretical model presented above has enabled us to examine two fundamental economic questions. First, what underlying economic environments may lead buyers and sellers to form networks of exchange? The answer has been that networks arise because they serve economic actors to pool uncertainty in demand. When sellers productive capacity is costly and buyers have uncertain valuations of goods, it is socially optimal for buyers to share the capacity of a limited number of sellers (realization of economies of sharing). Second, should we expect these networks to be efficient? Can buyers and sellers acting in their own interest build socially optimal networks? We have found that an efficient allocation mechanism (ex-post competitive environment) is sufficient to align the buyers' incentives to form ties with the social incentives provided that link costs are low. This alignment is sensitive to the manner in which links are built, unilaterally or bilaterally.

3.2 Networks versus vertical integration

We now move to examine the emergence of buyer-seller networks when sellers have an outsourcing motivation. In doing this we follow Kranton and Minehart (2000) and ask whether networks of buyers and sellers can perform better than vertically integrated markets or spot exchange markets. This work helps us understand why supply relations vary across industries. There exists evidence that firms, in some cases, procure standardized inputs from arm's length markets. In other cases, however, firms vertically integrate and produce their own specialized inputs. Finally, firms are also observed to establish a number of stable connections with input suppliers and create networks of buyers and sellers (examples are General Motors in the 1950s, the New York garment industry, Japanese electronics and automotive *keiretsus*, etc.).

We shall compare vertically integrated firms with networks of manufacturers and suppliers in a model of technology and industrial structure due to Kranton and Minehart (2000). Manufacturers can decide to build a dedicated asset to produce their own inputs or, alternatively, they can invest in links to external sellers from which they will buy specialized inputs. We shall concentrate on the conditions under which

[3]In settings where the competitive process is not efficient, the reduction in surplus would lead to a suboptimal investment in links; moreover, a costly investment decision of the sellers might lead to inefficiency (Kranton and Minehart, 2001). Similarly, Jackson (2001) shows that buyer-seller networks with two-sided link formation are typically inefficient.

industries are likely to be organized as networks. We shall establish a connection between industrial structure and uncertainty in demand: outsourcing networks appear to be more efficient than vertically integrated structures when uncertainty in demand is substantial. The intuition is that when dispersion in buyers' valuations is large, industrial welfare can be higher when buyers share the productive capacity of sellers (economies of sharing). In addition we shall discuss when networks organizations are socially desirable.

Consider the following model of technology and industrial structure. Suppose an economy where there is a set of buyers $B = \{b_1, b_2, ..., b_m\}$ and a set of sellers $S = \{s_1, s_2, ..., s_n\}$. Each buyer demands a single unit of an input. This input can come in two types: it can be a (normalized) zero-value *standard* input or a positive-value *specialized* input. There are three ways in which buyers can secure this input. First, they can buy a *standard* input from a competitive fringe of standardized input suppliers at a price normalized to zero. Second, buyers can vertically integrate and produce a *specialized* input themselves, by investing in a *dedicated* asset. The investment costs necessary to build a dedicated asset are $\alpha_d > 0$. A buyer who produces its own input is called a *vertically integrated firm.* Finally, a buyer can create links with sellers of *specialized* inputs ("specialists"). Each link costs $c > 0$ to a buyer, and enables the buyer to buy a specialized input from one of the sellers.

A buyer i has a zero valuation for a standardized input and a random valuation v_i for a specialized input, with $v_i = z+\varepsilon_i$, where z and ε_i are random variables. Assume that z is a common shock to all buyers, with mean $\overline{z}$. The variable ε_i is a buyer-specific shock with a continuous distribution $G_\sigma(\varepsilon) \equiv F(\varepsilon/\sigma)$ which has mean 0. Parameter $\sigma \geq 0$ measures the *dispersion* of buyers' idiosyncratic shocks, that is, if $\sigma_1 > \sigma_2$, then $G_{\sigma_1}(\varepsilon)$ is a mean-preserving spread of $G_{\sigma_2}(\varepsilon)$. Let $\mu^{n:B}(\sigma)$ be the expected n-th highest shock to the buyers, i.e., the n-th order statistic of B draws from $G_\sigma(\varepsilon)$. A property that proves useful immediately is that $\mu^{n:B}(\sigma)$ is a homogeneous function of degree 1 in σ; i.e., $\mu^{n:B}(\sigma) = \sigma\mu^{n:B}(1)$. Thus, the expectation of the n-th highest valuation of B buyers is $\overline{z} + \sigma\mu^{n:B}(1)$.

Specialized inputs can also be bought from *specialized sellers.* There are S *specialists* who can produce just one specialized input. A buyer who acquires the input from a seller needs to have a link with that seller. As mentioned above, building the link costs $c > 0$ to the buyer. A seller needs to invest $\alpha_f > 0$ in a *flexible* asset to be able to produce a specialized input. Assume $\alpha_f = \alpha_d = \alpha$ for simplicity. Once the investment is made, the seller can satisfy the needs of different buyers, i.e., the seller is a "*flexible specialist.*"

Buyers that build links with sellers and sellers who invest in the flexible production facility are called *a network of buyers and sellers.* As usual, we shall denote a network of buyers and sellers as $g = \{g_{i,j}\}_{i\in B, j\in S}$. An *industrial structure* is formed by the investments of the buyers and the sellers. Networks involve buyers' specific investments and sellers' quasi-specific investments. Vertical integration means ownership of the asset while investment in a network does not.

We first pay attention to the welfare maximizing network structures. Fix an industrial structure (or network) g and a vector $v = (v_1, v_2, ..., v_m)$ of realized buyers' valuations. Let A be an *allocation* of goods given buyers' valuations v and the industrial structure g. The economic surplus derived from an allocation of goods in an industrial structure g is $w(v, A(v, g))$. The allocation A^* is *efficient* if and only if $w(v, A^*(v, g)) > w(v, A(v, g))$, for all feasible $A(v, g)$. For an industrial structure g, the expected surplus is $E_v[w(v, A^*(v, g))]$ and expected welfare is then $W(g) = E_v[w(v, A^*(v, g))] - \alpha_d \sum_{i=1}^{B} \delta_i(g) - c \sum_{i=1}^{B} l_i(g) - \alpha_f \sum_{j=1}^{S} \kappa_j(g)$, where $\delta_i(g) = 1$ when buyer i is vertically integrated and equals zero otherwise, $l_i(g)$ is the number of links buyer i maintains, and $\kappa_j(g) = 1$ when seller j has invested in productive capacity and equals zero otherwise. We shall say that an industrial structure g is *efficient* if and only if $W(g) > W(g')$ for all $g' \neq g$.

The following result, due to Kranton and Minehart (2000), proves that there exist circumstances under which networks yield higher welfare than other industrial configurations.

Proposition 3.4 *Assume that buyers valuations are dispersed ($\sigma > 0$). Then there exist investment costs α and link costs c such that an industrial network structure is efficient.*

The intuition behind this result is that when there is dispersion in valuations, welfare can be higher when buyers share the productive capacity of various sellers. Once we have seen that networks can perform better than other industrial structures from the point of view of the society, the next question that arises is whether strategic buyers and sellers, acting non-cooperatively to maximize their own economic profit, have the right incentives to form efficient network structures. We examine this question within the context of the following two-stage game. In the first stage, buyers, taking as given sellers' decisions, choose whether to invest in a dedicated asset (which costs them α), or to create links (which costs them c), or not to invest at all. Likewise, sellers choose whether to invest in a flexible asset (which costs them α) or not to invest at all, taking as given buyers' decisions. In the second stage, buyers' valuations are realized and production and exchange takes place. Exchange can be competitive (modeled as in Kranton and Minehart, 2001, i.e., as having sellers holding simultaneously ascending-bid auctions and buyers bidding truthfully), or modeled through some sort of negotiation where agents to a relationship split evenly the gains obtained (agents get their Shapley values). We shall see that the mode of competition is crucial as to whether private incentives to form links are aligned with the social incentives.

Fix a network g and a vector of valuations $v = (v_1, v_2, ..., v_m)$. Let the revenue to a buyer be $r_i^b(v, g)$ and the revenue to a seller $r_j^s(v, g)$; let us normalize revenues when buyers and sellers do not invest to zero, i.e., $r_i^b(v, g) = r_j^s(v, g) = 0$. Assume further that the payoff to a vertically integrated buyer is $r_i^b(v, g) = v_i$ and that if a network is in place, the surplus generated by the network is fully distributed to its constituent firms. Thus, the payoff to a buyer i is $\pi_i^b(g) = E_v[r_i^b(v, g)] - \alpha\delta_i(g) - cl_i(g)$ while the payoff to a seller j is $\pi_j^s(g) = E_v[r_j^s(v, g)] - \alpha\kappa_j(g)$.

In this context a network g is a Nash equilibrium if and only if the following two conditions hold: (i) $\pi_i^b(g) \geq \pi_i^b(g')$ for all $g \neq g'$ and for all i, where g' only differs from g in the links or dedicated assets of buyer i. (ii) $\pi_j^s(g) \geq \pi_j^s(g')$ for all $g \neq g'$ and for all j, where g' only differs from g in the investment of seller j.

Firms must make investments considering how its actions affect future ability to obtain inputs and its competitive (or bargaining position). Investments somehow "lock-in" agents and opportunistic behavior may arise (hold-up problem). Thus, investment behavior is sensitive to the way the goods are allocated. The following result, due to Kranton and Minehart (2000) establishes that equilibria exist where industrial structures are either vertically integrated or of the spot exchange type. Moreover, these equilibria are efficient.

Proposition 3.5 *Assume that investment costs are low (high) relative to average buyers' valuations ($\alpha \leq (\geq)\overline{z}$). Then vertical integration (spot exchange) is an equilibrium. Moreover, suppose that vertical integration (or spot exchange) is efficient; then, it is the unique equilibrium of the game.*

The intuition is that under vertical integration a buyer expected payoff is exactly its contribution to social welfare. Therefore vertical integration is the unique equilibrium when it is efficient. The same holds for a competitive market structure.

We now move to address the question of whether privately motivated agents do form efficient networks. The answer to this question is negative. This is so for two reasons: (i) Agents' payoffs depend on the investment of other agents and coordination may fail to obtain. (ii) Agent's expected payoff needs not be equal to its contribution to social welfare.

We illustrate this issue using the competitive allocation mechanism outlined above. In a network, sellers simultaneously hold ascending-bid auctions. A buyer equilibrium strategy is to remain in the auction of all his linked sellers until the price reaches his valuation. Anticipating *competitive* revenues, we can ask what are buyers' incentives to create links? Given sellers' investments, a link contributes to the buyer the same amount it contributes to social welfare. Therefore, buyers' incentives are aligned with social incentives. The following result, due to Kranton and Minehart (2000), proves this is true in general.

Proposition 3.6 *When firms' revenues are competitive, buyers invest in links when it is socially desirable.*

Let us now look at the sellers' side. Anticipating *competitive* revenues, what are sellers' incentives to create links? They are indeed lower than social incentives. In fact, a seller's contribution to welfare equals the valuation of the buyer who gets the good. A seller's revenue is however the valuation of the buyer with the next highest valuation. The two observations above enable us to argue that under the competitive rule, the region of parameters for which a network is both efficient and equilibrium is only restricted by the sellers' equilibrium conditions. This result is weakened when agents are given ex post their Shapley values instead of competitive revenues. The reason is that the Shapley value does not give a buyer the marginal social value of its network links. As a result, a buyer may have an incentive to severe efficient links or to add inefficient links. The same applies to sellers. The Shapley value can be less than a seller's marginal contribution to social welfare. As a result, each seller has little incentive to invest in the productive assets.

In summary, we have examined a model of networks between buyers and sellers and asked if self-interested agents can form efficient networks. We have seen that large idiosyncratic shocks, costly productive capacity and low linking costs make denser networks to be efficient as compared to vertical integration or arm's length markets. Under these conditions buyers should have links to multiple buyers and share their productive capacity. Whether unilateral incentives of buyers and sellers can lead to efficient networks seem to depend on the ex-post surplus allocation mechanism. The agents' incentives appear to be quite distorted when they receive their Shapley values; by contrast, buyers incentives are fully aligned with the social ones if the allocation mechanism is competitive.

4 Concluding Remarks

Collaborations among firms take diverse forms and are very popular. There is some evidence to suggest that this collaborative activity has been growing in recent years. Empirical work also suggests that collaboration agreements are often bilateral and are embedded within a broader network of collaborations.

Collaborations are valuable and well connected firms have an advantage as compared to their less connected counterparts. This suggests that firms may have individual and collective incentives to form collaboration ties. In this survey we have explored the implications of this form of link formation activity undertaken by firms. We have examined two broad class of settings: one, where firms form collaborative relations with other firms who are potential competitors and two, where firms form links across the market, so buyers link with sellers (and vice-versa). The survey suggests that this line of enquiry yields interesting insights about the architecture of networks as well as about the welfare effects of link formation activity of individual players. It also brings out in an interesting way the two-way flow of influence between networks and market stage competition. One the one hand, markets shape the incentives to form links, while on the other hand, links between firms affect their competitive position and thereby shape the functioning of markets.

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