Implications of Small- and Large-Stakes Risk Aversion for Decision Theory

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By James C. Cox and Vjollca Sadiraj*

A growing literature reports the conclusion that expected utility theory does not provide a plausible theory of risk aversion for both small-stakes and large-stakes gambles and that this decision theory should be replaced with an alternative theory characterized by loss aversion. This paper explains that the arguments in previous literature fail to support that conclusion because they apply only to the expected utility of terminal wealth model. The extant concavity-calibration arguments have no implication for the widely-used expected utility of income model nor for a new model that we introduce, the expected utility of initial wealth and income model. In order to show that the new model should have fruitful applications, we extend the Arrow-Pratt characterization of comparative risk aversion to it. In order to explicate the actual implications of concavity calibration for decision theory, we introduce an alternative assumed pattern of global small-stakes risk aversion and calibrate its implications for prospect theory and three expected utility models.

Key Words: decision theory, game theory, risk aversion
JEL Classification Numbers: C90, D81

1. Introduction

In their seminal work on game theory, von Neumann and Morgenstern (1944, 1947) developed a theory of utility because it is an essential component of a theory of play for strategic games. Their utility theory, now known as expected utility theory, is based on a set of axioms for a preference ordering of probability distributions of "prizes." The set of axioms includes the independence axiom, which gives an expected utility functional representing the axioms its defining characteristic of linearity in probabilities. It was clearly understood in classic work (e.g., Luce and Raiffa, 1957, Ch. 2) that the axioms do not specify the identity of the prizes. A difference in the assumed identity of the prizes is the characteristic that distinguishes one expected utility model from another. Failure in recent literature to distinguish between expected utility *theory* − all models based on a set of axioms that includes the independence axiom − and a specific expected utility *model* has led to incorrect conclusions.

In a provocative essay, Rabin and Thaler (2001, p. 221) – building on earlier work by Rabin (2000) – state that they "… establish the implausibility of expected utility theory by showing that absurd large-stakes risk aversion … follow inherently from non-negligible modest-scale risk aversion." Validity of the Rabin-Thaler conclusion has been accepted in the academic literature (Kahneman, 2003; Camerer and Thaler, 2003) and general readership literature (*The Economist*, 2001) and in the award literature for the 2002 Nobel Prize in Economic Sciences (Royal Swedish Academy of Sciences, 2002, p. 16). This paper explains that Rabin's and Thaler's arguments do not logically support their conclusion.

In Section 2, our evaluation of their conclusion proceeds as follows. First, we recall that the Rabin-Thaler argument is based on the assumption that an agent will reject a small-stakes gamble with equal probabilities of 50% of winning or losing relatively small amounts, and that the agent will do this at all initial wealth levels in some large interval. Second, we explain why (for the case of a differentiable utility function) the Rabin-Thaler risk aversion assumption *does* imply implausible large-stakes risk aversion for the expected utility of terminal wealth model. Third, we explain that expected utility *theory* is not coincident with the expected utility of terminal wealth *model* but, instead, contains other models such as the expected utility of income model.[1] Fourth, we demonstrate that the Rabin-Thaler risk aversion assumption does *not* imply implausible large-stakes risk aversion for the expected utility of income model.[2] Therefore, the conclusion by Rabin and Thaler that they "establish the implausibility of expected utility theory" does not stand.

Although the expected utility of income model is widely used in the theory of auctions, this model does not provide an explanation of how an agent's initial wealth affects its attitude towards risk. The perceived importance of a model with this capacity is what accounted for the widespread use in non-auction theoretical papers of the expected utility of terminal wealth model after publication of the classic papers by Pratt (1964) and Arrow (1971). In order to be able to analyze the effects of wealth differences on risk-taking behavior, one needs a model in which risk attitude *does* depend on initial wealth. In order for a model to withstand the Rabin-Thaler critique, initial wealth must not be additive to income in the utility function. This is our motivation for introducing, in Section 3, an expected utility model in which the arguments of the utility function are ordered pairs of initial wealth and income.

In Section 3, we demonstrate that this new model, the expected utility of initial wealth *and* income model, should have fruitful applications by extending the Arrow and Pratt characterization of comparative risk aversion to the new model. We also demonstrate that the Rabin-Thaler risk aversion assumption does *not* imply implausible large-stakes risk aversion for the new expected utility model. Therefore, Rabin's and Thaler's arguments not only fail to establish the implausibility of expected utility theory, they also fail to show that expected utility theory does not provide a coherent explanation of how agents' initial wealth and their comparative risk aversion affect their willingness to bear small- and large-stakes risks.

Having demonstrated that the Rabin-Thaler small-stakes risk aversion assumption has implications for only one expected utility model, and hence has no general implication for expected utility theory, in Section 4 we ask whether there exists an alternative small-stakes risk aversion assumption that has more general implications. We identify an assumption that implies that all three expected utility models have implausible large-stakes risk aversion. Furthermore, we demonstrate that this assumed pattern of risk aversion also implies that cumulative prospect theory (Tversky and Kahneman, 1992) has implausible large-stakes risk aversion. Therefore, if expected utility theory were, in fact, to have implausible large-stakes risk aversion (because our risk aversion assumption had empirical validity) then so would cumulative prospect theory, a theory that is characterized by both loss aversion and isolation of each risky choice.[3] Thus the analysis in Section 4 contradicts the conclusion stated by Rabin and Thaler (2001, p. 230) in their last paragraph: "What should expected utility theory be replaced with? We think it is clear that loss aversion and the tendency to isolate each risky choice must both be key components of a good descriptive theory of risk attitudes." The correct conclusion is: If expected utility theory were to need to be replaced (because the Cox-Sadiraj alternative risk aversion assumption had empirical validity) then cumulative prospect theory would also need to be replaced *because* loss aversion and the tendency to isolate each risky choice would not provide a way around the problem.

We end, in Section 5, by summarizing the discussion and pointing out that further progress in resolving the implications of these assumed patterns of small-stakes risk aversion for decision theory is dependent on acquisition of credible data. It is currently unknown whether either the Rabin-Thaler risk aversion assumption or our alternative assumption has empirical validity.

2. Expected Utility Theory Withstands the Rabin-Thaler Critique

In this section we re-examine the Rabin-Thaler conclusion that their arguments "…establish the implausibility of expected utility theory…" We provide straightforward demonstrations that Rabin's and Thaler's assumed pattern of small-stakes risk aversion: (a) does imply implausible large-stakes risk aversion for the expected utility of terminal wealth model; and (b) does *not* have an analogous implausible risk aversion implication for the expected utility of income model.

The Rabin-Thaler analysis of the implications of small-stakes risk aversion is based on the assumption that an agent with (weakly) concave Bernoulli utility function will reject a small-stakes gamble with equal probabilities of 0.5 of winning or losing relatively small amounts, and that the agent will do this at all initial wealth levels in some large interval. We examine the implications of their assumption: (a) when the prize is terminal wealth; and (b) when the prize is income.

2.1. Implications for the Expected Utility of Terminal Wealth Model

Here we use the expected utility of terminal wealth model, the model based on the expected utility axioms *and* the assumption that the prizes are amounts of terminal wealth. Assume that an agent rejects the gamble in which he can lose 100 with probability 0.5 or gain 110 with probability 0.5 for all w between 100 and M, where $M > 100$ can be as large as one chooses. Letting u denote the agent's (weakly) concave Bernoulli utility function for terminal wealth, the sufficient condition for rejecting the 50-50 lose 100, gain 110 gamble is

(1) $u(w) > 0.5u(w-100)+0.5u(w+110)$, for all w such that $100 \le w \le M$.

We next calibrate the implications of the assumption contained in statement (1). In order to simplify the derivation we will assume that $u(\cdot)$ is differentiable.[4] Define $z_t = w + 210\,t$, for $t = 0,1,2\cdots$. Evaluate the inequality in statement (1) for $t = 0$ at $z_0 + 100 = w + 100 + 210 \times 0 = w + 100$, multiply both sides of the resulting inequality by 2, and rearrange terms to get

(2) $u(w+210) - u(w+100) < u(w+100) - u(w)$.

Inequality (2) and concavity imply the following inequalities for the marginal utility of terminal wealth, $u'(\cdot)$:

(3)
$$u'(w+210) \le \frac{u(w+210)-u(w+100)}{110}$$
$$< \left[\frac{100}{110}\right] \times \left[\frac{u(w+100)-u(w)}{100}\right] \le \frac{100}{110} u'(w).$$

The inequalities in statement (3) hold because: the left-most inequality follows from the fact that, with a concave function, marginal utility at the high end of an interval cannot be larger than average utility over the interval; the middle inequality follows immediately from (2); and the right-most inequality follows from the fact that, with a concave function, marginal utility at the low end of an interval cannot be less than average utility over the interval.

Evaluating the inequality in statement (1) for $t = 1$, at $z_1 + 100 = w + 100 + 210$, multiplying both sides by 2 and rearranging terms, and using the strict inequality between the left-most and right-most terms in statement (3), one gets:

$$(4) \qquad u'(w + 210 \times 2) = u'(w + 420) \leq \frac{u(w + 420) - u(w + 310)}{110}$$

$$< \left[\frac{100}{110}\right] \times \left[\frac{u(w + 310) - u(w + 210)}{100}\right] \leq \frac{100}{110} u'(w + 210) < \left[\frac{100}{110}\right]^2 u'(w).$$

Iteration over *t*, using the logic explained above, yields

$$(5) \qquad u'(w + 210t) < \left[\frac{100}{110}\right] u'(w + 210(t-1)) < ... < \left[\frac{100}{110}\right]^t u'(w).$$

The preceding derivation illustrates the core of concavity calibration. Statement (5) implies the pattern of rapidly decreasing marginal utility of terminal wealth that has the striking implications derived in part A of the appendix and reported in the second column of Table 1. As shown in Table 1, an agent who would reject the 50-50, lose 100, gain 110 gamble at all initial wealth levels weakly between 100 and $M = 300{,}000$ would also reject a gamble in which he would, with equal probability, lose 30,000 or gain 166,933,671,700 when his initial wealth is 290,000. Therefore, calibration of concavity for the expected utility of terminal wealth model implies that an agent with small-stakes risk preferences that are consistent with the assumption will have implausible large-stakes risk aversion.

2.2 Implications for the Expected Utility of Income Model

Here we use the expected utility of income model, the model based on the expected utility axioms *and* the assumption that the prizes are random amounts of income. Assume that the agent rejects the 50-50 lose 100, gain 110 gamble. Letting μ denote the agent's Bernoulli utility function for income, the sufficient condition for rejecting the 50-50 lose 100, gain 110 gamble is:

$$(6) \qquad \mu(0) > 0.5\mu(-100) + 0.5\mu(110).$$

We next present an example of a Bernoulli function utility μ that both satisfies inequality (6) and implies plausible large-stakes risk aversion. Consider the following utility function μ for an agent with random income y :

(7) $$\mu(y) = 0.9y + 1, \text{ for } y < 0$$

$$= (y+1)^{0.9}, \text{ otherwise,}$$

This function is globally concave. An expected utility of income maximizer with utility function (7) will reject the 50-50, lose 100 or gain 110 gamble. But the agent would also *accept* the gambles in the "Equation (7) Acceptances" column of Table 1. For example, the agent would accept a gamble in which she would, with equal probability, lose 20,000 or gain 53,469. Thus, the assumed pattern of risk aversion over small-stakes gambles does not imply implausible risk aversion over large-stakes gambles with this model.

In conclusion, the assumed pattern of small-stakes risk aversion underlying arguments in previous literature has no implication of implausible large-stakes risk aversion for the expected utility of income model. Therefore, Rabin's and Thaler's arguments fail to establish the implausibility of expected utility theory.

3. Expected Utility of Initial Wealth and Income Model

The expected utility of income model is widely used in the theory of auctions but this model does not provide an explanation of how an agent's initial wealth affects its attitude towards risk. Thus the income model can only be used to address a narrower range of questions than can the expected utility of terminal wealth model (Pratt, 1964; Arrow, 1971). What is needed is a model in which risk attitude depends on initial wealth but income is not additive to initial wealth. We next provide such a model in order to demonstrate that, not only do Rabin's and Thaler's arguments fail to establish the implausibility of expected utility theory, they also fail to show that expected utility theory does not provide a coherent explanation of how initial wealth might affect an agent's willingness to bear small- and large-stakes risks.

Assume that the arguments of the utility function are ordered pairs of initial wealth and income. Let υ denote the agent's "Bernoulli" utility function for initial wealth and income. For any integrable probability distribution function F for random income y, the expected utility functional for this model is written as

$$(8) \qquad \int \upsilon(w,y)dF = E_F(\upsilon(w,y)),$$

where the function υ is strictly increasing in both arguments and (resp. strictly) concave in its second argument *if* the agent is (resp. strictly) risk averse. Although, in this model, risk attitude depends on initial wealth, the model is not called into question by the type of global small-stakes risk aversion assumed in previous literature, as we shall now demonstrate.

3.1 *Rationalizing Small- and Large-Stakes Risk Aversion*

Assume that an agent rejects the 50-50 lose 100, gain 110 gamble for all w between 100 and M, where M > 100 can be as large as one chooses. The sufficient condition for rejecting the gamble is

$$(9) \qquad \upsilon(w,0) > 0.5\upsilon(w,-100) + 0.5\upsilon(w,110), \text{ for all } w \text{ such that } 100 \leq w \leq M.$$

The following example demonstrates that this model can rationalize small- and large-stakes risk aversion.

Assume that the agent's "Bernoulli" utility function is

$$(10) \qquad \upsilon(w,y) = (0.9y + w/M)(w/M)^{-0.1}, \text{ for } y < 0$$

$$= (y + w/M)^{0.9}, \quad \text{otherwise.}$$

Inequality (9) is satisfied by this utility function. But the agent with utility function given by statement (10) would also *accept* the gambles in the "Equation (10) Acceptances" column of

Table 1 when $M = 300{,}000$ and $w =$ 290,000.[5] For example, the agent would accept a gamble in which she would, with equal probability, lose $20,000 or gain $53,671. Therefore, the assumed pattern of risk aversion for small-stakes gambles does not imply implausible risk aversion for large-stakes gambles with this model.

3.2 <u>*Comparative Risk Aversion*</u>

Having observed that the expected utility of initial wealth and income model is not called into question by the type of global small-stakes risk aversion assumed in previous literature, the next question is whether this model can be used in applications in which the central questions are concerned with the implications of different attitudes towards risk and their possible dependence on initial wealth, an area of fruitful application of the expected utility of terminal wealth model. We address this question by extending the Arrow-Pratt characterization of comparative risk aversion to the new model.

The "Bernoulli" utility functions for two agents can be written as $\upsilon^j(w, y)$, for $j = a, b$. The measure of absolute risk aversion for this model is

(11) $$A^j(w, y) = -\frac{\upsilon_{22}^j(w, y)}{\upsilon_2^j(w, y)}.$$

Let $\bar{y}$ be the mean value of income for the distribution F; then the risk premium, π^j is defined by

(12) $$\upsilon^j(w, \bar{y} - \pi^j(w, F)) = E_F(\upsilon^j(w, y)).$$

Given that the function υ^j is strictly increasing in its second argument, *y* there exists a *y*-inverse function ϕ^j defined by

(13) $$y = \phi^j(w, \upsilon^j(w, y)).$$

Define the function g as follows:

(14) $$g(w,u) = \upsilon^a(w, \phi^b(w,u)).$$

The measures of comparative risk attitudes for agents a and b are as given in the following theorem, which states that: (*i*) the absolute risk aversion measure for agent a is greater than the absolute risk aversion measure for agent b, if and only if, (*ii*) the risk premium for agent a is greater than the risk premium for agent b, if and only if, (*iii*) the utility function for agent a is a strictly increasing and strictly concave transformation of the utility function of agent b of the form given by the definition in equation (15).

Theorem 1. *If υ^a and υ^b are strictly increasing in y and twice differentiable then the following statements are equivalent:*

(i) $A^a(w,y) > A^b(w,y)$, *for all* (w,y);

(ii) $\pi^a(w,F) > \pi^b(w,F)$, *for all* w *and F;*

(iii) $\upsilon^a(w,y) = g(w,\upsilon^b(w,y))$, $g_2(w,u) > 0$, $g_{22}(w,u) < 0$, *for all* (w,u).

Proof: See part B of the appendix.

Theorem 1 makes clear that the Arrow-Pratt characterization of agents' comparative risk aversion can be extended from the expected utility of terminal wealth model to the two-argument, expected utility of initial wealth and income model. Hence agents' risk-avoiding behavior can be modeled with the new model rather than the expected utility of terminal wealth model that is called into question by the critique in the literature. Therefore, Rabin's and Thaler's arguments not only fail to establish the implausibility of expected utility theory, they also fail to show that expected utility theory does not provide a coherent explanation of how agents' initial wealth and comparative risk aversion affect their willingness to bear small- and large-stakes risks.

4. Taking the Con Out of Concavity Calibration

In Section 2, we demonstrated that the Rabin-Thaler concavity-calibration arguments do not support their conclusion that expected utility theory should be replaced. We now re-examine their claim that a model with loss aversion and isolation of each risky choice is the solution to the problems associated with concavity calibration. We explain that a decision-theoretic model with loss aversion and isolation of each risky choice, such as prospect theory, will not provide a viable alternative to expected utility theory *if* reformulated concavity-calibration arguments do have problematic implications for expected utility theory.[6] Our approach will be to: (a) identify an alternative risk aversion assumption that would, *if* empirically valid, imply that all three of (the above) expected utility models had implausible large-stakes risk aversion; and (b) show that this same assumption would imply implausible large-stakes risk aversion for prospect theory.

Consider the following alternative to the risk aversion assumption underlying Rabin's and Thaler's arguments. Assume that an agent prefers the certain amount of income, x to playing a 50-50 bet with income payoffs $x-100$ and $x+110$, for all values of x between 100 and M, where M > 100 can be as large as one chooses. The assumed pattern of risk aversion implies the following inequality for the expected utility of terminal wealth model:

(15) $$u(w+x) > 0.5u(w+x-100)+0.5u(w+x+110)$$

for all x such that $100 \le x \le M$. The assumed pattern of risk aversion implies that, for the same income interval,

(16) $$\mu(x) > 0.5\mu(x-100)+0.5\mu(x+110)$$

for the expected utility of income model and

(17) $$W^+(1)v(x) > W^+(0.5-0)v(x-100)+W^+(1-0.5)v(x+110)$$

for cumulative prospect theory, where $W^+(\cdot)$ is the probability weighting function for gains and $v(\cdot)$ is the value function for income that is concave for gains (Tversky and Kahneman, 1992) .

First consider the expected utility of income model. The assumption that inequality (16) holds for $M = 300{,}000$ implies that the agent will reject the gambles in the second ("Inequality (1) Rejections") column of Table 1. Obviously, the same rejections are implied by inequality

(15) for the expected utility of terminal wealth model if $w = 0$. For $w > 0$, inequality (15) implies even larger minimum gain figures than the ones in the second column of Table 1. Furthermore, a similar argument to the one used here for the expected utility of income model will apply to the expected utility of initial wealth and income model.

Next, consider cumulative prospect theory and assume that inequality (17) holds for $M = 300{,}000$. Since the weighting function $W^{+}(\cdot)$ over-weights the lower payoff $(x-100)$ and under-weights the higher payoff $(x+110)$, and the $W^{+}(\cdot)$ weights on the right-hand-side of (17) sum to one (Tversky and Kahneman, 1992), the certainty equivalent predicted by prospect theory is not larger than the one predicted by expected utility. Therefore, the minimum gain figures for cumulative prospect theory are at least as large as the figures in the second column of Table 1.[7] Hence, if one believes that concavity-calibration arguments imply that expected utility theory does not provide a tenable theory of small-stakes and large-stakes risk aversion, then logic requires that he also believe that prospect theory does not provide such a tenable theory of risk aversion. Loss aversion will not rescue decision theories with concave utility or value functions for gains *if* the risk aversion assumption underlying the analysis in this section has empirical validity (and, hence, expected utility theory does have implausible large-stakes risk aversion).

5. Concluding Remarks

We explain that the type of global small-stakes risk aversion assumed in previous literature (Rabin, 2000; Rabin and Thaler, 2001) has no implication for the expected utility of income model, hence no general implication for expected utility theory. But, if credible empirical support were to be provided for the type of risk aversion assumed by Rabin and Thaler, then the expected utility of terminal wealth model would have been shown to imply implausible large-stakes risk aversion. This could be a problem because there have been many fruitful applications of the expected utility of terminal wealth model based on the characterization of comparative risk aversion developed by Arrow (1971) and Pratt (1964). This is our motivation for discussion of a two-argument model for which risk attitude does depend on initial wealth but initial wealth is not additive to income in the utility function. We show that this model is immune to the concavity-calibration critique in previous literature. In order to demonstrate that this two-argument model should have fruitful applications, we extend the Arrow-Pratt characterization of comparative risk aversion to it.

We also consider the implications of an alternative assumed type of risk aversion. This alternative assumption makes the concavity-calibration critique apply to both of the conventional expected utility models and the two-argument, expected utility of initial wealth and income model. But concavity-calibration and this type of risk aversion have similar implications for prospect theory. Therefore, if one believes that concavity-calibration arguments imply that expected utility theory does not provide a coherent theory of small- and large-stakes risk aversion then logic requires that he have the same negative belief about prospect theory (and other decision theories with concave value or utility functions).

It is important to realize that, to date, neither of the risk aversion assumptions discussed above has credible empirical support. Thus the actual implications of concavity calibration for decision theory are presently unknown. In ongoing empirical research, we are attempting to obtain data that can help resolve these important issues.

Endnotes

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1. Use of the expected utility of income model has been widespread in the theory of auctions beginning with Vickrey (1961). A few other examples from the large literature of papers that develop bidding theory with Bayesian-Nash equilibrium and the expected utility of income model are the following: Holt (1980), Harris and Raviv (1981), Riley and Samuelson (1981), Cox, Smith, and Walker (1982), Matthews (1983), Milgrom and Weber (1982), Maskin and Riley (1984), and Moore (1984).

2. Rubinstein (2001) also explains that the Rabin-Thaler arguments have no implication for the expected utility of income model.

3. Loss aversion, together with risk aversion, can be incorporated into a decision-theoretic model with a utility or value function for income that is strictly concave for gains and steeper in the loss domain.

4. See the appendix in Rabin (2000) for a derivation that does not use differentiability.

5. The assumption that an agent rejects the small-stakes gamble for wealth levels up to 300,000 and the evaluation in the table for the wealth level of 290,000 uses the same values as did Rabin (2000, p. 1284), for the expected utility of terminal wealth model.

6. Of course, loss aversion as an empirical phenomenon does not discriminate between prospect theory and expected utility theory. The expected utility of income model and the expected utility of initial wealth and income model are both consistent with loss aversion.

7. Evaluation for $x \geq 100$ makes loss aversion irrelevant to the argument.

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Table 1. Loss/Gain Bounds for Alternative Calibrations

	Inequality (1) Rejections EUW Model*	**Equation (7) Acceptances EUI Model**	**Equation (10) Acceptances EUW&I Model***
Loss	**Gain**	**Gain**	**Gain**
400	420	694	697
600	630	1,088	1,092
800	1,050	1,497	1,503
1,000	1,470	1,918	1,925
2,000	90,090	4,142	4,157
4,000	836,430	8,945	8,978
6,000	2,690,310	14,034	14,087
8,000	7,277,550	19,319	19,392
10,000	18,665,850	24,754	24,847
20,000	1,784,396,250	53,469	53,671
30,000	166,933,671,700	83,897	84,215

* Rejections and acceptances are derived for an initial wealth level of 290,000 given the assumption that the agent rejects the 50-50 lose 100, gain 110 gamble for all $w \in [100,300,000]$.

Appendix

Part A. Completion of the Concavity Calibration

Derivation of an inequality like statement (5) in the text that applies to for all 50-50 gambles is straightforward. Assume that an agent will reject a 50-50 bet with loss amount ℓ and gain amount g for all w such that $\ell \le w \le M$ and $M > \ell$. The logic used in the text implies

$$\text{(a.1)} \qquad u'(w+(g+\ell)t) < \left[\frac{\ell}{g}\right]^t u'(w)$$

for all integers $t \in$ $T = \{t \in Z \mid w+(g+\ell)t - g \in [\ell, M]\}$. We will show that statement (a.1) implies that the agent will reject the 50-50 bet with loss L and gain G,

$\forall L \in (0,w)$, $\forall G > 0$ such that

$$\text{(a.2)} \qquad F(g/\ell, L) \ge F(\ell/g, G)$$

where $F(r,X) = r^k\left(X/(g+\ell)-k\right) + \sum_{t=1}^{k} r^{t-1}$ and k is the largest integer in T less than or equal to $X/(g+\ell)$. Denote $z_x = w+(g+\ell)x$, for $x \in R$, and note that, for $r = g/\ell$, $X = L$, and $k = n$, one has

$$\text{(a.3)} \qquad \begin{aligned} \frac{u(w)-u(w-L)}{g+\ell} &= \int_{-L/(g+\ell)}^{-n} u'(z_x)dx + \sum_{t=1}^{n}\int_{-t}^{-t+1} u'(z_x)dx \ge \int_{-L/(g+\ell)}^{-n} u'(z_{-n})dx + \sum_{t=1}^{n}\int_{-t}^{-t+1} u'(z_{-t+1})dx \\ &> \left((g/\ell)^n \left(L/(g+\ell)-n\right) + \sum_{t=1}^{n} (g/\ell)^{t-1} \right) u'(w) = F(g/\ell, L) \end{aligned}$$

and, for $r = \ell/g$, $X = G$, and $k = m$, one has

$$\text{(a.4)} \qquad \begin{aligned} \frac{u(w+G)-u(w)}{g+\ell} &= \int_{m}^{G/(g+\ell)} u'(z_x)dx + \sum_{t=1}^{m}\int_{t-1}^{t} u'(z_x)dx \le \int_{m}^{G/(g+\ell)} u'(z_m)dx + \sum_{t=1}^{m}\int_{t-1}^{t} u'(z_{t-1})dx \\ &< \left((\ell/g)^m \left(G/(g+\ell)-m\right) + \sum_{t=1}^{m} (\ell/g)^{t-1} \right) u'(w) = F(\ell/g, G) \end{aligned}$$

where the weak inequalities follow from concavity and the strict inequalities follow from statement (a.1). Statements (a.2), (a.3) and (a.4) immediately imply rejection of lottery *(0.5;-L,+G)* at initial wealth *w*.

Note that for $M = \infty$, $F(\ell / g, \infty)$ is a finite number whereas $F(g / \ell, L)$ takes values as large as one wants at finite L. This implies that for some finite L and $G = \infty$ condition (a.2) is satisfied, hence the agent rejects lottery $(0.5; -L, +\infty)$.

Part B. Proof of Theorem 1

We first show that statements (*i*) and (*iii*) in Theorem 1 imply each other: (*i*) $\leftrightarrow$ (*iii*). Differentiation of the functions in statement (*iii*) with respect to with respect to y yields

(b.1) $\upsilon_2^\alpha = g_2 \upsilon_2^\beta$

and

(b.2) $\upsilon_{22}^\alpha = g_{22} (\upsilon_2^\beta)^2 + g_2 \upsilon_{22}^\beta$.

The definition of $A^j(w, y)$ and statements (b.1) and (b.2) imply

(b.3) $$g_{22} = \frac{\upsilon_{22}^\alpha - g_2 \upsilon_{22}^\beta}{\left(\upsilon_2^\beta\right)^2} = \left[\frac{\upsilon_{22}^\alpha}{\upsilon_2^\alpha} - \frac{\upsilon_{22}^\beta}{\upsilon_2^\beta}\right] \frac{\upsilon_2^\alpha}{\left(\upsilon_2^\beta\right)^2} = \left[\mathrm{A}^\beta - A^\alpha\right] \frac{\upsilon_2^\alpha}{\left(\upsilon_2^\beta\right)^2}.$$

Statement (b.1) implies that $g_2(w,u) > 0, \forall (w,u),$ because $\upsilon_2^\alpha(w, y) > 0$ and $\upsilon_2^\beta(w, y) > 0$, $\forall (w, y)$. Statement (b.3) implies that $g_{22}(w,u) < 0, \forall (w,u),$ if and only if

$A^\alpha(w, y) > A^\beta(w, y), \forall (w, y).$

We next show that statement (*iii*) in Theorem 1 implies statement (*ii*) in the theorem: (*iii*) $\rightarrow$ (*ii*). Jensen's inequality and the definitions imply

(b.4) $$\begin{aligned} &\upsilon^\alpha(w, \phi^\beta(w, E_F(\upsilon^\beta(w, y)))) = g(w, E_F(\upsilon^\beta(w, y))) \\ &> E(g(w, \upsilon^\beta(w, y))) = E(\upsilon^\alpha(w, \phi^\beta(w, \upsilon^\beta(w, y)))) = E(\upsilon^\alpha(w, y)). \end{aligned}$$

Therefore

(b.5) $$\begin{aligned} &E_F(y) - \pi^\beta(w, F) = \phi^\beta(w, E_F(\upsilon^\beta(w, y))) = \phi^\alpha(w, \upsilon^\alpha(w, \phi^\beta(w, E_F(\upsilon^\beta(w, y))))) \\ &> \phi^\alpha(w, E_F(\upsilon^\alpha(w, y))) = E_F(y) - \pi^\alpha(w, F). \end{aligned}$$

Therefore statement (*iii*) implies statement (*ii*) in Theorem 1.

We next show that statement (*ii*) in Theorem 1 implies statement (*iii*) in the theorem: (*ii*) $\rightarrow$ (*iii*). Statement (*ii*) and the definitions imply

$$\begin{aligned} &\upsilon^{\alpha}(w,\phi^{\beta}(w,E_F(\upsilon^{\beta}(w,y)))) = \upsilon^{\alpha}(w,E_F(y)-\pi^{\beta}(w,F)) \\ &> \upsilon^{\alpha}(w,E_F(y)-\pi^{\alpha}(w,F)) = \upsilon^{\alpha}(w,\phi^{\alpha}(w,E_F(\upsilon^{\alpha}(w,y)))) = E_F(\upsilon^{\alpha}(w,y)). \end{aligned} \tag{b.6}$$

Hence

$$\begin{aligned} &g(w,E_F(\upsilon^{\beta}(w,y))) = \upsilon^{\alpha}(w,\phi^{\beta}(w,E_F(\upsilon^{\beta}(w,y)))) \\ &> E_F(\upsilon^{\alpha}(w,y)) = E_F(\upsilon^{\alpha}(w,\phi^{\beta}(w,\upsilon^{\beta}(w,y)))) = E_F(g(w,\upsilon^{\beta}(w,y))). \end{aligned} \tag{b.7}$$

Therefore, g is strictly concave in u: $g_{22}(w,u) < 0, \forall (w,u)$.