THE AMSTERDAM AUCTION

BY JACOB K. GOEREE AND THEO OFFERMAN[1]

ABSTRACT

The Amsterdam auction has been used to sell real estate in the Dutch capital for centuries. By awarding a premium to the highest losing bidder, the Amsterdam auction favors weak bidders without having the implementation difficulties of Myerson's (1981) optimal auction. In a series of experiments, we compare the standard first-price and English auctions, the optimal auction, and two variants of the Amsterdam auction. With strongly asymmetric bidders, the second-price Amsterdam auction raises substantially more revenues than standard formats and only slightly less than the optimal auction.

JEL Classification: C72, D44.
Keywords: Auctions, experiments, asymmetries, premium, practical mechanism design.

1. INTRODUCTION

IN EUROPE, SELLERS OF HOUSES, land, boats, machinery and equipment regularly offer a premium to the highest losing bidder to promote competitive bidding. Many Dutch and Belgian towns have their own variant of premium auctions, some of which date back to the Middle Ages (Sikkel, 2001). A particularly prominent example is the "First Amsterdam Real Estate Auction," conducted every other week in the center of the Dutch capital.

In this paper, we investigate why sellers would use premium auctions such as the one in Amsterdam. The formats we consider involve two stages, as is common for most premium auctions observed in practice. In the first stage, the price level rises until all but two bidders have dropped out. The level at which the last bidder exits is called the bottom price, which acts as a reserve price for the second stage. In this stage, both finalists make sealed bids, the highest bidder wins the object, and both bidders receive a premium proportional to the difference between the lowest sealed bid and the bottom price. In the "first-price Amsterdam

[1] We would like to thank the editor and three anonymous referees for helpful comments, and Hessel Abbink Spaink, Jan Bouckaert, and Didi van den Elsaker, president of the First Amsterdam Real Estate Auction, for many historical details. We benefitted from helpful comments from seminar participants at Harvard Business School, New York University, University of Antwerp, University of Bonn, ETH Zurich, CERGE-EI Prague, Erasmus University Rotterdam, T.J. Watson Research Center and the University of Cologne. We gratefully acknowledge financial support from the Alfred P. Sloan Foundation, the Bankard Fund at the University of Virginia, the NSF (SBR 0094800), and the Royal Dutch Academy of the Sciences (KNAW).

auction" the winner pays her own bid, while she pays the other finalist's sealed bid in the "second-price Amsterdam auction."[2]

One plausible reason for introducing a premium is that real-estate auctions often display *ex ante* bidder-asymmetries: speculators out for a bargain compete with buyers that have a genuine interest in the house.[3] Below we show that in some cases of asymmetry, Amsterdam auctions raise more revenues than an English auction. The intuition is that a premium encourages weak bidders to set an endogenous reserve price for stronger rivals, while Bertrand competition between weak bidders dissipates the premium they are paid.

Of course, any (Amsterdam) auction is revenue dominated by Myerson's (1981) optimal auction. Optimal auctions are difficult to implement, however, which may explain why they are rarely used in practice. Indeed, sellers usually lack the detailed information needed to choose proper bidding credits and bidder-specific reserve prices. Furthermore, national or international law may prohibit sellers from discriminating among bidders (as in Europe). Finally, substantial reserve prices may be considered too risky or incredible.

We report the results of a series of experiments to compare the performance of standard first-price and English auctions with that of optimal and Amsterdam auctions. We consider both symmetric and asymmetric settings. In the latter case, multiple equilibria exist, and experiments may help identify the equilibria that are selected in practice. In addition, it is interesting to see whether weak bidders in Amsterdam auctions are willing to take risks for possibly small premiums, as presumed by the theory. A comparison with Myerson's optimal auction reveals how much revenue is potentially lost.[4]

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[2] See **http://www.troostwijk.be/de/alg_neen.htm** for a premium auction that differs slightly from the ones considered here. Klemperer (2002) considers two-stage auctions without a premium and introduces the terminology "Anglo-Dutch" and "Anglo-Anglo" auctions. Awarding a premium is the main feature of the auctions studied here, and we use the terminology first-price and second-price Amsterdam auctions to indicate the difference.

[3] Amsterdam auctions attract "premium hunters" who only participate to win the premium. If a premium hunters ends up winning the house for sale, he is called a "hanger." To prevent default of a hanger, bidders have to show a bank guarantee prior to the auction or they must bid via a broker with an established reputation. In former times, if a hanger could not pay for the house he won, he would be sent to prison for one or two months. If it happened twice, he would be tortured (Sikkel, 2001).

[4] Standard symmetric auctions have been tested extensively in the laboratory, see Kagel (1995) for an excellent survey. The following experiments concern asymmetric auctions: Corns and Schotter (1999), Kagel and Levin (1999), Pezanis-Christou (2002).

2. Theoretical Background

2.1. *The Symmetric Case*

There are n bidders with values drawn from a uniform distribution on [0,1]. The Amsterdam auctions involve two stages and their equilibria are derived via backward induction.[5] Consider, for instance, the first-price Amsterdam auction. Let B_i denote the optimal bidding function for stage i = 1, 2, and let v_3 be the value of the bidder that determined the bottom price X in the first stage, that is $X = B_1(v_3)$. The second-stage expected payoffs of a bidder with value v who bids as if her value is $w \geq v_3$ equal:

$$\pi_2^{\,e}(w\,|\,v) = (1 - v_3)^{-1}\{(v - \boldsymbol{B}_2(w))(w - v_3) + \alpha\int_{v_3}^{w}(\boldsymbol{B}_2(z) - X))\,dz + \alpha\,(\boldsymbol{B}_2(w) - X)(1 - w)\}, \tag{1}$$

where the first two terms apply when the bidder wins and the final term occurs when she loses. More specifically, the first term represents the high bidder's surplus from winning the object, the second term is a bidder's expected premium when she wins and the final term is the premium when she loses. The first-order condition for profit maximization becomes:

$$v - \boldsymbol{B}_2(v) + \boldsymbol{B}_2'(v)(\alpha - (1 + \alpha)v - v_3) = 0. \tag{2}$$

The solution to (2) is $B_2(v) = (v + v_3 + \alpha)/(2 + \alpha)$. Since a bidder with value v_3 expects zero payoffs when entering the second stage, (1) implies $B_2(v_3) = X$ so $v_3 = (2 + \alpha)X/2 - \alpha/2$. Hence, the second-stage optimal bid can be written as $B_2(v) = (v + \alpha/2)/(2 + \alpha) + X/2$. Finally, the optimal bids for the first stage follow from the condition $B_1(v_3) = B_2(v_3)$ for all possible realizations of v_3, which yields $B_1(v) = (2\,v + \alpha)/(2 + \alpha)$.

Following the same steps reveals that in the second-price Amsterdam auction the equilibrium bids are the same in the first and second stage: $B_1(v) = B_2(v) = (v + \alpha)/(1 + \alpha)$.

In the symmetric case, Amsterdam auctions yield the same revenues as the first-price and English auctions, as the assumptions underlying the revenue equivalence theorem are satisfied. Interestingly, awarding a premium results in a less variable revenue compared to an

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[5] The optimal bids for the standard first-price and English auctions are $B_{FP}(v) = (n\text{-}1)\,v/n$ and $B_E(v) = v$ respectively. Truthful bidding is also an equilibrium in the optimal auction, which can be implemented as a second-price auction with a reserve price of 1/2 (independent of the number of bidders, e.g. Riley and Samuelson, 1981).

English auction. The intuition is that with a positive premium, $\alpha > 0$, bidding functions become "less steep," which reduces the dependence of the revenue on the uncertain value draws.[6] The next proposition summarizes.

Proposition 1. The seller's revenue is highest for the optimal auction and is the same for the first-price, English, and Amsterdam auctions. The variances in revenues of the Amsterdam auctions are less than that of an English auction but no less than that of a first-price auction.

2.2. *The Asymmetric Case*

Next consider the case when n - 1 "weak" bidders with uniform values on [0,1] compete against a single "strong" bidder whose value is uniformly distributed on $[L, H]$ where $L > 0$ and $H > 1$. In the English auction truthful bidding remains an equilibrium. The same is true for the optimal auction, where weak bidders receive bidding credits equal to $(H - 1)/2$ (see Myerson, 1981). In other words, a weak bidder with bid b_W beats a strong bidder with bid b_S if $b_W > b_S - (H - 1)/2$. In addition, reserve prices are 1/2 for weak bidders and $\max(L, H/2)$ for strong bidders.

When $L > 0$, there exist no closed-form expression for the equilibrium bids in the first-price auction although its unique solution (Maskin and Riley, 1996) can be found using numerical techniques.[7] We have not been able to derive the equilibria of the Amsterdam auctions in the weakly asymmetric case $0<L<1$. However, the strongly asymmetric case, $L > 1$, can be used to demonstrate the potential revenue-generating virtues of the Amsterdam auctions. Consider, for instance, the second-price Amsterdam auction and suppose weak bidders drop out at $X < L$. Irrespective of a weak bidder's value, it then pays to stay in the auction somewhat longer. To see this, note that in the second stage the strong bidder has an incentive to bid at least her value: bids below value can only cause her to lose at a price she

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[6] The variances in revenues for the auctions are: $V_E = 2(n-1)/((n+1)^2\,(n+2))$, $V_{FP} = (n-1)/(2\,n)\,V_E$, $V_{SPA} = V_E - 2\alpha(2-\alpha)/((1+\alpha)^2\,(n+1)\,(n+2))$, and $V_{FPA} = V_E - 2(1+2\,\alpha-\alpha^2)/((2+\alpha)^2\,(n+1)\,(n+2))$ for the English, first-price, and Amsterdam auctions respectively. It is readily verified that the claims of Proposition 1 hold whenever $n \geq 3$. It is also straightforward to show that the variance in revenue from an optimal auction exceeds that of an English auction.

[7] The experiment employs discrete values, which facilitates numerical computations.

would have liked to win, and they may lower the premium she receives. The weak bidder can therefore safely bid L, in which case she earns $\alpha(L - X) > 0$ independent of her value. Of course, this positive expected profit will attract other weak bidders as well. In equilibrium, Bertrand competition between weak bidders dissipates this potential profit, while their bids create an "endogenous reserve price" for the strong bidder.[8]

Proposition 2. The following is an equilibrium outcome of the second-price Amsterdam auction if $\alpha \leq (L - 1)/(H - L)$ and $L > 1$. In the first stage, the strong bidder bids up to her value and weak bidders bid up to L, at which point $n - 2$ weak bidders drop out. In the second stage, weak bidders bid L and the strong bidder bids her value.

The outcome of Proposition 2 is not the unique equilibrium outcome.[9] For example, another equilibrium is for all weak bidders to bid up to some common price level X in the first stage, with $1 \leq X \leq L$, at which point $n - 2$ weak bidders drop out (and the strong bidder bids up to her value). In the second stage, weak bidders bid X and the strong bidder bids (slightly above) X.[10] Note, however, that in this equilibrium the *strong* bidder's strategy is weakly dominated. By bidding below her value she may lose at a price level she would have like to win, and she may reduce the premium. The outcome of Proposition 2 does not involve weakly dominated strategies for the strong bidder, and therefore seems most natural for the second-price Amsterdam auction.

In the first-price Amsterdam auction, however, the strong bidder has an incentive to "shade" her bid in the second stage and no outcome seems particularly focal.

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[8] In the experiment, the premium parameter α is chosen such that it satisfies the condition in Proposition 2.

[9] Likewise, there exist multiple equilibria in the asymmetric English auction, e.g. strong bids truthfully while weak bidders bid any amount between 0 and $L > 1$. Hence, when $L > 1$ any revenue between 0 and L can be sustained in equilibrium.

[10] This is an equilibrium if the tie-breaking rule favors the strong bidder. In the experiment, bids were discrete so "bidding slightly above X" is well defined.

Proposition 3. Let X be any price level between 1 and L > 1. The following is an equilibrium outcome of the first-price Amsterdam auction. In the first stage, the strong bidder bids up to her value and all weak bidders bid up to X, at which point n - 2 weak bidders drop out. In the second stage, weak bidders bid X and the strong bidder bids (slightly above) X.

3. Experimental Design

The computerized experiments consisted of three parts of twelve periods each (plus two practice periods): a symmetric environment, a weakly asymmetric environment and an asymmetric environment. Only after one part of the experiment was completed did subjects receive instructions for a new part. Subjects received a starting capital of 60 "points". They also earned points in the experiment, which were exchanged into local currency afterwards.[11] Table I summarizes the four treatments. For statistical reasons subjects were allocated to the same group of 4 bidders in each period.[12] In each period, subjects received information about their own private values only. Values were independent across bidders and periods. The role of strong bidder rotated each period and subjects were informed whether they were weak or strong. The rules and the procedure to generate valuations were common knowledge.

In the first-price and optimal auctions, subjects simultaneously submitted their bids and the winning bidder received a profit equal to her valuation minus the price. All other formats had an ascending phase, implemented by a "thermometer" that rose point-by-point starting at zero. The thermometer’s "temperature" showed the price that active bidders were willing to pay. A bidder’s "stop" decision was irrevocable. Other bidders in the group were immediately informed when one of their rivals had dropped out (and, when applicable, whether the bidder was weak or strong). In the English auction, the one bidder who did not drop out received a profit equal to her valuation minus the price level at which the last rival had dropped out.

[11] For each point a subject received the equivalent of about $0.12. The optimal auction was run in December 2002, while the other treatments were run in February 2001.

[12] Subjects were not informed about this aspect to avoid repeated game considerations.

In the first stage of an Amsterdam auction, the thermometer kept rising until two of the four bidders had dropped out: the bottom price. In the second stage, the two remaining bidders simultaneously submitted sealed bids no less than the bottom price.[13] Bidders knew whether their opponent was weak or strong in this final stage. Both finalists received a premium equal to 30 percent of the difference between the lowest sealed bid and the bottom price. The highest bidder received her value and paid her own bid in the first-price Amsterdam auction and the other's sealed bid in the second-price Amsterdam auction.

In the optimal auction subjects had the option to not submit a bid. Submitted bids had to be no less than the reserve price. The first part of the experiment was a second-price auction with a reserve price of 30. In the other parts, submitted bids were weighted. The strong bidder's weighted bid was equal to her submitted bid minus 20, while the weak bidders' weighted bids equaled their submitted bids. The winner was the bidder with the highest weighted bid. The winner paid a price equal to the lowest possible submitted bid with which she would have won. The reserve price for weak bidders was equal to 30 in parts 2 and 3, while the reserve price for the strong bidder was 50 in part 2 and 70 in part 3. When none of the bidders submitted a bid, all bidders earned 0 profits.[14] [15]

4. Results

4.1. *Revenue, Variance, and Efficiency*

Figure 1 shows revenue histograms for all three parts of all treatments.[16] In the symmetric case, revenues seem highest for the first-price auction, followed by the English, the first-price Amsterdam and optimal auction. Small changes occur when weak asymmetries are

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[13] Sealed bids were not allowed to be higher than 60 (100) points in the symmetric (weakly asymmetric and asymmetric) environment. In case of a tie, the computer randomly chose a winner.

[14] At the end of each period, subjects were informed about their profits (but not about that of others). In the first-price auction, subjects were told all bids in their group (ranked from low to high) and which bid was made by the strong bidder. Similar information was automatically available to subjects in the other formats.

[15] The instructions for the Second-Price Amsterdam auction are presented in the extended version of this paper: **http://www.fee.uva.nl/creed/pdffiles/adam.pdf**. The extended version also discusses the bankruptcy procedure. The experiment was programmed using the *Rat-Image* toolbox (Abbink and Sadrieh, 1995).

[16] We consider *net* revenues of the Amsterdam auctions, defined as the winner's payment minus the premiums paid to the bidders.

introduced. The first-price, first-price Amsterdam and optimal auctions perform best, although the latter does so at a high variance. However, the picture is completely different for the asymmetric case. Now the optimal auction generates the highest revenue at a low variance. Both variants of the Amsterdam auction revenue dominate standard auctions, with the second-price Amsterdam auction raising the most. The first-price auction again outperforms the English auction, which quite frequently yields very low revenues.

Table II presents further evidence for these claims and a comparison with Nash predictions.[17] In the symmetric case, the first-price auction generates significantly higher revenues than all other formats.[18] With weak asymmetries, the observed revenues of all formats increase while their variances decrease. Here, the first-price auction, the first-price Amsterdam auction and the optimal auction all raise significantly more revenue than the other two formats (the differences between them are insignificant). In the symmetric and weakly asymmetric bidding environments, Nash predictions trace actual revenues quite well, except for the first-price auction where (risk-neutral) Nash underpredicts.

Consistent with Nash predictions, the first-price auction also does well when strong asymmetries are introduced. Revenue rises and its variance falls. In contrast, the English auction performs much worse. First, the observed variance in revenues jumps sharply (the standard deviation of the revenue is twice as high as predicted). Second, its revenues are low (although comparable to predicted levels). The Amsterdam auctions perform better: they yield high revenues with low variability. In the set of formats that do not discriminate among bidders, the second-price Amsterdam auction yields significantly higher revenues than other formats (the difference with the first-price Amsterdam auction is only significant at the 10% level), and it is the only non-discriminating format for which revenues exceed the weak bidders' maximum value of 60. The second-price Amsterdam auction raises only 7% less revenue than the optimal auction, which significantly outperforms all other formats.

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[17] The Nash predictions reflect that the draws were slightly unfavorable for the first-price auction and the optimal auction.

[18] The text mentions Mann-Whitney test results at a 5% significance level. The tests use independent average data per group as observations (n_{FP}=9;n_{En}=8;n_{FPA}=9;n_{SPA}=8;n_{Opt}=9).

While the English auction yields low revenues it may still be preferred for efficiency reasons. Table III, which displays average efficiency levels in all treatments for symmetric and asymmetric settings, shows that the English auction is most efficient. Surprisingly, efficiency losses are small when other formats are used. Indeed, the first-price auction yields efficiency levels comparable to the English auction (although the small differences are significant), and also the first-price Amsterdam auction yields efficiency levels of 94 percent. Efficiency losses are more pronounced only for the optimal auction (in the symmetric case) and the second-price Amsterdam auction (in the asymmetric case).

Sometimes it is argued that ascending auctions discourage entry in asymmetric situations because entrants anticipate they have no chance of winning (Klemperer, 2002). In our experiment, bidders did not make a formal entry decision but they could drop out at low prices. Table IV reports the proportion of almost-zero bids. In the first-price auction this percentage is roughly constant across bidding environments. In the Amsterdam auctions, the low percentages of almost-zero bids in the symmetric case fall when asymmetries are introduced. The English auction, however, shows a reverse pattern. The percentage of almost-zero bids is small in symmetric settings but dramatically increases to almost 40% in the asymmetric case. In the optimal auction, the substantial reserve prices lead to relatively little entry. Table V demonstrates that competition between the bidders dissipates the premium available in the Amsterdam auctions to a large extent.

4.2. *Individual Bids and Equilibrium Selection*

In the English auction, the proportion of bids equal (or close) to values decreases dramatically when asymmetries are introduced. In the symmetric environment, bidders' drop-out levels do not differ more than 1 point from their values in 73.4% of the cases, versus 44.8% and 18.3% in the weakly asymmetric and asymmetric environments respectively. In the asymmetric case, bidders more often drop out at very low levels (34.9% of the bids are 20 or more points below value) than at very high levels (14.2% of the bids are 20 or more points above value). Thus, the equilibrium where weak bidders drop out early is selected more often.

In line with previous symmetric first-price auctions (e.g., Cox, Smith, and Walker, 1988), observed bids exceed risk-neutral Nash predictions. In the weakly asymmetric setting, the Nash equilibrium fits the bids of weak bidders quite well but it underpredicts for strong bidders. In the asymmetric part, bids conform nicely to the predictions although the Nash equilibrium slightly overpredicts for both weak and strong bidders. This is intuitive as weak bidders make no profits in the Nash equilibrium and any noise in behavior will push their bids downward, which may be profitable if the strong bidder is noisy too.

The optimal auction elicits truthful bidding in all three parts. Overall, submitted bids are only slightly higher than predicted (on average 32.7 versus 32.0): 74.7% of the submitted bids are equal to Nash bids, while an additional 10.6% differs by only 1 point.

In the first-price Amsterdam auction, bidders tend to exit somewhat earlier than the risk-neutral Nash prediction in the symmetric case (Figure 2). With asymmetries (periods 25-36), weak bidders correctly anticipate that they can remain active longer and exit functions become flatter. There are some discrepancies with Nash predictions. Recall that in an equilibrium of the first-price Amsterdam auction, weak bidders are predicted to bid up to a level between 60 and 70, independent of their value. Figure 2 shows that weak bidders quit before 60 and that their drop-out levels are increasing in their values. These behavioral corrections to the "knife-edge" Nash predictions are intuitive since the Nash equilibrium requires weak bidders to take large risks for a small premium. In accordance with Nash predictions, observed bidding functions of the two finalists in the asymmetric part are nearly flat for weak and strong bidders. There appear to be two focal equilibrium outcomes for the strong bidders in this auction: a small majority of the strong bidders bid the maximum of the weak bidders' values (60) while the rest bid the minimum of the strong bidders' values (70).

In the second-price Amsterdam auction, weak bidders' behavior in the final stage contrasts with their cautious exit choices in the first stage (Figure 2). Average sealed bids at 70 and rise to 75 for high values.[19] Observed bids of strong bidders are close to their values with a slight downward bias. This small bias may be due to random errors or a conscious attempt to punish overly aggressive weak bidders. Indeed, for a strong bidder it is not very

[19] One possible explanation is that the auction selects the more risk averse bidders to exit early, while the risk loving types stay until the second stage. Alternatively, a weak bidder who makes it to the final stage could submit a low bid but then the resulting premium is small and the risk taken in vain. To be consistent with her choice not to drop out in the first stage, a weak bidder may therefore be inclined to submit a substantial sealed bid.

costly to bid slightly below value while the punishment for a weak bidder may be severe. In the experiment, some strong bidders with relatively low values decided to quit early (at prices between 60 and 70), making sure a weak bidder got "burned."[20] Nevertheless, such deviations are rare and observed second stage bids are relatively close to the equilibrium where the strong bidder bids her value.

5. CONCLUSIONS

Amsterdam auctions stimulate weak bidders to compete aggressively without having the implementation difficulties of Myerson's (1981) optimal auction. In a series of experiments, we compare the standard first-price and English formats with the optimal auction and two variants of the Amsterdam auction. When asymmetries between bidders exist, the second-price Amsterdam auction raises substantially more revenue than the standard formats and only slightly less revenue than the optimal auction.

Bidders in our experiment did not make a formal entry decision but they could drop out at very low prices. We find that much fewer weak bidders drop out at low prices in Amsterdam auctions than in the English auction. With more entry Amsterdam auctions are less prone to collusion. If weak bidders suspect the auction proceeds will be divided among the members of a cartel they have a strong incentive to take part and pursue the premium. Perhaps the most persuasive argument for the success of the Amsterdam auction is its century-long use as a means to sell real estate. Amsterdam auctions are prime examples of practical mechanism design and the introduction of a premium is a robust and costless way to enhance revenues in an otherwise uncompetitive asymmetric setting.

CREED, University of Amsterdam, Roetersstraat 11, 1018 WB Amsterdam, The Netherlands

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[20] In some sense, weak bidders are "cursed" in the second-price Amsterdam auction, which stimulates aggressive bidding too much and leaves them with negative earnings (-1.18 points per period).

REFERENCES

ABBINK, K., AND A. SADRIEH (1995): "Ratimage, Research Assistance Toolbox for Computer-Aided Human Behavior Experiments," Discussion paper B325, University of Bonn.

CORNS, A., AND A. SCHOTTER (1999): "Can Affirmative Action Be Cost Effective? An Experimental Examination of Price-Preference Auctions," *American Economic Review*, 89, 291-305.

COX, J.C., V.L. SMITH, AND J.L. WALKER (1988): "Theory and Individual Behavior of First-Price Auctions," *Journal of Risk and Uncertainty*, 1, 61-99.

KAGEL, J.H. (1995): "Auctions: A Survey of Experimental Research," in J.H. Kagel and A.E. Roth, eds., *The Handbook of Experimental Economics*, Princeton: Princeton University Press, 501-585.

KAGEL, J.H., AND D. LEVIN (1999): "Common Value Auctions with Insider Information," *Econometrica*, 67, 1219-1238.

KLEMPERER, P.D. (2002): "What Really Matters in Auction Design," *Journal of Economic Perspectives*, 16, 169-189.

MASKIN, E., AND J. RILEY (1996): "Uniqueness in Sealed High Bid Auctions," Mimeo.

MYERSON, R.B. (1981): "Optimal Auction Design," *Mathematics of Operations Research*, 6, 58-73.

PEZANIS-CHRISTOU, P. (2002): "On the Impact of Low-Balling: Experimental Results in Asymmetric Auctions," forthcoming in *International Journal of Game Theory*.

RILEY, J.G., AND W.F. SAMUELSON (1981): "Optimal Auctions," *American Economic Review*, 71, 381-392.

SIKKEL, M. (2001): "Plokken en Mijnen," *Parool* (main Dutch newspaper), April 3, 2.

Appendix: Proofs of Propositions 2 and 3

Proof of Proposition 2. First, consider the final stage where one weak and one strong bidder face a minimum price of L. When the strong bidder bids her value, the expected payoffs for a weak bidder with value v_w of bidding $b \geq L$ are:

$$\pi_2^{\,e}(b|v_w) \;=\; (H - L)^{-1}\,\{\int_L^b (v_w - z + \alpha\,(z - L))\,dz \;+\; \alpha\,(b - L)(H - b)\}, \qquad (3)$$

where the first (second) term in the curly brackets corresponds to the case where the weak bidder wins (loses). The derivative of the expected profit with respect to b is proportional to $v_w - b + \alpha\,(H - b)$, and since $v_w \leq 1$ this derivative is negative all $b > L$ if $\alpha \leq (L - 1)/(H - L)$. Hence it is optimal for a weak bidder to bid L in the second stage. The strong bidder can do no better than bidding her value in the second stage, since other bids either yield the same payoff (when strong wins) or may result in lower payoffs (when strong loses). For the same reason, the strong bidder can do no better than bidding up to her value in the first stage. Finally, in the first stage, weak bidders do not profit from dropping out earlier than L (since it would result in the same zero payoffs they receive in the proposed equilibrium). Furthermore, a single weak bidder cannot change the price level where the first stage ends when she is willing to bid up to levels higher than L. Q.E.D.

Proof of Proposition 3. First, consider the final stage where one weak and one strong bidder face a minimum price of X. When the strong bidder bids slightly above X, the weak bidder's expected payoffs for bids greater than X are $v_w - b$, which is negative since $v_w \leq 1$ and $X \geq 1$. Hence it is optimal for a weak bidder to bid X in the second stage. The strong bidder can do no better than bidding slightly above X in the second stage, since higher bids would only raise the amount she has to pay. In the first stage, dropping out below value is weakly dominated for the strong bidder. Finally, in the first stage, weak bidders do not profit from dropping out earlier than X (since it would result in the same zero payoffs they receive in the proposed equilibrium). Furthermore, a single weak bidder cannot change the price level where the first stage ends when she is willing to bid up to levels higher than X. Q.E.D.

TABLE I
EXPERIMENTAL DESIGN

Treatment	First-Price	English	First-Price Amsterdam	Second-Price Amsterdam	Optimal
Number of groups	9	8	9	8	9
Group Size	4	4	4	4	4
Premium 2 Finalists	-	-	0.3 $(b_2 - X)$	0.3 $(b_2 - X)$	-

Bidding Environment	All Treatments Bidders' Types	Valuations
Symmetric (periods 1-12)	4 symmetric	U[0,60]
Weakly Asymmetric (periods 13-24)	3 weak and 1 strong	weak: U[0,60] strong: U[40,100]
Asymmetric (periods 25-36)	3 weak and 1 strong	weak: U[0,60] strong: U[70,100]

TABLE II
REVENUES[a]

		Symmetric	Weakly Asymmetric	Asymmetric	All
First-Price	actual	41.5 *9.7*	54.2 *6.1*	57.4 *3.8*	51.0 *9.8*
	Nash	35.4 *8.5*	50.2 *4.6*	58.5 *0.8*	48.0 *11.1*
English	actual	38.2 *13.2*	45.4 *10.3*	44.1 *22.7*	42.6 *16.5*
	Nash	38.4 *13.0*	42.4 *11.2*	44.7 *11.1*	41.8 *12.1*
First-Price Amsterdam	actual	37.7 *10.8*	51.8 *9.7*	60.1 *8.2*	49.8 *13.5*
	Nash	36.2 *9.3*		60.0-70.0 *0.0*	
Second-Price Amsterdam	actual	34.7 *13.7*	48.3 *12.0*	66.0 *9.9*	49.7 *17.5*
	Nash	37.1 *9.3*		70.0 *0.0*	
Optimal	actual	37.0 *16.5*	53.9 *14.6*	70.7 *5.8*	53.9 *19.0*
	Nash	36.6 *16.6*	52.2 *14.3*	70.0 *5.4*	52.9 *18.9*

[a]The first entry in a cell displays the average revenue and the second entry displays the standard deviation (in italics). For the English auction, the Nash predictions of the equilibrium where players bid value are shown. For the asymmetric second-price Amsterdam auction Nash predictions concern the equilibrium where the strong bidder bids value in the second stage.

TABLE III
EFFICIENCIES[a]

	Symmetric	Weakly Asymmetric	Asymmetric	All
First-Price	96.5 *16.3*	97.8 *9.0*	95.9 *13.2*	96.7 *13.2*
English	99.7 *1.8*	99.2 *8.1*	96.7 *15.0*	98.5 *9.9*
First-Price Amsterdam	92.2 *23.7*	94.3 *18.2*	95.6 *18.8*	94.0 *20.4*
Second-Price Amsterdam	92.4 *21.0*	90.8 *25.2*	87.7 *27.1*	90.3 *24.5*
Optimal	83.4 *36.4*	93.4 *15.2*	96.8 *11.2*	91.2 *24.3*

[a]The first entry in a cell displays the average efficiency in % and the second entry displays the standard deviation (in italics). Efficiency is defined as $(v_{winner}-v_{min})/(v_{max}-v_{min})*100\%$, i.e. the winner's value minus the lowest of the bidders' values as a percentage of the difference between the highest and lowest of the bidders' values.

TABLE IV
PROPORTION OF ALMOST-ZERO BIDS[a]

	Symmetric	Weakly Asymmetric	Asymmetric	All
First-Price	10.4	10.7	13.0	11.3
English	7.8	11.5	39.1	19.4
First-Price Amsterdam	8.8	7.4	4.2	6.9
Second-Price Amsterdam	8.3	7.0	6.5	7.3
Optimal	47.0	40.7	32.6	40.1

[a]Each cell shows the observed frequency (in %) with which a bidder bids 5 or less (or does not bid at all in the optimal auction).

TABLE V
PREMIUMS[a]

		Symmetric	Weakly Asymmetric	Asymmetric	All
First-Price Amsterdam	actual	2.20 *2.10*	2.77 *2.93*	1.86 *2.25*	2.29 *2.48*
	Nash	1.61 *1.45*		0.0 *0.0*	
Second-Price Amsterdam	actual	2.63 *3.11*	2.52 *3.18*	2.20 *3.58*	2.45 *3.30*
	Nash	2.85 *2.54*		0.0 *0.0*	

[a]The first entry in a cell displays the average premium and the second entry the standard deviation (in italics).

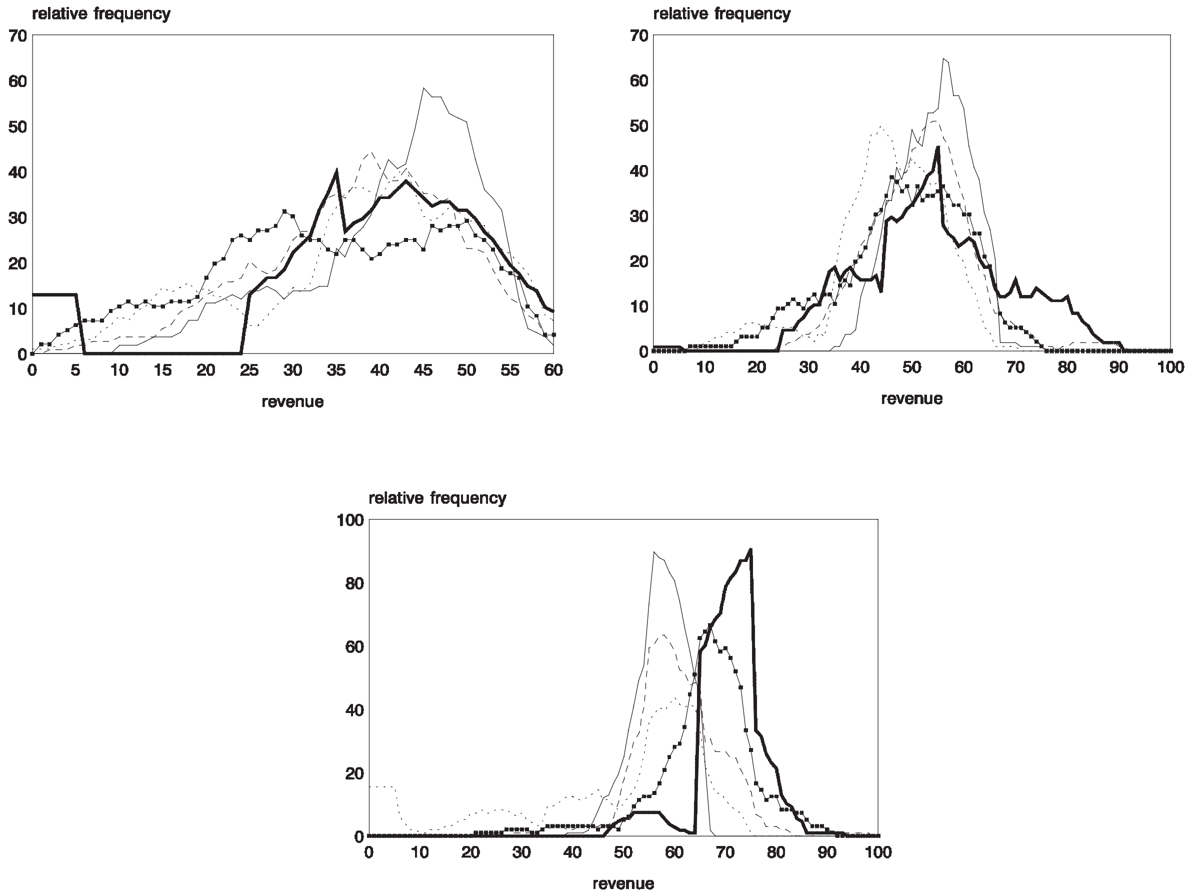

FIGURE 1.- *Revenue histograms.* The upper-left, the upper-right and the lower Figure show the revenue histograms for the symmetric case, the weakly asymmetric case and the asymmetric case respectively. In each Figure the thin solid line represents the first-price auction, the dotted line the English auction, the broken line the first-price Amsterdam auction, the solid line with knots the second-price Amsterdam auction and the thick solid line the optimal auction. For every revenue level the percentage of outcomes that fall in the interval [revenue-5, revenue+5] is shown.

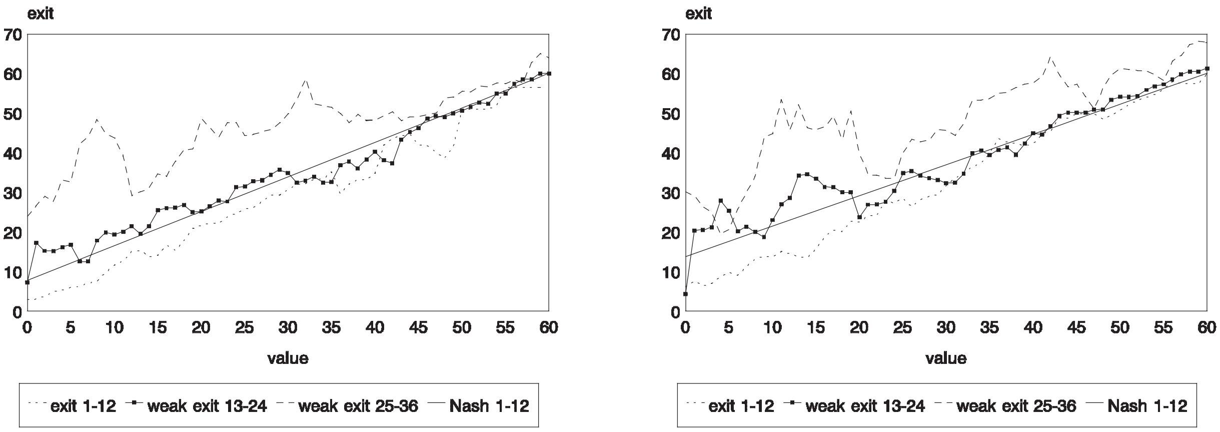

FIGURE 2.- *Exit decisions in first-price (left) and second-price (right) Amsterdam auction.* For each value the average of exit decisions in the interval [value-2, value+2] is reported.

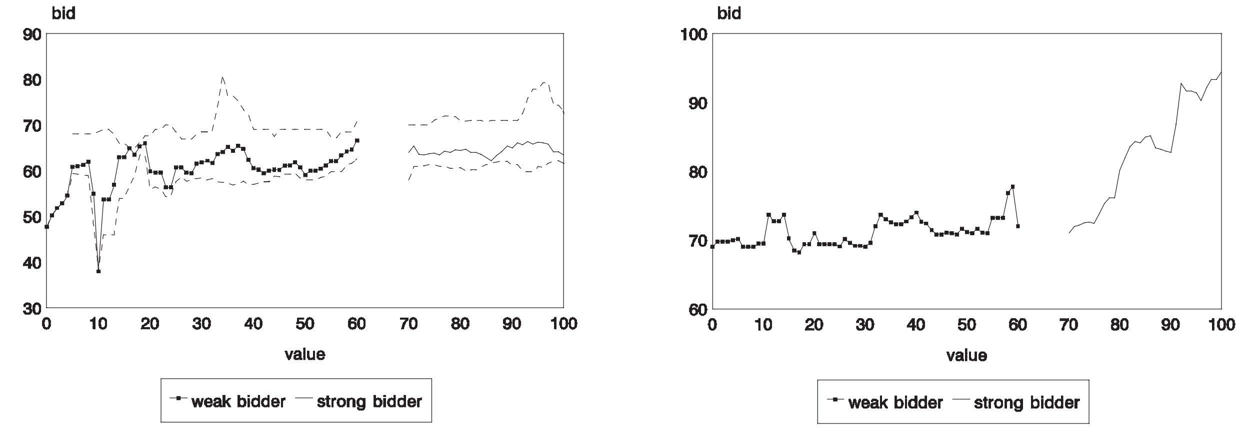

FIGURE 3.- *Sealed bids in the first-price (left) and second-price (right) Amsterdam auction.* For each value the average of bids that fall in the interval [value-2,value+2] are graphed. In the first-price Amsterdam auction the distribution of sealed bids is bi-modal: the broken lines represent the sealed bids of the groups that focused on 60 (6 groups) and 70 (6 groups) separately.